Communication-Failure-Resilient Distributed Frequency Control in Smart Grids: Part I: Architecture and Distributed Algorithms

Masoud H. Nazari ©, Member, IEEE, Le Yi Wang ©, Fellow, IEEE, Santiago Grijalva, Senior Member, IEEE, and Magnus Egerstedt ©, Fellow, IEEE

Abstract—Distributed algorithms have been proposed as options to scale control propositions to the massive number of intelligent energy devices, sub-systems, and distributed energy resources being integrated into the electricity grid. Distributed algorithms rely on the communication network for exchanging information. Failures in the communication network can jeopardize distributed decision-making and in the worst-case scenario can lead to system-level stability problems. This paper proposes a communication-failure-resilient architecture for distributed operation and control in smart grids with hybrid producer/consumer (prosumer) agents. We describe the relations between system-wide performance and communication failures and identify topological conditions on the cyber-physical network, under which prosumers can perform key operating tasks, such as distributed frequency regulation through an imperfect communication network.

Index Terms—Communication-failure-resilient architecture, cyber-physical systems, smart grid, distributed frequency regulation, multi-agent network, prosumer.

I. INTRODUCTION

The current operation of electric power infrastructures is based on centralized decision-making and traditional SCADA system paradigms [1]. The centralized control architecture exhibits the following limitations that affect the industry’s capabilities for modernization through consumer empowerment, and limit the integration of distributed energy resources (DERs):

1) Centralized communication and control algorithms are not scalable to the control of massive numbers of renewable energy sources and storage devices needed to achieve sustainability objectives while maintaining reliability and economic optimality;

2) The control center constitutes a single point of failure, which is a cyber and physical security target; and,

3) They primarily use dedicated communication links that are not appropriate for networks with large numbers of users or system components.

It is envisioned that the future smart power grids will be populated with thousands or millions of active hybrid producer/consumer (prosumer) agents, which can make strategic decisions, such as producing, consuming, and/or storing electricity, and providing services to the grid. Distributed algorithms have been proposed to address both system reliable functionality and scalability of prosumer power grids [2]–[5]. Here, distributed algorithms are referred to the scenario where inter-agent communication is required to arrive at appropriate control decisions.

Fully decentralized control actions, where each sub-system unilaterally demands its local states, provide sub-optimal solutions and can cause instability. As an example, currently various regions of the grid control the local frequency and unilaterally respond to it without considering other regions actions, often causing inter-area oscillations [6]–[9].

In future power grids, prosumers have both physical and cyber connections. Thus, the action of each prosumer will affect the rest of the system. Distributed algorithms allow prosumers to include the effect of other prosumers in the decision-making process and decompose large-scale system-wide optimization and control problems into sub-problems [10], [11]. For convex problems, it is provably guaranteed that distributed algorithms, such as Accelerated Gradient methods or ADMM (Alternating Direction Method of Multipliers) can achieve the global optimal solutions [12], [13].

Distributed algorithms rely on the communication network for exchanging information. Thus, the communication network performance and reliability are key elements in the overall success of the distributed algorithms [14], [15]. Failures in the communication network, caused by disconnection of communication links, cyber-attacks, delay and other channel imperfections can jeopardize the performance of the algorithms and lead to system-level reliability problems. In order to ensure the reliable operation of physical smart grids, it is important that the cooperative algorithms be designed to withstand malfunctioning of communication links, and still guarantee correct behavior. This is often called resilient multi-agent networks [16]. The problem of reaching consensus in the presence of adversaries has been studied in distributed computing [17], [18],...
communication networks [15], [19], and mobile robotics [20]–[22]. But, this problem has not received much attention in smart power grids. This paper aims to address this problem for a particular distributed algorithm, namely distributed frequency regulation (DFR). In particular, we explore the effects of information isolation at the prosumer level on the performance of the DFR algorithm. Other communication failures scenarios such as malicious agents are subject of the future research endeavor.

DFR involves bringing the quasi-steady state system frequency to 60 Hz, while minimizing overall control effort, using only local information and sparse exchange among the agents [2], [23]–[32]. We propose a resilient architecture for distributed control of prosumers to ensure stable operation of the grid in the presence of single or multiple communication contingencies at the prosumer level. We explore a scenario under which all communication links connecting a set of prosumers to the rest of the grid are disconnected. During this abnormal condition, information isolation, denoted as “muteness,” occurs. When a prosumer or multiple prosumers face information isolation, they need to take an algorithmic contingency-based action. A course of action is that the isolated prosumers do not participate in frequency regulation due to lack of information, letting other prosumers control their states. But, this can result in loss of actuation and may cause stability problems. We explore this relation between system-wide stability and communication failures in prosumer-based smart grids. We identify topological conditions for $N - k$ cyber network reliability of prosumer power grids. This paper focuses on the theoretical findings, particularly the system architecture and distributed algorithms. The algorithmic implementation and simulations will be presented in Part II.

The rest of the paper is structured as follows: Section II introduces the smart power grid model from the graph-theory point of view, followed by sub-sections, exploring the controllability and stability of prosumer power grids in the presence of communication failures. In Section III, a framework for distributed frequency regulation is introduced for systems with isolated prosumers. The paper concludes in Section IV with the discussion of overall findings.

II. BACKGROUND

A. Graph Theoretic Model of Smart Grid

We first review the quasi-steady state model of prosumer power grids from the graph-theory point of view. In a power grid, each prosumer is considered as a node, which contains distributed energy resources, including renewable sources, flexible loads, and/or energy storage. Fig. 1 shows the internal schematics of a prosumer.

Prosumer power grids form a multi-agent network. Tie-lines connect neighboring prosumers. The tie-lines are the edges of the power grid graph. From a physical interconnection point of view, the prosumer power grid can be represented by a graph $G_P = (V, E_P)$, where $V = \{1, 2, \ldots, n\}$ is the set of all prosumers and $E_P$ is the set of edges or power systems tie-lines. The presence of a tie-line $(v_i, v_j)$ indicates that prosumer $i$ is electrically adjacent to prosumer $j$. We begin by assuming that electrically adjacent prosumers can directly communicate and have both physical and cyber connections. Thus, under normal conditions the network control topology is the same as the prosumer power grid tie-line topology, so we can model the network control by a graph $G_C = (V, E_C)$, where $G_C \simeq G_P$ (graphs $G_C$ and $G_P$ are homomorphic). As discussed in [2], [13], the neighbor-to-neighbor communication between prosumers is essential for the operation of distributed optimization algorithms.

In the timeframe relevant to frequency regulation, the quasi-stationary deviations in frequency are related to prosumers’ power deviations through the first-order droop constant equation [33]–[35]. Thus, the dynamic evolution of the prosumers with respect to discrete time $k$ can be modeled as [2]:

$$p(k+1) = Ap(k) + Bu(k),$$

where $p = [p_1, p_2, \ldots, p_n]^T$ is the vector of prosumers’ real power deviation from the scheduled value and $u = [u_1, u_2, \ldots, u_n]^T$ is the vector of prosumers’ frequency control variables. The control variable depends on the nature of prosumers. For instance, if a prosumer provides frequency control through its fast ramping generators, such as in a microgrid with micro gas turbines, the control variable will be the set point of the generators. Or, if a prosumer provides frequency regulation through demand response, such as a cluster of smart buildings, the control variable will be the set-point of the controllable loads.

In (1), matrices $A$ and $B$ represent the system matrix and control matrix, defined respectively as:

$$A = I - T_s JS$$

(2)
where \( I \) is the \( n \)-dimension identity matrix, \( J \) is the sub-matrix of the power flow Jacobian matrix of the prosumer power grid, which is an \( n \)-dimension square matrix, representing the sensitivity of the power deviation of prosumers (\( p \)) with respect to changes in the voltage angle of prosumers (\( \delta \)), and \( T_z \) is the sampling time, which should be less than 1 min to satisfy the NERC reliability criteria. Note that the NERC B2 criterion requires prosumers to start returning power deviations to zero within 1 min after the beginning of a disturbance [14], [36]. In addition, \( S \) and \( \beta \) are diagonal matrices of dimension \( n \times n \) corresponding to the droop constants and control coefficients of prosumers [2].

Note that matrices \( J, A \) and \( B \) have the same sparsity pattern\(^1\) and represent the underlying electrical topology of the prosumer power grid [37]. The inter-connection between the cyber-physical power network allows developing topological conditions, under which prosumers can maintain controllability, frequency stability, as well as economically, fairly optimal performance during communication failures.

\[ B = T_z J \beta \]  \hspace{1cm} (3)

B. Problem Formulation

The objective of frequency regulation is to bring the quasi-steady state frequency deviations to zero after a disturbance, while minimizing overall control effort throughout the power grid. This is an optimal control problem with two objectives: 1) system stability; and, 2) cost minimization. For convenience, we have formulated the general structure of the frequency regulation problem for a system with \( n \) prosumers in (4).

\[
\begin{align*}
\min_{u} & \quad p(k + 1)^T Q p(k + 1) + u(k)^T R u(k) \\
\text{s.t.} & \quad p(k + 1) = A p(k) + B u(k),
\end{align*}
\]

where \( Q = \text{diag}(Q_1, Q_2, \ldots, Q_n) \) and \( R = \text{diag}(R_1, R_2, \ldots, R_n) \) are positive definite diagonal matrices. For each prosumer \( i \), \( Q_i \) represents the cost of power deviation and \( R_i \) represents the cost of frequency control. The objective of (4) is to minimize power deviations at time \( k + 1 \), using minimal control effort at time \( k \). Note that by increasing the optimization time horizon \((K + 2, K + 3, \ldots)\), prosumers need to predict further into the future, which requires multi-hop communication among prosumers [38]. One of the open problems that should be explored as a future research topic is to understand the trade of between information exchange (one-hop vs. multi-hop) and the system-wide performance (economic optimality) for the DFR and other distributed optimization algorithms.

We assume that under normal conditions, all prosumers can participate in frequency regulation and the control matrix \( B \) is full rank. Thus, the system is completely controllable. Now, if a set of prosumers, \( \mathcal{M} \), is isolated due to communication failures, the prosumers are not able to communicate with the rest of the system. These prosumers are referred to as “mute”. This lack of communication can create critical problems for distributed algorithms, such as distributed frequency regulation, as mute prosumers cannot participate in the consensus protocols. One possible course of action for mute prosumers is to act according to a “zero-bias” control strategy and allow other prosumers to minimize their power deviations. The following equation summarizes this control law

\[ v_i \in \mathcal{M} \Rightarrow u_i(k) = 0 \forall k \in \mathbb{N}. \]  \hspace{1cm} (5)

For notation convenience, let \( \mathcal{M} \) be the last \( m \) prosumers, i.e., \( \mathcal{M} = \{ n - m + 1, n - m + 2, \ldots, n \} \) where \( |\mathcal{M}| = m \). Also, the set of non-mute prosumers is denoted by \( \mathcal{N} \), where \( |\mathcal{N}| = n - m \). Thus, the overall set of prosumers is \( \mathcal{M} \cup \mathcal{N} \).

After substituting the control strategy adopted by the mute prosumers and implementing the constraint in the objective function, (4) becomes:

\[
\begin{align*}
\min_{u_N} & \quad p(k + 1)^T Q p(k + 1) + u_N(k)^T R u_N(k) \\
& \quad + \begin{bmatrix} u_N^T \ 0_{1 \times m} \end{bmatrix} \begin{bmatrix} R_N & 0_{(n - m) \times m} \\ 0_{m \times (n - m)} & R_M \end{bmatrix} \begin{bmatrix} u_N \\ 0_{m \times 1} \end{bmatrix} \\
\text{s.t.} & \quad p(k + 1) = A p(k) + [B_N B_M] u_N(k),
\end{align*}
\]

where \( B_N \) is the matrix comprising of the first \( n - m \) columns of \( B \), and the \( B_M \) matrix contains the rest of the columns. In addition, \( R_N \) and \( R_M \) are square matrices corresponding to the cost matrix of the non-mute prosumers (\( \mathcal{N} \)) and that of the mute prosumers (\( \mathcal{M} \)), respectively. For convenience, we set \( p(k) = p \) and \( u(k) = u \). Since the mute prosumers do not participate in the frequency regulation, the last \( m \) elements of \( u \) are zero. Thus, the cost function can be simplified and recast as an unconstrained optimization problem

\[
\begin{align*}
\min_{u_N} & \quad J(u_N, p) = \min_{u_N} u_N^T R_N u_N \\
& \quad + \left( A p + B_N u_N \right)^T Q \left( A p + B_N u_N \right),
\end{align*}
\]

where

\[
\begin{align*}
J(u_N, p) & = \min_{u_N} u_N^T R_N u_N \\
& \quad + \left( A p + B_N u_N \right)^T Q \left( A p + B_N u_N \right).
\end{align*}
\]

(8)

The questions that we would like to further address in this paper are:

1) Under what conditions is the system with mute prosumers, expressed in (7), controllable?

2) Under what conditions can non-mute prosumers maintain stability and minimize the system-wide cost function, expressed in (8), through distributed optimization protocols?

C. Single Communication Network Contingency

In this section, we establish theoretical conditions to address the controllability of the system with mute prosumers. The controllability matrix of the system described by (7) is expressed as

\[
\Gamma = \begin{bmatrix} B_N & AB_N & \ldots & A^{n - 1} B_N \end{bmatrix}.
\]

As the original control matrix, \( B \), is full rank, the rank of the truncated control matrix, \( B_N \), is \( n - m \). Thus, to ensure the system-wide controllability, it is sufficient to check the rank

\[ |\mathcal{N}| \geq m \].

1The same sparsity pattern implies that both \( A \) and \( B \) matrices have the same coordination of nonzero entries.
of the first \( m + 1 \) entries of (9) or reduce the controllability matrix [37] as follows
\[
\Gamma_m = \begin{bmatrix} B_N & AB_N & \cdots & A^m B_N \end{bmatrix}.
\] (10)

Higher-order entries of the original controllability matrix \( \Gamma \) will be linear combinations of the columns of the reduced controllability matrix \( \Gamma_m \). Thus, the rank of matrix \( \Gamma \) is the same as that of matrix \( \Gamma_m \). For detailed theoretical discussions on this trend see [37].

Next, we explore physical conditions, under which a system with a single mute prosumer is controllable. Or the controllability matrix, expressed below, is full rank.
\[
\Gamma_1 = \begin{bmatrix} B_N & AB_N \end{bmatrix}.
\] (11)

We can simplify the controllability matrix \( \Gamma_1 \) and draw connections to the underlying physical topology by exploiting the structure of the matrices \( A \) and \( B_N \). We recall that \( A = I - T_s JS \) and \( B_N = T_s J \hat{\beta} \), where \( \hat{\beta} = [b_1, b_2, \ldots, b_{n-m}] \), where \( b_i \) is the \( i \)-th column of matrix \( \beta \). Thus, \( \Gamma_1 \) can be recast as
\[
\Gamma_1 = \begin{bmatrix} T_s J \hat{\beta} \quad (I - T_s JS) T_s J \hat{\beta} \end{bmatrix}.
\] (12)

The sampling time \( T_s \) does not affect the rank analysis. So, we can set \( T_s = 1 \) to simplify the analysis. Also, the Jacobian matrix \( J \) is full rank and does not reduce the rank of the controllability matrix. Thus, the rank is purely determined by the second term of the product. That is,
\[
\rho(\Gamma_1) = \rho\left( \begin{bmatrix} \beta \quad (I - JS) \hat{\beta} \end{bmatrix} \right).
\] (13)

where, \( \rho(\cdot) \) is a rank operator. The term \( \hat{\beta} \) can be dropped from the second entry as it is equal to the first entry and will not affect the rank operation. Also, the sign does not affect the rank operation. Thus we can simplify (13) as
\[
\rho(\Gamma_1) = \rho\left( \begin{bmatrix} \beta \quad JS \hat{\beta} \end{bmatrix} \right).
\] (14)

Note that \( \hat{\beta} \) is a truncated diagonal matrix of the form
\[
\hat{\beta} = \begin{bmatrix} D \\ 0_{1 \times (n-1)} \end{bmatrix},
\]
where \( D \) is a diagonal matrix of dimension \((n - 1) \times (n - 1)\). We can use this to express the simplified controllability matrix \( \begin{bmatrix} \beta \quad JS \hat{\beta} \end{bmatrix} \) as a block matrix of the form
\[
\begin{bmatrix} \beta \quad JS \hat{\beta} \end{bmatrix} = \begin{bmatrix} D \\ 0_{1 \times (n-1)} \end{bmatrix} G.
\]
where \( G \) is a matrix of dimension \((n - 1) \times (n - 1)\) and \( F \) is a matrix of dimension \(1 \times (n - 1)\).
Since \( D \) is a diagonal matrix, we can take linear combinations of its columns to eliminate the entries of the matrix \( G \). This allows us to conclude that
\[
\rho(\Gamma_1) = \rho\left( \begin{bmatrix} D \\ 0_{1 \times (n-1)} \end{bmatrix} \right).
\] (15)

In order for the system with a single mute prosumer to be completely controllable, we require \( \rho(F) = 1 \). The rank test provided by equation (15) involves inspecting the matrix \( \begin{bmatrix} \beta \quad JS \hat{\beta} \end{bmatrix} \).

Next, we establish controllability by extracting specific linear sub-matrices and interpret the results from a graph-theoretic viewpoint.

When there is a single mute prosumer, we want to show that the controllability of (7) can be directly related to the connectivity of the physical network, represented by the graph \( G_p \).

**Theorem 1:** For \( |\mathcal{M}| = 1 \), the pair \((A, B_N)\) is always completely controllable, if the physical grid, \( G_p \), is strongly connected.

**Proof:** When the number of mute prosumers is equal to 1, \( F \) is a \( 1 \times (n - 1) \) matrix. Note that \( F \) is just the last row of the matrix \( JS \hat{\beta} \). Since both \( \beta \) and \( S \) are diagonal matrices, they do not affect the sparsity structure of the matrix \( JS \hat{\beta} \). So, \( JS \hat{\beta} \) inherits its sparsity structure from the \( J \) matrix. Let \( u = n \) denote the single element of the set \( \mathcal{M} \). Since \( G_p \) is strongly connected, there exists at least one prosumer \( v \) in \( V \setminus \mathcal{M} \) such that \((v, u) \in E\). This implies that the element in the \( v \)-th position in the vector \( F \) is non-zero. Since \( F \) is a row vector with a non-zero entry, we conclude that the rank of \( F \) is equal to 1 which is the number of mute prosumers in the system. Recalling equation (15), we can conclude that the pair \((A, B_N)\) is completely controllable.

Theorem 1 implies that as long as the mute prosumer is electrically connected to the system, other prosumers can control its states, irrespective of its position in the network topology. We compare this condition with \( N - 1 \) power reliability, which requires that the power system withstands loss of any single system component. Thus, \( N - 1 \) power reliability requires constraint system operation. But, \( N - 1 \) cyber reliability for frequency regulation can be warranted by prosumer power system connectivity.

**D. Multiple Communication Network Contingencies**

When multiple prosumers face communication contingencies, \( |\mathcal{M}| > 1 \), we provide a sufficient condition for controllability, which relies on the notion of electrical separation. We show that if any mute prosumer is neighbored with at least one free non-mute prosumer, which is not neighboring any other mute prosumers, then the system with multiple mute prosumers is controllable. In order to show this, we define the set \( \mathcal{N} = V \setminus \mathcal{M} \), where \( V \) is the set of all prosumers and \( \mathcal{M} \) is the set of mute prosumers. Next, we identify topological conditions on the set \( \mathcal{M} \), which renders the pair \((A, B_N)\) controllable.

**Theorem 2:** If there exists an injective map \( \phi : \mathcal{M} \to \mathcal{N} \) such that each mute prosumer is electrically adjacent to at least one free non-mute prosumer as follows
\[
\phi(m) = n \leftrightarrow (m, n) \in E \wedge (v, n) \notin E \forall v \in \mathcal{N} \setminus \{m\},
\] (17)
then the pair \((A, B_N)\) is completely controllable.
Proof: Assume that there exists \( \phi : \mathcal{M} \rightarrow \mathcal{N} \) which satisfies the condition given by (17). Physically speaking, the existence of the map \( \phi \) implies that every mute prosumer \( v \in \mathcal{M} \) is electrically connected to a free non-mute prosumer \( \phi(v) = m \), which is not connected to any prosumers in the set \( \mathcal{M} \setminus \{v\} \).

We restrict our attention to matrix \( JS\hat{\beta} \) and express it as

\[
JS\hat{\beta} = \begin{bmatrix} \hat{G} \\ \hat{F} \end{bmatrix}
\]

(18)

where \( \hat{F} \) is a matrix of \( m \times (n-m) \) dimension. As in the case of the single mute prosumer, the matrix \( \hat{F} \) encodes the relationship between mute prosumers and non-mute prosumers. Allowing \( \hat{F}_{(i,j)} \) to stand for the element located along the \( i \)-th row and the \( j \)-th column of the matrix \( \hat{F} \), we can say that

\[
\hat{F}_{(i,j)} = 0 \Leftrightarrow (i, j) \notin E \wedge (i \in \mathcal{M}) \wedge (j \in \mathcal{N}).
\]

(19)

Since \( \phi \) satisfies the condition (17), for every mute prosumer \( m \in \mathcal{M} \), there exists a prosumer \( n = \phi(m) \in \mathcal{N} \) such that the column \( \hat{F}_{\phi(m)} \) contains zero at all locations except \( \hat{F}_{m,\phi(m)} \). Then, the collection of columns \( \{ \hat{F}_{\phi(m)} \mid m \in \mathcal{M} \} \) are mutually orthogonal and therefore linearly independent. Therefore, the matrix \( \hat{F} \) contains \( |\mathcal{M}| \) linearly independent columns and the rank of \( \hat{F} \) is equal to \( m = |\mathcal{M}| \). Now, we write the controllability matrix \( \Gamma_m \) as

\[
\Gamma_m = \begin{bmatrix} T_sJ\hat{\beta} & (I - T_sJS)T_sJ\hat{\beta} & \ldots & (I - T_sJS)^mT_sJ\hat{\beta} \end{bmatrix}.
\]

(20)

Recalling from the previous section, the sampling time \( T_s \) does not affect the rank analysis and can be set to \( T_s = 1 \). Also, the Jacobian matrix \( J \) is full rank and does not reduce the rank of the controllability matrix. Thus,

\[
\rho(\Gamma_m) = \rho \left( \begin{bmatrix} \hat{\beta} & (I - JS)\hat{\beta} & \ldots & (I - JS)^m\hat{\beta} \end{bmatrix} \right).
\]

(21)

In addition, the term \( \hat{\beta} \) can be dropped from the second entry as it is equal to the first entry and will not affect the rank operation. Also, the sign does not affect the rank operation. Thus we can simplify (22) as

\[
\rho(\Gamma_m) = \rho \left( \begin{bmatrix} (I - JS)\hat{\beta} & \ldots & (I - JS)^m\hat{\beta} \end{bmatrix} \right).
\]

(22)

Note that \( \hat{\beta} \) is a truncated diagonal matrix of the form

\[
\hat{\beta} = \begin{bmatrix} D \\ 0_{m \times (n-m)} \end{bmatrix}
\]

where \( D \) is a diagonal matrix of dimension \( (n-m) \times (n-m) \). Thus, we can use this to express the simplified controllability matrix as a block matrix of the form

\[
\rho(\Gamma_m) = \begin{bmatrix} D & \hat{G} & \hat{G} \\ 0_{m \times (n-m)} & \hat{F} & \hat{F} \end{bmatrix}.
\]

where \( \hat{G} \) and \( \hat{F} \) represent the third and higher-order entries of the controllability matrix. Since \( D \) is a diagonal matrix, we can take linear combinations of its columns to eliminate the entries of the matrices \( \hat{G} \) and \( \hat{F} \). This allows us to conclude that

\[
\rho(\Gamma_m) = \rho(D) + \rho \left( \begin{bmatrix} \hat{F} \\ \hat{F} \end{bmatrix} \right) = (n - m) + m = n,
\]

(23)

which was to be shown. Thus, the pair \( (A, B_N) \) is completely controllable.

Theorems 1 and 2 connect the topology of the physical power network to the controllability of the underlying cyber-physical system. The topological tests established in this paper can be a valuable aid when it comes to designing cyber-physical power network topologies as they can be used to identify problematic cyber network configurations and restructure them so that the power system is more reliable and resilient.

Theorem 2 also identifies upper bound on the number of mute prosumers. Since there is a one-to-one mapping between mute prosumers and free non-mute prosumers, the number of mute prosumers should be always \( m \leq \frac{1}{2}N \). In other words, the upper bound for \( N - k \) cyber network reliability is \( N - \frac{1}{2}N = \frac{1}{2}N \). For instance, Figures 2 and 3 display two different configurations of a line prosumer power graph with four prosumers. The white prosumers are mute and the black ones are the non-mute prosumers. In the configuration shown in Fig. 2, we can define a map \( \phi \) as follows:

\[
\phi(2) = 1
\]
\[
\phi(3) = 4
\]

The above map satisfies the conditions in Theorem 2 and infer that the prosumer power grid is controllable. We can also define a similar \( \phi \) for the configuration given in Fig. 3 by mapping the Prosumer 1 to 2 and Prosumer 4 to 3. In both combinations, the number of mute prosumers is equal to the number of non-mute prosumers, but the system is still controllable. The controllability of these systems ensures that there exists an optimal stabilizing control law to the frequency regulation problem. But, obtaining that control law is a challenging task, which will be explored in the next section.

E. System-Wide Stability During Communication Contingencies

We recall that as long as the pair \( (A, B_N) \) is controllable, there exists a positive definite \( Q \) that renders the closed-loop system described in (7) stable when driven by the minimizing solution [39]. In order to obtain the stabilizing \( Q \), we solve the below discrete-time Lyapunov equation

\[
(A - B_NK)^TQ(A - B_NK) - Q = -K^TR_NK,
\]

where \( u_N(k) = -Kp(k) \) is a stabilizing optimal solution to the system, \( p(k + 1) = Ap(k) + B_Nu_N(k) \). Once the \( Q \) has been obtained, the optimizing control law can be computed. However, this requires a centralized communication/computation. But, we can calculate the minimizing \( u_N \) by letting

\[
0 = \frac{\partial J}{\partial u_N} = 2B_N^TQAx + 2(B_N^TQB_N^T + R_N)u_N
\]

(24)

i.e.,

\[
u_N = -(B_N^TQB_N^T + R_N)^{-1}B_N^TQAx.
\]

(25)
The minimization controller asymptotically stabilizes the system, $p(k + 1) = Ap(k) + B_{N}u_{i}(k)$, if and only if $|\lambda_{i}| < 1$ for all eigenvalues $\lambda_{i}$ to

\[
(I - B_{N}(B_{N}^{T}Q_{N}B_{N} + R_{N})^{-1}B_{N}^{T}Q_{N})A.
\]

Thus, as long as the topological conditions discussed in Theorems 1 and 2 are satisfied, the stabilizing minimizing controller can be obtained. But, it does not follow that prosumers can obtain the minimizing controller in a distributed fashion. The problem that each prosumer $i$ needs to solve is

\[
\begin{align*}
\min_{u_{i}} J_{i}(x_{i}, u_{i}) &= \min_{u_{i}} Q_{i}p_{i}(k + 1)^{2} + R_{i}u_{i}(k)^{2}, \\
\text{s.t.} \quad p_{i}(k + 1) &= a_{ii}p_{i}(k) + b_{ii}u_{i}(k) \\
&\quad + \sum_{j \in N_{i}} a_{ij}p_{j}(k) + b_{ij}u_{j}(k)
\end{align*}
\]

where $p_{j}$ is obtained through direct communication between prosumers and $u_{ij}$ is obtained through a consensus-based distributed optimization protocol. In the next section, we explore topological conditions on the cyber-physical system under which the distributed optimal control solutions can be obtained during communication contingencies.

III. PROPOSED APPROACH: RESILIENT DISTRIBUTED FREQUENCY REGULATION

Because the problem in (8) is convex, several methods can be used for distributed optimization. This paper uses a consensus-based ADMM algorithm [13], which exhibits guaranteed convergence for convex separable optimization problems. We first re-formulate (4) and decompose the problem into $n$ sub-problems, each corresponding to the optimization of a prosumer. Through the communication network, each prosumer $i$ has a copy (perception) of the control action of its neighbor $j$, as $u_{ij}$. Therefore, a new control matrix can be formed as $U = [u_{ij}]_{n \times n}$, where

\[
U_{ij} = \begin{cases} 
  u_{ij} & \text{if } i = j \\
  u_{ij} & \text{if } i \neq j \text{ and } (u_{i}, v_{j}) \in E_{P} \\
  0 & \text{Otherwise .}
\end{cases}
\]

Similarly, a new power deviations matrix is formed as $X = [X_{ij}]_{n \times n}$, where

\[
P_{ij} = \begin{cases} 
  p_{i} & \text{if } i = j \\
  p_{j} & \text{if } i \neq j \text{ and } (v_{i}, v_{j}) \in E_{P} \\
  0 & \text{Otherwise .}
\end{cases}
\]

It follows from (29) and (30) that $u = [U_{11}, U_{12}, \ldots, U_{nn}]^{T} = \delta(U)$ and $p = \delta(P)$, where $\delta(.)$ is the main-diagonal operator. Thus, the original centralized problem, (4) takes as

\[
\begin{align*}
\min_{\mathcal{U}} \mathcal{J}(\mathcal{U}, \mathcal{P}) &= \min_{\mathcal{U}} \delta(\mathcal{U})^{T}R\delta(\mathcal{U}) \\
&\quad + \left(\mathbf{A}\delta(\mathcal{P}) + \mathbf{B}\delta(\mathcal{U})\right)^{T}Q_{1}\left(\mathbf{A}\delta(\mathcal{P}) + \mathbf{B}\delta(\mathcal{U})\right) \\
&= \min_{\mathcal{U}_{i}, \ldots, \mathcal{U}_{n}} \sum_{i=1}^{n} \mathcal{J}_{i}(\mathcal{U}_{i}, \mathcal{P}_{i}) \\
\text{s.t.} \quad U_{ij} &= U_{ij} \forall j \in N_{i} \\
&= \sum_{i=1}^{n} \min_{\mathcal{U}_{i}} \mathcal{U}_{i}^{T}R_{i}\mathcal{U}_{ii} \\
&\quad + \left(A_{i}\mathcal{P}_{i}^{T} + B_{i}\mathcal{U}_{i}^{T}\right)^{T}Q_{i}\left(A_{i}\mathcal{X}_{i}^{T} + B_{i}\mathcal{U}_{i}^{T}\right) \\
\text{s.t.} \quad U_{ij} &= U_{ij} \forall j \in N_{i}.
\end{align*}
\]

where $A_{i}$ and $B_{i}$ are the $i$-th row of the $A$ and $B$ matrices, and $\mathcal{U}_{i}$ and $\mathcal{P}_{i}$ are the $i$-th row of $\mathcal{U}$ and $\mathcal{P}$, respectively. As discussed
earlier, we assume that when prosumers become mute, they take zero-bias action and consequently $\mathcal{U}_{mm} = 0$, $\forall m \in \mathcal{M}$. This implies that the perception of neighbors from the control action of the mute prosumers is $\mathcal{U}_{mi} = 0$, $\forall m \in \mathcal{M} \land \forall i \in \mathcal{N}_m$, where $\mathcal{N}_m$ is the set of mute prosumers’ neighbors. Note that a mute prosumer can be neighbored by another mute prosumer or by a non-mute prosumer. Although mute prosumers are not participating in frequency regulation, the control action of their neighbors can still affect their states. In other words, $\mathcal{U}_{mj} \neq 0$, $\forall j \in \mathcal{N}_m \land j \notin \mathcal{M}$. Thus, the centralized cost function, represented in (31), recasts as

$$
\min_{\mathcal{U}_{Ni}} J(\mathcal{U}_{Ni}, \mathcal{P}) = \sum_{i=1}^{n-m} \min_{\mathcal{U}_{ii}} Q_i \left( A_i \mathcal{P}_i^T + B_i \mathcal{U}_i^T \right)^2 + R_i \left( \mathcal{U}_i^* \right)^2 \\
\text{s.t. } \mathcal{U}_{ij} = \mathcal{U}_{ji} \ \forall j \in \mathcal{N}_i \\
\mathcal{U}_{mm} = 0 \ \forall m \in \mathcal{M} \\
+ \sum_{i=n-m+1}^{n} \min_{\mathcal{U}_{ii}} Q_i \left( A_i \mathcal{P}_i^T + B_i \mathcal{U}_i^T \right)^2 \\
\text{s.t. } \mathcal{U}_{ij} = \mathcal{U}_{ji} \ \forall j \in \mathcal{N}_i \\
\mathcal{U}_{ii} = 0.
$$

(32)

The next step is to develop conditions under which prosumers can obtain a minimizing stabilizing control law to (32) in a distributed way, with only local information exchange. The following Theorem satisfies these conditions for a single-mute-prosumer system ($|\mathcal{M}| = 1$).

**Theorem 3:** If the following conditions hold, a system with $|\mathcal{M}| = 1$ can minimize (32) in a distributed way.

1) The graph $\mathcal{G}_P$ is strongly connected.

2) $\mathcal{G}_C \supseteq \mathcal{G}_P^2$

**Proof:** As shown in Theorem 1, the connectivity of the power grid $\mathcal{G}_P$ satisfies the controllability of the pair $(A, B_N)$. But, obtaining the minimizing controller needs centralized communication/computation, as shown in (25).

In order to remedy this problem, each prosumer $i$ solves its sub-problem $J_i(\mathcal{U}_i, \mathcal{P}_i)$ through a consensus-based protocol [2]. The mute prosumer cannot participate in the process, but its cost function ($J_m$) is a non-separable quadratic function of the control actions of its neighbors. Thus, if the neighboring prosumers include $J_m$ in their optimization problem, they can obtain minimizing control laws to $J_m$.

Now, let us assume that prosumer $i^*$, where $i^* \in \mathcal{N}_m$, is minimizing the states of prosumer $n$. Then, the DFR problem is recast as

$$
\min_{\mathcal{U}_{Ni}} J = \sum_{i=1}^{n-1} \min_{\mathcal{U}_{ii}} Q_i \left( A_i \mathcal{P}_i^T + B_i \mathcal{U}_i^T \right)^2 + R_i \left( \mathcal{U}_i^* \right)^2 \\
\text{s.t. } \mathcal{U}_{ij} = \mathcal{U}_{ji} \ \forall j \in \mathcal{N}_i \\
\mathcal{U}_{nn} = 0 \\
+ \min_{\mathcal{U}_{i^*}, \mathcal{U}_n} Q_{i^*} \left( A_{i^*} \mathcal{P}_{i^*}^T + B_{i^*} \mathcal{U}_{i^*}^T \right)^2 + \mathbf{R}_{i^*} \left( \mathcal{U}_{i^*} \right)^2 + Q_n \left( A_n \mathcal{P}_n^T + B_n \mathcal{U}_n^T \right)^2 \\
\text{s.t. } \mathcal{U}_{nj} = \mathcal{U}_{jn} \ \forall j \in \mathcal{N}_n \\
\mathcal{U}_{nj} = \mathcal{U}_{jj} \ \forall j \in \mathcal{N}_i \\
\mathcal{U}_{nn} = 0.
$$

(33)

Prosumer $i^*$ can minimize $J_n$ as long as it has access to the control action of the neighbors of prosumer $n$, since $\mathcal{U}_n$ and $\mathcal{P}_n$ include the control actions and states of all neighbors. This implies that, prosumer $i^*$ requires two-hop information exchange. Thus the communication graph, $\mathcal{G}_C$ should be at least the superset of $\mathcal{G}_P^2$, which was to be shown.

Theorem 3 shows that as long as the physical grid is strongly connected and the cyber network is designed for two-hop information exchange, other prosumers can stabilize the state of the mute prosumer irrespective of its position in the network topology. To ensure stability, at least a neighbor of the mute prosumer should minimize its states. During communication failure, this prosumer takes three algorithmic contingency-based actions. 1) It extends its information exchange to ensure that the mute prosumer is still physically connected to the grid. 2) Through the two-hop communication, the prosumer determines the states of the mute prosumer, $\mathcal{P}_n$. This is done by measuring the tie-line flows of the mute prosumer. 3) The prosumer controls $\mathcal{P}_n$ by minimizing the cost function of the mute prosumer. To this end, the prosumer needs two-hop communication to receive the necessary information from the neighboring prosumers. Then, each non-mute prosumer $i$ solves its cost function and through a consensus-based protocol shares its control strategy and its perception from the control action of its neighbors. The distributed optimization iterations continue until the overall error becomes smaller than a desired value [2].

The results of Theorem 3 can be extended to provide sufficient conditions for stability of systems with multiple mute prosumers. The proposed stability conditions rely on the notion of electrical separation.

**Theorem 4:** If the following conditions hold, a system with $|\mathcal{M}| > 1$ can minimize (32) in a distributed way.

1) The graph $\mathcal{G}_P$ is strongly connected.

2) There exists an injective map from the set of mute prosumers to non-mute prosumers, $\phi : \mathcal{M} \rightarrow \mathcal{N}$, such that $\phi(m) = n \Rightarrow (m, n) \in \mathcal{E}_P$ and $(v, n) \notin \mathcal{E}_P \ \forall v \in \mathcal{M}$

3) $\mathcal{G}_C \supseteq \mathcal{G}_P^{m+1}$

**Proof:** As shown in Theorem 2, the connectivity of the graph $\mathcal{G}_P$ and the existence of the injective map $\phi : \mathcal{M} \rightarrow \mathcal{N}$ satisfy the controllability of the pair $(A, B_N)$. Therefore, there exists a minimizing controller to (32), which stabilizes the system. The second condition also guarantees that every mute prosumer $i$ is connected to at least a non-mute prosumer $i^*$, where $i^* \in \phi(\mathcal{M})$. Thus, if each prosumer $i^*$ minimizes the states of prosumer $i$, the overall cost function ($J(\mathcal{P}, \mathcal{U}_{Ni})$) can be minimized. And, due to the controllability of $(A, B_N)$, the minimizing control law can stabilize the system. But, each prosumer $i^*$ needs access to the control action of the neighbors of the prosumer $i$, since $J_i$ includes $\mathcal{P}_i$ and $\mathcal{U}_i$. This implies that the communication network should be designed for multi-hop information exchange. In the extreme scenario that all $m$ mute prosumers are in series,
the upper bound of communication hubs is \( m + 1 \), which was to be shown.

Fig. 3 shows a scenario, under which two series prosumers are mute (\( k = 2 \)). Thus, Prosumers 1 and 4 should have 3-hop communication to stabilize the states of the mute prosumers (\( m + 1 = 3 \)).

The results of Theorems 3 and 4 provide a guide-line for designing a resilient communication/control network, which can ensure stable operation of prosumers after several communication failures scenarios. For instance, Fig. 4 illustrates the schematics of an \( N - 1 \) communication-failure-resilient architecture for a loop power grid with eight prosumers.

A. Practical Barriers for Implementing Resilient Distributed Frequency Regulation

This paper described some of the architectural requirements and tenets for distributed frequency regulation. The fact that distributed resources, all the way to smart appliances, can contribute to frequency regulation poses the challenge of broad scalability: potentially billions of devices distributed across the electricity interconnection and across thousands of utilities and possibly millions of customers, can contribute to frequency regulation. This has several barriers:

1) Prosumer resource modeling: Each customer, prosumer, microgrid, etc must know the devices that it owns or operates, and their characteristics. This sounds easy but is not trivial since usually vendor devices do not incorporate documentation on the models or provide seamless interfaces to obtain the models.

2) Communication networks are not real-time: In particular towards the edge of the grid, communications rely on multiple protocols, leased networks, or the internet, which do not have real-time guarantees. Thus, delay becomes a very relevant practical consideration.

3) Cyber-security: Attack surface and methods for its mitigation and defense need to be study.

4) Regulatory: Frequency is a global system quantity, subject to Federal (FERC and NERC) standards and regulation. However, resources at the distribution level and grid edge are regulated by each states Commission. It will take time for regulatory bodies to develop the appropriate orders for implementation of distributed frequency regulation approaches. Some regions such as PJM allow frequency regulation services from small resources or subsystems.

IV. Conclusion

This paper proposed a communication-failure-resilient architecture for distributed frequency regulation in prosumer power grids. We developed topological conditions, under which the system is controllable with a single or multiple communication failures. The connectivity of the physical power grid allows non-mute prosumers to affect the states of mute prosumers. For instance, during single communication failure scenarios, \( N - 1 \) reliability in the communication network, the physical system connectivity guarantees the controllability of the system. During multiple communication failures, we proposed an inject map, which guarantees system controllability. The inject map also provides an upper bound on \( N - k \) cyber network reliability, \( k_{\text{max}} = \frac{1}{2} N \).

Next, we connected the controllability to system-wide stability and defined sufficient conditions for stability during different scenarios. We also developed topological conditions that allow non-mute prosumers to stabilize the states of the mute prosumers and minimize system-wide cost function in a distributed way. We showed that two-hop communication between prosumers was essential for obtaining the stabilizing minimizer during \( N - 1 \) cyber reliability. We also discussed about the practical barriers of implementing the resilient DFR algorithm on today’s power systems. In Part II, we will illustrate the theoretical findings on two practical prosumer power systems with different scale and complexity. We will show that prosumers could indeed stabilize the system during several communication failure scenarios.

The future research endeavor is to explore the interconnection between the cyber network contingency and the physical network contingency, when the physical connections are lost. Particularly, we are interested to understand how the connectivity of the network influences the control strategy during both cyber and physical contingencies.

REFERENCES


Dr. Santiago Grijalva received the graduate degrees in electrical and computer engineering, the M.Sc. degree in 1999, and the Ph.D. degree in 2002 from the University of Illinois at Urbana-Champaign, Urbana, IL, USA.

From 2002 to 2009 he was with PowerWorld Corporation. From 2013 to 2014 he was with the National Renewable Energy Laboratory (NREL) as Founding Director of the Power System Engineering Center (PSEC). He is the Southern Company Distinguished Professor of Electrical and Computer Engineering and Director of the Advanced Computational Electricity Systems (ACES) Laboratory with Georgia Tech, Atlanta, GA, USA. His research interests include decentralized power system control, cyber-physical security, and economics.

Dr. Grijalva is a Member of the NIST Federal Smart Grid Advisory Committee.

Dr. Magnus Egerstedt (F’02) received the B.A. degree in philosophy from Stockholm University, Stockholm, Sweden, and the M.S. degree in engineering physics and the Ph.D. degree in applied mathematics from the Royal Institute of Technology, Stockholm.

He was a Postdoctoral Scholar at Harvard University. He is currently the Steve W. Chaddick School Chair and Professor with the School of ECE at the Georgia Institute of Technology, Atlanta, GA, USA, where he also holds secondary appointments in Mechanical Engineering, Aerospace Engineering, and Interactive Computing. He conducts research in the areas of control theory and robotics, with particular focus on control and coordination of complex networks, such as multirobot systems, mobile sensor networks, and cyber-physical systems.

Dr. Egerstedt is a Fellow of IFAC, and is a foreign member of the Royal Swedish Academy of Engineering Sciences. He has received a number of teaching and research awards for his work, including the John R. Ragazzini Award from the American Automatic Control Council, the O. Hugo Schuck Best Paper Award from American Control Conference, and the Alumni of the Year Award from the Royal Institute of Technology.