# Air Traffic Maximization for the Terminal Phase of Flight Under FAA's NextGen Framework

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## Abstract

The NextGen program is the FAA's response to the ever increasing air traffic, that provides tools to increase the capacity of national airspace, while ensuring the safety of aircraft. In support of this vision, this paper provides a decentralized algorithm based on dual decomposition for safe merging and spacing of aircraft at the terminal phase of the flight. Aircraft negotiate optimal merging times that ensure safety, while penalizing deviations from the nominal path. We provide feasibility conditions for the safe merging of all incoming legs of flight and put the viability of the proposed algorithm to the test through simulations.

## Introduction

Conventional air traffic management practices are too rigid to accommodate the projected increase in air traffic. Next Generation Air Transportation System (NextGen) is the FAA's vision to address the impact of air traffic growth by increasing the National Airspace System capacity and efficiency, while improving safety and reducing environmental impacts [14]. One of NextGen's goals is to explore improvements to terminal area operations, namely automatic merging and spacing of the incoming traffic paths, to increase the traffic capacity of the terminal area and save fuel by reducing maneuvers such as holding patterns.

In this work, we present an optimal decentralized negotiation algorithm to safely merge incoming traffic in a terminal area. Current systems completely rely on air traffic controllers to safely route aircraft, who sometimes identify the conflicts in merging routes too late and ask merging aircraft to hold or redirect to wait for an opening, thus creating a large separation between the aircraft. Our approach aims at achieving maximum throughput by identifying and resolving conflicts ahead of time in a decentralized manner. The advantage of decentralization is that it makes the procedure robust and scalable by not requiring all-to-all communication or reliance on a supervisor.

Safe and efficient merging and spacing techniques in support of the FAA's NextGen is an active area of research and the subject of a number of large-scale tests of developed systems that are based on Automatic Dependent Surveillance-Broadcast (ADS-B) information [15]. SafeRoute and Point Merge are examples of proposed large-scale centralized solutions to this problem. SafeRoute is a centralized, large scale system implemented on UPS aircraft. Air traffic controllers instruct the pilot to follow a particular aircraft, while an onboard system actively computes and displays a recommended aircraft velocity such that a safe distance is maintained with the leading aircraft and safe merging is guaranteed at the merge points [16]. In Point Merge, another centralized merging and spacing solution, aircraft approaching the terminal area achieve the desired separation by flying on one of the vertically spaced sequencing legs [3]. NASA is also actively involved in air traffic management research [2]. NASA's Aviation Systems Division is focusing on hi-flow airports [12], high density en route operations, and automated separation assurance by using trajectory based tactical air traffic management [5].

A number of academic research projects have also addressed different aspects of decentralized conflict detection and resolution in air traffic management. Tomlin *et al.* use a game theoretic approach for conflict resolution of noncooperative aircraft [11]. Mao *et al.* provide sufficient conditions for stable conflict avoidance of two intersecting aircraft flows [4]. Rahmani *et al.* propose a decentralized deconfliction algorithm based on artificial potential functions [8]. Wollkind *et al.* use the bargaining technique of Monotonic Concession Protocol to detect and pseudo-optimally resolve conflicts [13]. Roy and Tomlin suggest a slot-based model where en-route aircraft select an available slot and then maintain its positioning in the traffic flow, hence guaranteeing safety-of-flight [10].

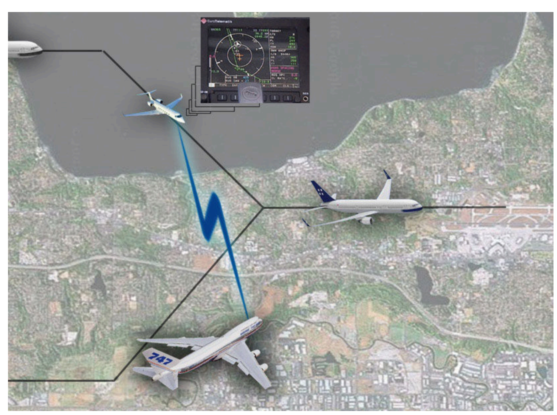



**Figure 1: Autonomous merging and spacing can increase the capacity of national airspace, while ensuring safety of the aircraft.**

In this paper we focus on autonomous merging and spacing for the terminal phase of flight (i.e., descents and approaches). Spacing aircraft as closely as possible and avoiding holding patterns would increase the air traffic capacity. In the proposed algorithm, air traffic controllers only monitor the overall performance of the system and if they sense the safety of aircraft is in jeopardy due to unforeseen circumstances, they will ask the aircraft to perform appropriate emergency evasive maneuvers.

We stay true to “safe-operation” practices (e.g., flying between predefined waypoints) and we use the information available to aircraft through ADS-B protocol with the addition of a few extra negotiation parameters. Specifically, we consider binary merging trees where at each branch, two paths merge into one at the merge point. Figure 1 illustrates this concept.

We use dual decomposition, a pair-wise optimal decentralized negotiation algorithm (e.g. [9]), to determine the achievable yet optimal arrival time of each aircraft at the merge point. In turn, these merging times determine the optimal velocity and path deviation during the “execution phase”, which is defined to be the segment between each aircraft’s next waypoint and the merge point. The pair-wise cost is composed of a maneuvering and arrival delay cost for each aircraft, and a separation cost involving both aircraft. Maneuvering costs penalize deviations on the path and changes in velocity when merging in order to conserve fuel. Arrival delay costs penalize changes in the estimated time of arrival (ETA) at the merge point for each aircraft. The separation cost penalizes spacing achieved after merging that is greater than the separation required to reach the optimal throughput.

We provide sufficient conditions for the attainability of safe separations, and derive maximum throughput controllers based on the characteristics of the traffic on merging flight paths. The viability of the proposed algorithm and derived feasibility conditions are validated through simulations.

## Trajectory Based Operations

In future air traffic systems using the ADS-B communication protocol, it is vital for aircraft approaching terminal areas to be able to maintain a safe separation from other aircraft, while also merging onto the same designated routes. Current systems completely rely on air traffic controllers to safely route aircraft, which often result in an excessive separation between aircraft. It is foreseen that ADS-B will furnish air traffic control systems with the ability to create tighter spacing amongst aircraft approaching terminals, resulting in increased throughput.

This paper presents a distributed algorithm for coordinating multiple aircraft such that adequate spacing is maintained between aircraft during traffic merging maneuvers and the approach to the terminal. Specifically, the NextGen framework will be used to accomplish this coordination and ADS-B communication will be utilized by each aircraft to communicate with other aircraft and the ground station, as shown in Figure 2. Each aircraft negotiates flight plans with the other aircraft in the terminal area to ensure proper spacing. ADS-B type messages passed between the aircraft (inter-aircraft messages) are used to communicate aircraft states, identification, negotiation parameters, and intended flight plans. Additionally, they are used to negotiate changes in these flight plans and coordinate merging and spacing actions.

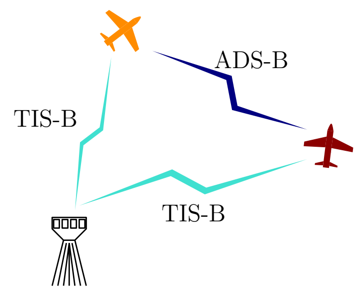



**Figure 2: NextGen’s system level communication framework.**

The software module for the merging and spacing algorithm within the NextGen/ADS-B framework is shown in Figure 3 and was designed while referring to [15]. Within this module, each aircraft determines if immediate collisions or potential conflicts are projected using state information and flight plan received from other aircraft in the terminal flight area. For immediate conflicts, the ownship plans evasive actions and sets a priority flag in its outgoing ADS-B message. For potential conflicts, the ownship uses the negotiation parameters passed through the ADS-B messages to re-plan its route and suggest new flight plans for other aircraft in conflict. This information is packaged and sent out via ADS-B as well as being sent to the pilot via CDTI (Cockpit Display of Traffic Information) and executed by the aircraft’s control laws. In the event of a potential collision, the Negotiate and Re-plan block within the Route Planning sub-module takes into account state, flight plan, suggested flight plan, and negotiation parameters passed by other aircraft in order to negotiate flight plan changes that result in proper merging and spacing. The resulting flight plan, suggested flight plans for other aircraft, updated cost parameters are then sent to the Data Packet Preparation sub-module. The updated flight plan is also sent to the Command Generation block.

The remainder of this paper will focus on a distributed implementation of the Negotiate and Replan Block as shown in Figure 3. Specifically, the following problem description will detail the problem followed by a distributed solution.

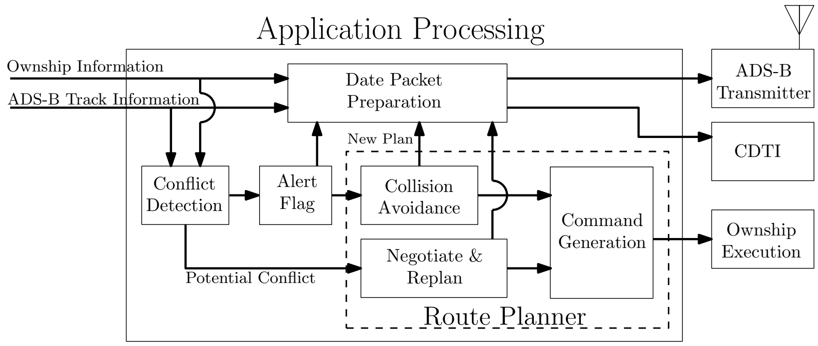



**Figure 3: Application processing, avoiding conflicts by negotiating and replanning.**

## Problem Definition

In this paper, we will leverage the inter-aircraft communication capabilities provided by the NextGen framework to design a distributed Negotiate and Replan module that will allow for the merging and spacing of aircraft on multiple legs of flight approaching an airport terminal. Compared to current methods for merging air traffic during a terminal approach, which relies heavily on instructions from the air traffic controller, the proposed automated Negotiate and Replan module will increase the air traffic throughput and minimize fuel consumption, all while ensuring that aircraft maintain a specified minimum spacing from one another throughout the entire procedure.

### *Two-Track Merging Scenario*

We will now focus on the two-track merging scenario for air traffic approaching a terminal. Consider a terminal area where the incoming legs of flight have a ground track configuration with an overhead view as shown by the fork in Figure 4. It will be shown later that our proposed solution can be generalized to merging incoming air traffic on multiple legs of flight into one using a binary tree configuration. In general, the goal is to merge air traffic coming from the legs on the left to the terminal leg on the right in a distributed manner, while making sure that all aircraft are spaced at least $\Delta_{\text{III}}$ apart from one another at all times. The proposed merging and spacing procedure consists of three main phases of operation:

- Phase I: Negotiation Phase
- Phase II: Action Phase
- Phase III: Terminal Approach Phase.

### *Phase I: Negotiation Phase*

In the *Negotiation Phase* (Phase I), incoming aircraft are assumed to be traveling on one of two parallel legs of flight: leg 1 or leg 2, approaching waypoints WP1 and WP2

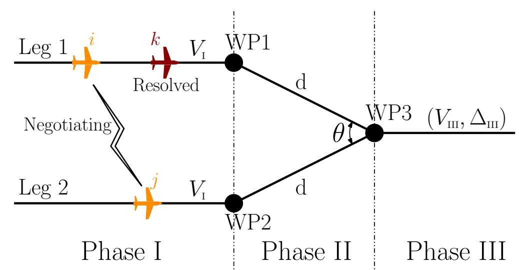



**Figure 4: An overhead view of two-track merging and spacing at the terminal phase of flight.**

respectively. We assume that all aircraft in this phase are traveling at a constant ground track speed of $V_{\text{I}}$ and have at least a spatial separation of $\Delta_{\text{I}}$ from each other, where $\Delta_{\text{I}} \geq \Delta_{\text{III}}$. During the approach to their respective waypoints, aircraft will negotiate with opposing aircraft on the opposite leg for flight plans (consisting of a velocity and path deviation assignment) to execute in the next phase of flight (Phase II), which will ensure that the aircraft maintain the required minimal spatial separation, while minimizing the fuel consumption and deviation from the original estimated time of arrival (ETA).

### *Phase II: Action Phase*

In the *Action Phase* (Phase II), aircraft approaching waypoints WP1 or WP2 travel towards WP3 using the negotiated flight plan from Phase I. A flight plan is parameterized by $V_{\text{II}}$ and $h$ where, as shown in Figure 5, $V_{\text{II}} \in [V_{min}, V_{max}]$ parameterizes the ground track speed of the aircraft and $h \in [0, h_{max}]$ parameterizes the path deviation. Furthermore, the geometry of the fork for merging is parameterized by $d$, the distance between WP3 to WP1 and WP2, as well as $\theta$, the angle made by the three waypoints, as seen in Figure 4. By modulating the speed and deviating from the straight line path to WP3, aircraft can adjust their separation from one another or equivalently, change their arrival times at WP3. However, changing the speed and increasing the distance flown between waypoints results in a change in the fuel consumption. Aircraft negotiating for flight plans in Phase I should therefore choose flight plans that minimize the fuel consumption and deviation from the original ETA at WP3 during Phase II. It should be noted that since the purpose of the triangular path deviation $h$ is to simply elongate the path to WP3, in practice it can be implemented as a constant curvature arc. Furthermore, the path deviations should be in the direction opposite to the opposing leg.

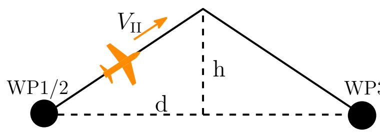



**Figure 5: An overhead view of the velocity and path deviations during Phase II.**

### *Phase III: Terminal Approach Phase*

In the *Terminal Approach Phase* (Phase III), aircraft on legs 1 and 2 have already merged at WP3 and are flying on a single leg in a straight path to the terminal. An overhead view of Phase III is shown in Figure 6. The ground track speed in this phase is $V_{\mathrm{III}}$ for all aircraft and it is assumed that the negotiated flight plans had been executed successfully during Phase II so as to ensure that aircraft have a spacing of at least $\Delta_{\mathrm{III}}$ throughout this entire phase. Therefore, Phase III is simply the leg resulting from the merging of legs 1 and 2. It will be shown later that the leg of merged air traffic in Phase III can be treated as Phase I of a new fork and be further merged with additional legs of air traffic using a binary tree configuration.

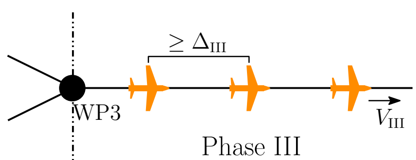



**Figure 6: Velocity and aircraft separation during Phase III.**

### *Pairwise Negotiations*

During the Negotiation Phase (Phase I), aircraft on one leg need to negotiate for flight plans with aircraft on the opposing leg to ensure that they can merge during the Action Phase (Phase II), while satisfying a minimum spacing requirement. We wish to design the negotiation process such that it occurs in a decentralized manner. Decentralized strategies require each aircraft to make a decision using limited information obtained by communicating only with the aircraft in its vicinity. The advantage of decentralization is that it makes the procedure robust and scalable by not requiring all-to-all communication or reliance on a supervisor. For our problem, we will consider a particular form of decentralized negotiation involving only two aircraft on opposing legs at a time, which we will call a *pairwise negotiation*.

The main purpose of having aircraft negotiate for flight plans to execute during Phase II is to create spacing between them for Phase III. Since the velocity for Phase III, $V_{\mathrm{III}}$, is constant, requiring aircraft to be spaced at least $\Delta_{\mathrm{III}}$ apart in Phase III is equivalent to having the arrival times at WP3 be at least $\frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}}$ apart. Therefore, we will have aircraft negotiate for arrival times at WP3, in which each proposed arrival time has a set of corresponding flight plans (with different levels of fuel consumption) that allow the particular aircraft to meet it. Furthermore, since there are limits imposed on the flight plan's velocity $V_{\mathrm{II}}$ and deviation $h$, there is only a limited range of reachable arrival times that each aircraft can hope to achieve. Each aircraft therefore has a set of reachable arrival times at WP3 and must negotiate in Phase I so that the arrival times are all at least $\frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}}$ apart from one another.

The pairwise negotiation procedure is as follows:

1. Each aircraft in Phase I is initially marked as *unresolved* and knows its own set of reachable arrival times at WP3. Furthermore, each aircraft's knowledge of the most recently resolved aircraft's arrival time at WP3 is initially set to $\infty$.

2. Pairwise negotiations occur between the unresolved aircraft on leg 1 and leg 2 that are closest to WP3 on their respective legs to determine a pair of reachable arrival times at WP3 that are at least $\frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}}$ apart from each other, as well as from the most recently resolved aircraft's arrival time at WP3.

3. After a pair of reachable arrival times at WP3 have been negotiated, the aircraft with the earliest arrival time marks itself as being *resolved* and communicates its planned arrival time at WP3 to the unresolved aircraft on both legs that are closest to WP3.

4. The aircraft with the later negotiated arrival time remains marked as unresolved and repeats the procedure by negotiating for a reachable arrival time with the unresolved aircraft on the opposite leg that is closest to WP3.

To illustrate this procedure, consider the three aircraft in Phase I of Figure 4: $i$, $j$, and $k$. Suppose all three aircraft are initially unresolved, then aircraft $j$ and $k$ will be the first pair to perform a pairwise negotiation for an arrival time at WP3 that will ensure the two meet the minimum spacing requirements. If the result of the negotiation determines that aircraft $k$ should go first, then aircraft $k$ marks itself as resolved and aims to reach WP3 at the negotiated time. Since aircraft $j$ has the later negotiated arrival time, it must now negotiate with aircraft $i$ for a set of arrival times that ensure not only that aircraft $i$ and $j$ will meet the minimum spatial separation, but that they also each maintain a separation from the most recently resolved aircraft: $k$.

### *Problem Statement*

Having presented all three phases of flight and the procedure for pairwise negotiations, we can now pose the merging and spacing problem to be solved throughout the rest of the paper in detail:

**Problem 3.1** *Given the fork setup for two-track merging as shown in Figure 4, determine:*

1. *Sufficient feasibility conditions on the geometry of the fork ($d$ and $\theta$), ground track speeds ($V_{I}$ and $V_{III}$), allowable flight plans ($V_{min}$, $V_{max}$, and $h_{max}$), and the density of the incoming air traffic ($\Delta_{I}$), such that pairwise negotiations between aircraft on opposing legs in Phase I will always result in arrival times at WP3 where all aircraft are guaranteed a spacing of at least $\Delta_{III}$ from each other.*
2. *A decentralized solution for pairwise negotiation in Phase I that will not only guarantee the minimum spatial separation of $\Delta_{III}$, but will also minimize fuel consumption and deviations from each aircraft's original ETA.*

## Feasibility Conditions for Separation

In this section, we will address the first part of Problem 3.1: finding sufficient feasibility conditions on the geometry of the fork and parameters defining allowable aircraft maneuvers, such that pairwise negotiations will guarantee a spacing of at least $\Delta_{III}$ amongst all aircraft. Feasibility conditions for Phases I, II, and III will each be treated separately. Note that only the main feasibility results will be stated and that for a more rigorous derivation of the conditions, the reader is referred to [1].

### *Phase III Conditions*

The feasibility conditions for Phase III require that given a pair of negotiating aircraft in Phase I, a pair of reachable arrival times that are at least $\frac{\Delta_{III}}{V_{III}}$ apart can always be found, thus ensuring that the two aircraft will have a spacing of at least $\Delta_{III}$ in Phase III. Such a condition can be satisfied as long as the range of reachable arrival times that can be achieved by an aircraft, which we will call $|R|$, as given by

$$|R| = \frac{2}{V_{min}}\sqrt{h_{max}^2 + \frac{d^2}{4}} - \frac{d}{V_{max}}, \tag{1}$$

is large enough. However, since a pair of aircraft also has to maintain a spatial separation from the most recently resolved aircraft, an aircraft's choice of arrival time will limit the choice of arrival times for other aircraft following behind it. To alleviate this, we require that the aircraft in Phase I, and hence their sets of reachable arrival times, be spaced sufficiently apart from one another. Combining both conditions guarantees that pairwise negotiation in Phase I will find reachable arrival times for all aircraft to be spaced at least $\Delta_{III}$ apart in Phase III and is summarized by the inequality:

$$\Delta_{I} \geq |R| V_{I} \geq 2\frac{\Delta_{III}}{V_{III}}. \tag{2}$$

### *Phase II Conditions*

Next, we will look at the feasibility conditions that guarantees spacing amongst aircraft in Phase II. The first condition concerns the transition between Phases II and III. Suppose there are two aircraft, $i$ and $j$, both in Phase III with aircraft $i$ located on WP3 and aircraft $j$ ahead of it by exactly $\Delta_{III}$. Tracing their trajectories "back in time" and assuming that aircraft $i$ travels at the worst case ground track speed of $V_{min}$ during Phase II, we see in Figure 7 that when aircraft $j$ was at WP3, aircraft $i$ was at a distance of $\frac{\Delta_{III}}{V_{III}} V_{min}$ behind it. Therefore, it leads us to conclude that

$$V_{min} \geq V_{III} \tag{3}$$

is necessary to guarantee a spatial separation of at least $\Delta_{III}$ during the transition between Phases II and III.

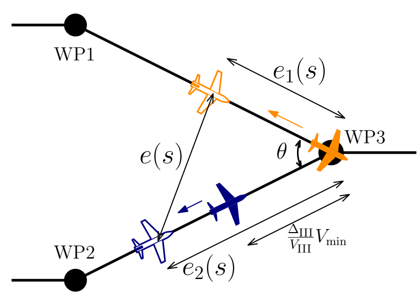



**Figure 7: Overhead diagram of Phase II used for feasibilty proof.**

We continue with the scenario to derive conditions for when both aircraft are in Phase II. Suppose aircraft $j$ is located at WP3 and aircraft $i$ is in Phase II but will reach WP3 in exactly $\frac{\Delta_{III}}{V_{III}}$ time, as illustrated in Figure 7. Since the allowed trajectory deviations in Phase II curve away from the opposing leg, the worst case scenario is when both aircraft are flying with no deviation, where aircraft $i$ is traveling at ground track speed $V_{min}$ and aircraft $j$ is traveling at ground track speed $V_{max}$. Letting $s$ parameterize time traveling backwards, we want to calculate the spatial separation $e(s)$ between aircraft $i$ and $j$ as a function of time. Solving for the time $s^*$ that minimizes the spatial separation between the two aircraft, we get that

$$s^* = \frac{V_{max} V_{min} \frac{\Delta_{\text{III}}}{V_{\text{III}}} \cos\theta - V_{min}^2 \frac{\Delta_{\text{III}}}{V_{\text{III}}}}{V_{min}^2 + V_{max}^2 - 2V_{max}V_{min}\cos\theta}. \quad (4)$$

Notice that $s^*$, the time at which the separation between aircraft is minimized, is a function of the fork angle $\theta$. Therefore, for every angle $\theta$, there is an associated worst case minimum spatial separation, $e^*(\theta)$, between the aircraft given by

$$e^*(\theta) = e(s^*(\theta)). \quad (5)$$

The angle of the fork in Phase II, $\theta$, must be chosen so as to ensure that the worst case minimum spatial separation amongst two aircraft is at least $\Delta_{\text{III}}$, as summarized by the inequality

$$e^*(\theta) \geq \Delta_{\text{III}}. \quad (6)$$

### Phase I Conditions

Spacing in Phase I along the same leg is already guaranteed by the requirement that $\Delta_{\text{I}} \geq \Delta_{\text{III}}$. To ensure a spatial separation amongst aircraft on opposing legs, we just require that the legs be spaced at least $\Delta_{\text{III}}$ apart, which is achieved when the length of the legs in Phase II, $d$, as well as the fork angle, $\theta$, satisfy

$$2d\sin\left(\frac{\theta}{2}\right) \geq \Delta_{\text{III}}. \quad (7)$$

### Sufficient Feasibility Conditions

Having gone through and derived feasibility conditions for all three phases of flight in this section, we summarize the results in the theorem below:

**Theorem 4.1** *A set of sufficient feasibility conditions which solves Part 1 of Problem 3.1 is given by equations (2), (3), (6), and (7), which are restated below:*

1. $\Delta_{I} \geq |R| V_{I} \geq 2\frac{\Delta_{III}}{V_{III}}$
2. $V_{min} \geq V_{III}$
3. $e^*(\theta) \geq \Delta_{III}$
4. $2d\sin\left(\frac{\theta}{2}\right) \geq \Delta_{III}$.

*Together, they ensure that when all aircraft use pairwise negotiations to determine arrival times at WP3 for the two-track merging setup in Figure 4, the resulting arrival times will allow all aircraft maintain a spacing of at least $\Delta_{III}$ from each other throughout all three phases of flight.*

### Merging Multiple Legs

The proposed two-track merging fork, as shown in Figure 4, allows for air traffic from two separate legs to safely merge into one with guarantees that all aircraft will maintain a safe spacing from one another at all times. The feasibility results derived thus far in this section can be used to generalize the two-track merging fork to allow for the merging of multiple legs of air traffic using a binary tree configuration as shown in Figure 8.

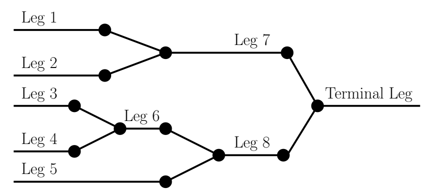



**Figure 8: Binary tree structure for merging multiple tracks.**

In the figure, air traffic from legs 1 through 5 on the left all merge onto to the terminal leg on the right, making use of intermediate legs 6, 7, and 8. The binary tree can be treated as a collection of two-track merging forks, where each leg in Phase I of a fork can be viewed as Phase III of another fork consisting of that leg and the two merging into it. Thus, a designer can propagate the groundtrack speed and separation requirements on the terminal leg backwards throughout the branches of the tree until parameters for all legs have been determined.

As an example, let legs 1, 2, and 7 of Figure 8 be Fork A, while legs 7, 8, and the terminal leg form Fork B. The desired conditions $\Delta_{\text{III}}^B$ and $V_{\text{III}}^B$ on the terminal leg will determine $\Delta_{\text{I}}^B$ and $V_{\text{I}}^B$ on legs 7 and 8 of Fork B. However, leg 7 is both Phase I of Fork B and Phase III of Fork A, so we let $\Delta_{\text{III}}^A = \Delta_{\text{I}}^B$ and $V_{\text{III}}^A = V_{\text{I}}^B$. With the conditions for Phase III of Fork A established, the feasibility conditions can then be used to determine valid choices of $\Delta_{\text{I}}^A$ and $V_{\text{I}}^A$ on legs 1 and 2.

To guarantee that all aircraft maintain a spacing of at least some distance $\Delta^*$ away from one another within each fork, one simply needs to let $\Delta_{\text{III}} \geq \Delta^*$ in the fork containing the terminal leg. Since we require that $\Delta_{\text{I}} \geq \Delta_{\text{III}}$ for each fork, and the $\Delta_{\text{I}}$ of a fork is chosen to equal $\Delta_{\text{III}}$ of the fork preceeding it, we are guaranteed that $\Delta_{\text{III}} \geq \Delta^*$ for every fork in the binary tree. It should be noted that the discussion above only addresses how to maintain a safe spacing amongst aircraft in the same fork. Additional care must be made in choosing the geometry of the fork ($d$ and $\theta$) so as to ensure that air traffic traveling on parallel forks, such as those on legs 2 and 3, are also able to maintain a spatial separation of at least $\Delta^*$.

## Pairwise Optimization Problem

The pairwise negotiations for arrival times at WP3 will minimize a pairwise cost for both aircraft, consisting of the sum of a maneuvering and delay cost for each aircraft, as well as a joint separation cost. For an Aircraft $i$ moving into Phase II, its ETA at WP3, which we will call $t_{i,0}^{\text{WP3}}$, is the time it would have arrived at WP3 had it flown in a straight line from WP1/WP2 to WP3 using the same ground track speed as in Phase I. We use the notation, $t_i^{\text{WP3}}$, to denote the ETA of Aircraft $i$ at WP3. Any additional deviation in the path or change in ground track speed corresponds to an increase in fuel consumption and is penalized.

Given an arrival time at WP3, the associated maneuvering and arrival delay cost for an Aircraft $i$ is given by:

$$J_i(t_i^{\text{WP3}}) = \min_{(V_{\text{II}},h)} (k_1 h^2 + k_2(V_{\text{II}} - V_{\text{I}})^2) + k_3(t_i^{\text{WP3}} - t_{i,0}^{\text{WP3}})^2,$$

such that the weights $k_1$, $k_2$, $k_3 \in \mathbb{R}_+$ may be chosen differently for each aircraft. The minimum term chooses the best $V_{\text{II}}$ and $h$ pair to arrive at WP3 at time $t_i^{\text{WP3}}$, which minimizes the penalty on deviations in path and speed. The last term penalizes changes in the arrival time so as to minimize aircraft delays in reaching the airport terminal.

The separation cost penalizes a proposed pair of arrival times for Aircraft $i$ and $j$ if it causes them to have a separation greater than exactly $\Delta_{\text{III}}$ in Phase III, and is given by:

$$J_{ij}\left(t_j^{\text{WP3}}, t_i^{\text{WP3}}\right) = \gamma_{ij}(|t_j^{\text{WP3}} - t_i^{\text{WP3}}| - \frac{\Delta_{\text{III}}}{V_{\text{III}}})^2,\ \gamma_{ij} > 0.$$

The purpose of having such a separation cost is to encourage the merging aircraft to space themselves as close together as possible, while still being $\Delta_{\text{III}}$ apart from one another. We will refer to this cost as being a joint cost since it relies on both $t_i^{\text{WP3}}$ and $t_j^{\text{WP3}}$.

There are two constraints on the allowable choices of WP3 arrival times. The first is that they must be feasible for the aircraft, i.e., the aircraft can actually reach WP3 at the specified ETA given its limits on ground track speed and path deviation, while maintaining a separation of $\Delta_{\text{III}}$ from the most recently resolved aircraft. The second is that the negotiated arrival times for Aircraft $i$ and $j$ must ensure that a minimum separation of $\Delta_{\text{III}}$ is achieved in Phase III, which is accomplished by the constraint $|t_j^{\text{WP3}} - t_i^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$.

Letting each aircraft be responsible for its own maneuvering and arrival delay cost as well as half of the separation cost, the individual costs for a negotiating pair of Aircraft $i$ and $j$ are:

$$U_i(t_i^{\text{WP3}}, t_j^{\text{WP3}}) = J_i(t_i^{\text{WP3}}) + \frac{1}{2}J_{ij}(t_j^{\text{WP3}}, t_i^{\text{WP3}}),$$
$$U_j(t_i^{\text{WP3}}, t_j^{\text{WP3}}) = J_j(t_j^{\text{WP3}}) + \frac{1}{2}J_{ij}(t_j^{\text{WP3}}, t_i^{\text{WP3}}).$$

These costs can be combined to create the pairwise cost, and hence the following pairwise optimization problem:

**Problem 5.2** *The pairwise negotiation problem encountred when performing two-track merging can be formulated as the following optimization problem:*

$$\min_{t_i^{WP3}, t_j^{WP3}} (U_i(t_i^{WP3}, t_j^{WP3}) + U_j(t_i^{WP3}, t_j^{WP3})),$$

*such that* $|t_j^{WP3} - t_i^{WP3}| \geq \frac{\Delta_{III}}{V_{III}}$.

## Distributed Solution

Dual decomposition is proposed to solve Problem 5.2 by allowing a pair of aircraft to each initially have opinions on what each other's arrival times at WP3 should be, but then negotiate to reach an agreement (as seen in [9]) on which arrival times minimizes the pairwise cost between them, while satisfying the separation constraint. First, we introduce the notion of Aircraft $i$'s opinion of what Aircraft $j$'s arrival time should be, given by $t_{ij}^{\text{WP3}}$. A dual optimization version of Problem 5.2 can then be formed. The new problem is split up such that each aircraft will update its estimate of the other aircraft's arrival time based on information it receives from the other aircraft, as seen in [1]. An artifact of the dual optimization problem are the parameters: $\lambda_1$ and $\lambda_2$, associated with the mismatch between estimates of arrival times.

Problem 5.2 has a bounded non-convex cost, meaning that the dual problem has weak duality and so its solution cannot be guaranteed to result in a global minimum. We therefore seek arrival times which achieve local minima for the pairwise constrained optimization problem.

### *Dual Decomposition Solution*

In [7] and [9], methods are presented for decomposing this dual optimization problem into subproblems that each agent can solve. As a result, the negotiation is broken down into steps. First, each aircraft solves a minimization problem based on its own arrival time estimates and given $\lambda$ values. Arrival time estimates are then communicated between the aircraft, and each aircraft takes a gradient step to update its value of $\lambda$. Finally, the updated $\lambda$ values are communicated to the other aircraft and the cycle begins again. These steps repeat until the other aircraft's arrival time estimates agree with the aircraft's own calculated arrival time.

Thorough formulations of the subproblems in the dual problem that are solved at each of these steps are detailed in [1]. In order to solve these problems, Aircraft $i$ must communicate $t_{ij}^{\text{WP3}}$ and $\lambda_1$ to Aircraft $j$, while Aircraft $j$ must communicate $t_{ji}^{\text{WP3}}$ and $\lambda_2$ to Aircraft $i$. As the negotiations proceed, $||t_{ij}^{\text{WP3}} - t_{jj}^{\text{WP3}} \to 0||$ and $||t_{ii}^{\text{WP3}} - t_{ji}^{\text{WP3}} \to 0||$, i.e., Aircraft $i$ agrees with Aircraft $j$ on what it should do in Phase II and vice versa. Each aircraft must solve the minimization problem once for when Aircraft $i$ arrives first, and again for when Aircraft $j$ arrives first. The scenario with the lowest pairwise cost determines which of the two aircraft will be scheduled to fly to the waypoint WP3 first.

# Simulations

Having explained the proposed merging and pairwise negotiation protocol in detail, we now will showcase its performance in a series of numerical simulations. The first simulation will give an example of how to use the binary tree configuration to merge air traffic from three legs onto a single terminal leg. A set of feasible parameters for the three legs are given and acceptable spacing amongst aircraft is verified. The second simulation examines pairwise negotiations in detail by showing how a pair of negotiating aircraft's opinions on arrival times converge throughout the negotiation procedure.

## *Binary Tree Simulation*

To showcase our algorithm for merging multiple legs of air traffic into one, we will look at a subtree of the binary tree in Figure 8, where only legs 1, 2, 7, 8, and the terminal leg are present. Let the two-track merging fork consisting of legs 1, 2, and 7 be called Fork A, while the fork consisting of legs 7, 8, and the terminal leg will be called Fork B. The set of parameters for each fork are chosen so as to satisfy the feasibility conditions. Recall that when using multiple forks to create a binary tree, the parameters chosen in one fork will then determine the parameters of the fork preceding it. The parameters must therefore be defined starting with Fork B:

$V_{\text{I}}^B = 1$ $\qquad$ $\Delta_{\text{I}}^B = 8.1$

$V_{min}^B = 0.8$ $\qquad$ $V_{max}^B = 1.8$

$h_{max}^B = 1$ $\qquad$ $V_{\text{III}}^B = 0.8$

$\Delta_{\text{III}}^B = 2$ $\qquad$ $d^B = 10$

$\theta^B = \frac{\pi}{2}$

Now that Fork B satisfies the feasibility conditions, we can define a set of parameters for Fork A. Notice however, that leg 7 is shared between the two forks so it must be that $V_{\text{III}}^A = V_{\text{I}}^B$ and $\Delta_{\text{III}}^A = \Delta_{\text{I}}^B$. The parameters for Fork A are therefore:

$V_{\text{I}}^A = 1$ $\qquad$ $\Delta_{\text{I}}^A = 20$

$V_{min}^A = 1$ $\qquad$ $V_{max}^A = 20$

$h_{max}^A = 4$ $\qquad$ $V_{\text{III}}^A = 1$

$\Delta_{\text{III}}^A = 8.1$ $\qquad$ $d^A = 15$

$\theta^A = \frac{\pi}{2}$

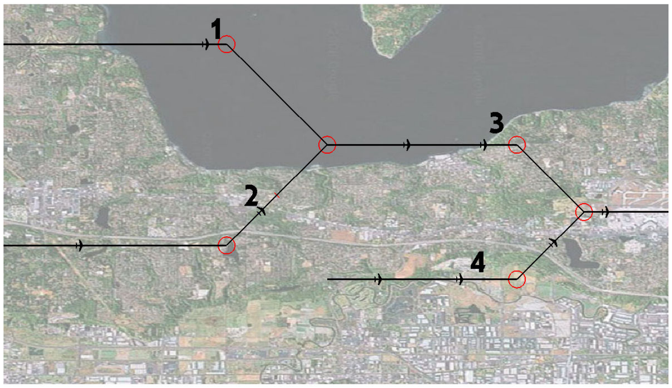
Since the parameters for each fork were chosen to satisfy the derived feasibility conditions, using the proposed pairwise negotiation protocol amongst merging aircraft will guarantee that aircraft will maintain a spacing of at least $\Delta_{\text{III}}^B = 2$ from each other at all times. A simulation of the binary tree was performed where incoming aircraft was randomly inserted into legs 1 and 2 of Fork A with at least a separation of $\Delta_{\text{I}}^A$ and leg 8 of Fork B with at least a separation of $\Delta_{\text{I}}^B$ from other aircraft on the same leg. Screenshots from the simulation showing how pairwise negotiation successfully merges the three legs of air traffic into one are shown in Figure 9.

Although there are many aircraft seen in the simulation, we will only focus on the actions taken by Aircraft 1 through 4, as marked accordingly in the figures. In Figure 9(a), both Aircraft 1 and 2 are approaching the merge point for Fork A. Similarly, Aircraft 3 and 4 are approaching the merge point for Fork B. Both pairs of aircraft are conflicting and must act accordingly if they wish to maintain a safe separation when merging. Figure 9(b) shows that Aircraft 1 and 2's negotiation resulted in Aircraft 1 taking a path deviation to delay its arrival time at the merge point. Aircraft 3 and 4's negotiation, on the other hand, determined that the best course of action was for Aircraft 3 to increase while Aircraft 4 decreases its ground track speed. By Figure 9(c), one aircraft on each fork has already merged. Figure 9(d) shows that both pairs of aircraft have merged successfully and have maintained a safe spacing with other aircraft. Aircraft 1 and 2 have merged onto the same leg in Fork B and must now negotiate with aircraft on the opposing leg of Fork B to determine how to engage the next merge point. Aircraft 3 and 4 have both merged onto the terminal leg and can proceed to land in the terminal.

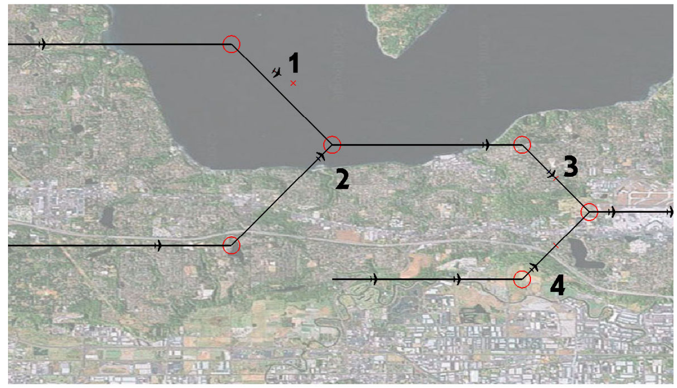
To verify concretely that pairwise negotiation in the preceding simulation had succeeded in maintaining the desired spatial separation, a plot of inter-aircraft spacing at the merge point of Fork A is shown in Figure 10(a), while a similar plot for the merge point in Fork B is shown in Figure 10(b). As expected, all pairs of aircraft have at least the desired separation upon reaching the waypoint of each fork, thus confirming that pairwise negotiation was successful.

## *Pairwise Negotiation Simulation*

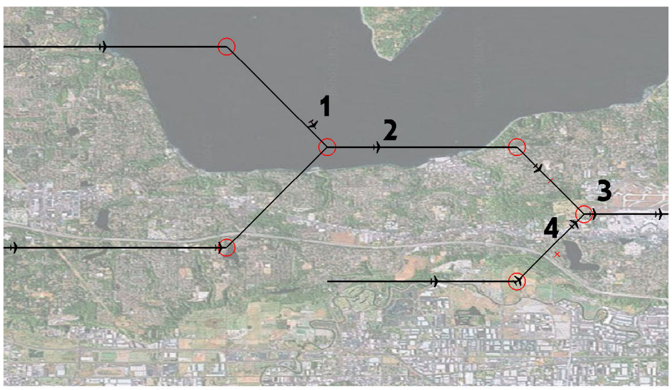
The previous simulation showed aircraft on three separate legs merging into a single leg using a binary tree made from two-track merging forks. Each time a pair of opposing



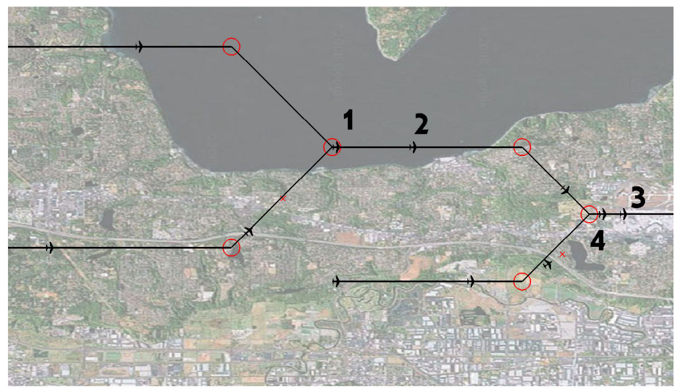

**a)** Aircraft 1 and 2 approach the merge point in Fork A, Aircraft 3 and 4 similarly approach the merge point in Fork B.




**b)** Aircraft 1 yields to Aircraft 2 by deviating its path. Fork B is resolved by Aircraft 3 increasing its speed, while Aircraft 4 decreases its speed.




**c)** Aircraft 2 has merged in Fork A, Aircraft 1 follows behind at a safe distance. Similarly, Aircraft 3 has already merged in Fork B and Aircraft 4 is about to enter the merge point.




**d)** Both Aircraft 1 and 2 have merged in Fork A and are now both on the same leg in Fork B. Aircraft 3 and 4 successfully merged in Fork B onto the terminal leg.

**Figure 9: A simulation of a binary tree structure merging three legs of air traffic into a single terminal leg.**

aircraft approach a merge point, they must perform a pairwise negotiation to determine flight plans that will ensure a safe spacing is maintained between each other. We will now investigate the negotiation process in detail by looking at the results of a particular pairwise negotiation.

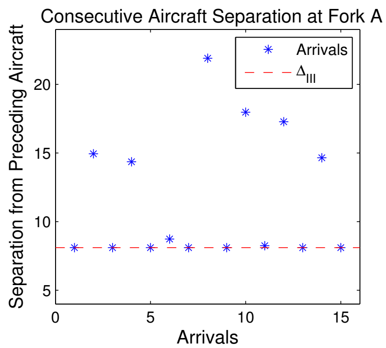
Consider a pair of aircraft on a two-track merging fork, such as in Figure 4, where the parameters of the fork satisfy the derived feasibility conditions and are given by:

$$V_{\text{I}} = 1 \qquad \Delta_{\text{I}} = 8.1$$
$$V_{min} = 0.5 \qquad V_{max} = 1.81$$
$$h_{max} = 1 \qquad V_{\text{III}} = 0.5$$
$$\Delta_{\text{III}} = 2 \qquad d = 5$$
$$\theta = \frac{\pi}{2}$$

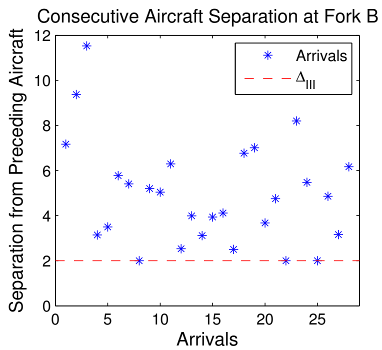
Note that since the derived feasibility conditions are already met, pairs of aircraft that satisfy the spacing constraints in Phase III will also satisfy them for Phase II.

Suppose Aircraft 1 and 2 are both on Phase I of legs 1 and 2, respectively, with no other aicraft preceding them. Aircraft 1's arrival time at WP1 is $t_1^{\text{WP1}} = 12$, while Aircraft 2's arrival time at WP2 is $t_2^{\text{WP2}} = 13$. Notice that if no negotiation occured and the two aircraft proceeded into Phase II with the default flight plan of $V_{\text{II}} = V_{\text{I}}$ and $h = 0$, then they would reach WP3 within one time unit of each other. Since the ground track speed in Phase III is $V_{\text{III}} = 0.5$, the two aircraft would only have a spatial separation of 0.5 in Phase III, which is much less than the required separation of $\Delta_{\text{III}} = 2$. Therefore, pairwise negotiations are needed to resolve this conflict.

The goal of the pairwise negotiation is to find a pair of feasible arrival times within the feasible time sets $\tau_1 = [14.76\ 22.77]$ and $\tau_2 = [15.76\ 23.77]$, for Aircraft 1 and 2




**a)** Spacing between consecutive aircraft arriving at the merge point for Fork A.




**b)** Spacing between consecutive aircraft arriving at the merge point for Fork B.

**Figure 10: Plots of spatial separation amongst consecutive aircraft for each two-track merging fork's merge point in the binary tree merging simulation.**

respectively. The pair of arrival times should allow the two aircraft to maintain a spacing of at least $\Delta_{\text{III}} = 2$ from each another at all times. Since $V_{\text{III}} = 0.5$, maintaining spacing in Phase III is equivalent to the arrival times at WP3 being at least $\frac{\Delta_{\text{III}}}{V_{\text{III}}} = 4$ apart from one another.

Pairwise negotiation cost weights for Aircraft 1 are $k_1 = 10$, $k_2 = 2$, and $k_3 = 1$, while the weights for Aircraft 2 are $k_1 = 3$, $k_2 = 8$, and $k_3 = 3$. Recall that $k_1$ penalizes deviations in path, $k_2$ penalizes changes in ground track speed, and $k_3$ penalizes deviations from an aircraft's ETA at the merge point. Therefore, from the values of the weights, we can deduce the preferences of each aircraft. Aircraft 1 would rather perform a change in its ground track speed than deviate its path, as seen by it weighing path deviations more in its cost. Aircraft 2 has opposite preferences and would rather deviate its path than change its ground track speed. Furthermore, since the $k_3$ term is larger for Aircraft 2, it wishes to arrive at WP3 at its ETA (had it used $V_{\text{II}} = V_{\text{I}}$ and $h = 0$ in Phase II) more than Aircraft 1. Lastly, the weight in the joint cost was chosen to be $\gamma = 10$ to give some incentive for the two aircraft to space themselves as closely as possible without violating the spatial constraint of being at least $\Delta_{\text{III}}$ apart.

Performing a pairwise negotiation through dual decomposition on the two aircraft yields arrival times that evolve as shown in Figures 11(a) and 11(b). The first case to consider is if Aircraft 1 is chosen to go first. In Figure 11(a), we see that the negotiated arrival times are $t_{11}^{\text{WP3}} = 14.92$ and $t_{22}^{\text{WP3}} = 18.92$. Next, in Figure 11(b), the case when Aircraft 2 is chosen to go first results in arrival times $t_{11}^{\text{WP3}} = 20.92$ and $t_{22}^{\text{WP3}} = 16.92$. Notice that in both cases, the pairwise negotiation converges in that $||t_{11}^{\text{WP3}} - t_{21}^{\text{WP3}}|| \to 0$ and $||t_{22}^{\text{WP3}} - t_{12}^{\text{WP3}}|| \to 0$, i.e., both Aircraft 1 and 2 agree on what Aircraft 1 should do during Phase II, and vice versa. More importantly, the agreed arrival times are at least 4 time units apart and so will ensure a safe spacing in Phase III.

The pairwise costs for the two cases, when Aircraft 1 goes first and when Aircraft 2 goes first, are both shown in Figure 11(c). It is important to note that the pairwise costs do not have to be monotonically decreasing during the pairwise negotiation, since forcing the arrival times to ensure a safe separation amongst aircraft may increase the cost at times. The final pairwise cost for when Aircraft 1 arrives first is $J = 8.074$, and for when Aircraft 2 arrives first is $J = 19.86$.

Having performed the dual decomposition for both cases, we see that both yield flight plans that maintain the desired spacing. However, the scenario where Aircraft 1 arrives at the merge point first yields the lower pairwise cost. As a consequence, Aircraft 1 is marked as resolved and is scheduled to take the merge point first, while choosing $V_{\text{II}} = 1.7123$ and $h = 0$ in Phase II. The negotiated flight plan for Aircraft 1 corresponds to an increase in its ground track speed with no deviation in its path in order to move away from Aircraft 2. Such a result is not surprising since as mentioned before, Aircraft 1's weights were chosen such that it penalized deviations in its path heavily. Finally, Aircraft 2 is still unresolved and so must now negotiate with the next unresolved aircraft on Leg 1 for an arrival time at the merge point.

# Conclusions

We provide a distributed algorithm that uses pairwise negotiations over merging legs to guarantee a minimum separation amongst aircraft and minimize an associated cost related to the fuel consumption and deviations from the estimated time of arrival (ETA). We develop a simulation and visualization environment to demonstrate the viability of the proposed method. Under the assumptions made throughout the paper and the feasibility conditions provided, the safety of the autonomous operation is guaranteed.

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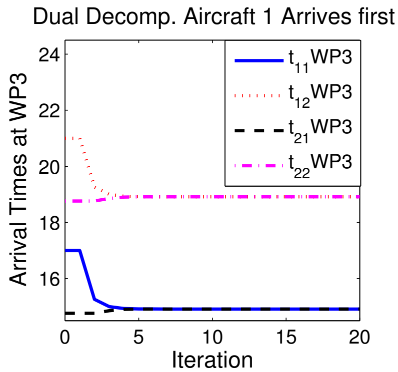
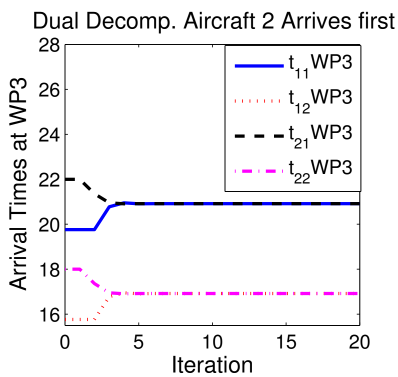
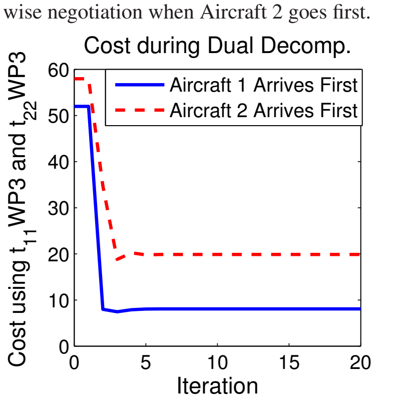



**a)** Arrival time estimate convergence for pairwise negotiation when Aircraft 1 goes first.




**b)** Arrival time estimate convergence for pairwise negotiation when Aircraft 2 goes first.




**c)** Pairwise cost trajectory using negotiated arrival times per iteration.

**Figure 11: Arrival time agreement and pairwise cost minimization.**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