A Control Lyapunov Function Approach to Human-Swarm Interactions

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***Abstract*— In this paper, we seek to establish formal guarantees for whether or not a given human-swarm interaction (HSI) is appropriate for achieving multi-robot tasks. Examples of such tasks include guiding a swarm of robots into a particular geometric configuration. In doing so, we define what it means to impose a HSI control structure on a multi-robot system. Control Lyapunov functions (CLFs) are used to prove that it is feasible for a user to achieve a particular geometric configuration with a multi-robot system under some selected HSI control structure. Several examples of multi-robot systems with unique HSI control structures are provided to illustrated the use of CLFs to establish feasibility.**

I. INTRODUCTION

Many applications require human intervention to guide autonomous robots through complicated tasks. For example, we often rely on and benefit from a human operator's ability to decide where robots should focus their efforts [1]. And even if autonomous robots do not require human guidance, humans and robots will continue to coexist in most environments (e.g., manufacturing floors [2], disaster areas [3]) and to interact with each other. Existing interfaces have focused on supporting human interactions with one or a few robots (for example, [4]); however, as the number of robots involved in the task grows large, such interfaces can become less effective or even unusable due to a lack of scalability in the corresponding interaction [5]. Therefore, there has been a growing effort to understand human-swarm interactions (HSI) and devise interactions that are amenable to having humans interact with swarms of robots easily and effectively (for example, see [6], [7], and other types of interactions cited below).

We are interested in establishing if a provided HSI is appropriate for a given task, because selecting an inappropriate HSI could frustrate and discourage users from working with robotic swarms. In particular, we investigate if the provided HSI allows a user to guide a swarm of robots into some geometric configuration. Human-swarm interactions come in a variety of flavors, such as gesture-based methods [8], [9], mode selection [10], music instrument interfaces [11], broadcast control [12], [13], deformable media [14], and biologically inspired interactions [15], just to cite and name a few. The commonalities amongst these interactions are two-fold: the user's interaction with the swarm happens alongside the interactions amongst the robots in the swarm, and each HSI imposes a specialized structure on the possible inputs to limit the complexity of the interaction. Some of these HSI control structures are tailored to make it possible to achieve particular types of geometric configurations with swarms of robots, such as rendezvous, flocking, coverage, and formations, which beckons the question, *if we are given a particular HSI control structure for a multi-robot system, is it feasible to use the corresponding interaction to achieve a desired geometric configuration with a swarm of robots?*

In this paper, we will show that control Lyapunov functions can be used to answer this question. While providing proofs of convergence of a HSI-enabled multi-agent systems to a geometric configuration is standard (see, for example, [16]), the novelty in our work is that we provide a formal definition of what it means to impose a HSI control structure on the dynamics of the multi-robot system that represents a swarm of robots. And then, we use a CLF approach to show convergence of the HSI control structured multi-robot system to some geometric configuration to demonstrate that it is feasible for the user to achieve the desired geometric configuration with such a swarm of robots.

II. DEFINITIONS

Our objective is to determine whether it is possible for a human operator to use a particular human-swarm interaction (HSI) to achieve some geometric configuration with a swarm of mobile robots. To establish feasibility, we first need to know what a HSI represents in terms of the structure it imposes on a multi-robot system, and what it means for a human operator to achieve a particular geometric configuration with the robotic swarm.

A. Human-Swarm Interaction Structure

In general, we consider continuous-time, time-invariant systems with inputs, which represent robotic swarms that can be externally controlled (or interacted with) by a user. The dynamics of such multi-robot systems can be defined as $\dot{x}(t) = f(x(t), u(t))$, where $x(t) \in \mathcal{X}$ is the state of the system at time t and $u(t) \in \mathcal{U}$ is the input to the system at time t. In fact, $x(t)$ will represent the stacked vector of the states belonging to all robots at time t, while $x_i(t)$ will refer to the state of robot i at time t. For example, $x(t)$ will typically represent the position or pose of all robots together at time t.

More importantly, the differentiable function $f : \mathcal{X} \times \mathcal{U} \to T\mathcal{X}$, where $T\mathcal{X}$ is a tangent space, is structured according to the network topology of the multi-robot system. The network topology is given by a graph $\mathcal{G} = (V, E)$, where V is the set of vertices representing the agents, and E is the set of edges representing information exchange between agents

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via communication links or due to sensor footprints (see, for example, [17]). Specifically, $f \in \text{sparse}_{\mathcal{X}}(\mathcal{G})$ conveys that state information in the multi-robot system can only flow between agents that are linked in the network topology. Consequently,

$$f \in \text{sparse}_{\mathcal{X}}(\mathcal{G}) \Leftrightarrow \left(j \notin \overline{N}(i) \Rightarrow \frac{\partial f_i(x,u)}{\partial x_j} = 0,\ \forall x, u \right), \tag{1}$$

where $N(i)$ is the so-called neighborhood of robot i, i.e., $j \in N(i)$ if $(i,j) \in E, i, j \in V$, and $\overline{N}(i) = N(i) \cup \{i\}$.

By picking a particular HSI control structure, we are being specific about the structure of $\mathcal{U}$, i.e., how the user can interact with the robotic swarm. Our definition is as follows:

Definition 1: A HSI control structure is a map

$$H : \mathcal{X} \times \mathcal{V} \rightarrow \mathcal{U}, \tag{2}$$

where $\mathcal{V}$ is some set of admissible inputs to make the corresponding robotic swarm more amenable to human control. Additionally,

$$f(x, H(x,v)) = f_H(x,v) \in \text{sparse}_{\mathcal{X}}(\mathcal{G}), \tag{3}$$

which means that the dynamics f under this map H needs to observe the sparsity structure imposed by the network topology.

This definition of a HSI control structure implies that the control input to the system is really a combination of state feedback and a restricted set of inputs from the user, which respects the constraints imposed by the network topology. Consequently, the dynamics of a multi-robot system under such a HSI control structure are

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ &= f(x(t), H(x(t), v(t)) \\ &= f_H(x(t), v(t)). \end{aligned} \tag{4}$$

Therefore, a HSI control structure is a very specific way in which the user controls the multi-robot system, i.e., interacts with the robotic swarm.

For example, suppose that a robotic swarm consists of n mobile robots positioned on a rail ($x_i(t) \in \mathbf{R}$) with single-integrator dynamics,

$$\dot{x}_i(t) = u_i(t),\ i = \{1, \ldots, n\}, \tag{5}$$

where the control input for the first $n-1$ robots is

$$u_i(t) = \sum_{j \in N(i)} (x_j(t) - x_i(t)). \tag{6}$$

$N(i)$ denotes the neighborhood of robot i, which is the set of all its immediate neighbors in the network topology derived from communication links or sensor footprints.

The control input for the n-th robot is

$$u_n(t) = v(t),\ v(t) \in \mathcal{V}, \tag{7}$$

which corresponds to the user directly controlling the position of the n-th robot. This HSI control structure is commonly referred to as a *single-leader network* (see, for example, [18]), because the user interacts with the swarm of robots by guiding a “leader” robot, while the other robots follow the leader and each other according to the consensus dynamics in (6) (see [19] for more on consensus).

If we stack all $x_i(t)$’s into a state vector $x(t) \in \mathbf{R}^n$ and all $u_i(t)$’s into an input vector $u(t) \in \mathbf{R}^n$, then the ensemble dynamics of our example system are

$$\dot{x}(t) = u(t) = -L_f x(t) + l v(t),\ l = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbf{R}^n \tag{8}$$

where L_f is a version of the graph Laplacian L as defined in [17] (and commonly used in multi-robot control) with all entries in the n-th row of L equal to zero. Consequently, the single-leader network HSI control structure is a particular structuring of the control input $u(t)$ in (8) given by the function H, such that

$$u(t) = H(x(t), v(t)) = -L_f x(t) + l v(t), \tag{9}$$

where $v(t) \in \mathbf{R}$ is the user input.

B. Achieving Geometric Configurations

Definition 2: When a multi-robot system under some HSI control structure can asymptotically converge to a state, a subset of states, or all states in a specification set $\mathcal{S}$ and stay in this set, then

$$\limsup_{t \to \infty} d(x(t), \mathcal{S}) = 0, \tag{10}$$

where,

$$d(x(t), \mathcal{S}) = \inf_{s \in \mathcal{S}} \|x(t) - s\|. \tag{11}$$

If this is true, then we say that the user can *achieve* some or all of the geometric configurations described by $\mathcal{S}$ with the robotic swarm.

The *specification set* is the set of geometric configurations that we want the user to achieve with the robotic swarm, in the sense that the user should be able to form a geometric configuration with the swarm and keep it in this configuration. For example, a specification set could be defined as

$$\mathcal{S} = \{ x \in \mathbf{R}^n \mid x_i = x_j,\ i, j = 1, \ldots, n \}, \tag{12}$$

which merely states that all components of the state should be equal, or $\mathcal{S} = \text{span}\{\mathbf{1}\}$. For example, the specification set for consensus problems with multi-robot teams is typically defined in this way. Or, we may want the user to guide a single-leader network, such that all robots in the swarm rendezvous at a specific location, i.e., $\alpha \in \mathbf{R}, \mathcal{S} = \alpha\mathbf{1}$.

III. Feasibility

We have shown that the function $H : \mathcal{X} \times \mathcal{V} \rightarrow \mathcal{U}$ encodes a particular HSI control structure into the dynamics of a multi-robot system, and that if this combination of multi-robot system and HSI control structure, $(\mathcal{X}, \mathcal{V}, f_H, x_0)$, can asymptotically converge to a specification set $\mathcal{S}$ (or a subset thereof), then we say that it is feasible for a user to use this HSI control structures to achieve some desired geometric configuration from an initial geometric configuration, x_0, with a robotic swarm. More formally,

Definition 3: It is feasible to achieve a specification set $\mathcal{S}$ under a HSI control structures defined by H if there exists $v(t)$ such that, $v(t) \in \mathcal{V},\ \forall t \geq t_0$, and

$$\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0,$$

when $\dot{x}(t) = f_H(x(t), v(t)), x(t_0) = x_0$.

We will use control Lyapunov functions (CLFs) [20] to determine this feasibility.

A. Control Lyapunov Functions

Let us denote $D \subset \mathcal{X}$ as a domain of the state space containing the quasi-static equilibrium point z for some $w \in \mathcal{V}$, such that $\dot{x}(t) = f_H(z, w) = 0$.

Definition 4: A continuously differentiable $V : D \to \mathbf{R}$ with

$$V(z) = 0 \text{ and } V(x) > 0 \text{ in } D - \{z\}$$

is a control Lyapunov function (CLF), if there exists a $v \in \mathcal{V}$ for each $x \in D$, such that

$$\dot{V}(x, v) = \nabla V(x) \cdot f_H(x, v) < 0 \text{ in } D - \{z\} \quad (13)$$

and $\dot{V}(z, w) = 0$.

If such a control Lyapunov function exists, then any trajectory starting in some compact subset $\Omega_c = \{x \in \mathcal{X} \mid V(x) \leq c,\ c > 0\} \subset D$ will approach z as $t \to \infty$.

Theorem 1: If there exists a CLF as defined in Definition 4 for the system $(\mathcal{X}, \mathcal{V}, f_H, x_0)$ and the specification set $\mathcal{S}$ is some quasi-static equilibrium point $z \in D$, then it is *feasible* to converge to z as $t \to \infty$.

Proof: By Definition 4, the existence of a CLF guarantees that if $x_0 \in \Omega_c$, then there exists $v(t) \in \mathcal{V}$, such that the multi-robot system converges to z asymptotically, i.e. $\lim_{t\to\infty} x(t) = z$. Since $z = \mathcal{S}$, it is true that

$$\left(\limsup_{t\to\infty} d(x(t), z) = 0\right) \Rightarrow \left(\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0\right),$$

which by Definition 3 confirms that for this particular multi-robot system and HSI control structure, the user can achieve the geometric configuration in the specification set $\mathcal{S}$ with the corresponding robotic swarm. ∎

Using this formulation of CLFs allows us to test the feasibility of achieving, for example, rendezvous at a specific location or a formation at a specific location with a specific rotation and assignment to positions. However, we would also like to capture formations that can translate and rotate, like cyclic pursuit, or rendezvous at any arbitrary location. Therefore, our definition of CLFs needs to include sets of quasi-static equilibrium points and limit cycles.

Suppose that $D \subset \mathcal{X}$ is a domain of the state space that all or part of the specification set $\mathcal{S}$.

Definition 5: A continuously differentiable $V : D \to \mathbf{R}$ (and locally positive definite as before) is a control Lyapunov function, if there exists $v \in \mathcal{V}$ such that

$$\dot{V}(x, v) = \nabla V(x) \cdot f_H(x, v) \leq 0 \quad (14)$$

for each x in some compact set $\Omega \subset D$, for example, Ω_c. By LaSalle's invariance principle [21], if M is the largest invariant set in $\left\{x \in \Omega \,\middle|\, \dot{V}(x, v) = 0,\ v \in \mathcal{V}\right\}$, then any trajectory starting in Ω will approach M as $t \to \infty$.

Consequently, we must ensure that our choice of CFL satisfies $M \subseteq \mathcal{S}$, otherwise we cannot show that it is feasible to achieve any of the geometric configurations in the specification set $\mathcal{S}$.

Theorem 2: If there exists a CLF as defined in Definition 5 for the system defined by $(\mathcal{X}, \mathcal{V}, f_H, x_0)$ and $M \subseteq \mathcal{S}$, then it is *feasible* to asymptotically converge to M from any $x(t_0) \in \Omega$.

Proof: The proof is similar to what was shown in the first theorem. By Definition 5, the existence of a CLF guarantees that if $x_0 \in \Omega$, then there exists $v(t) \in \mathcal{V}$, such that the multi-robot system converges to the invariant set M asymptotically. Therefore,

$$\limsup_{t\to\infty} d(x(t), M) = 0$$
$$\limsup_{t\to\infty} \inf_{m \in M} \|x(t) - m\| = 0.$$

If $M \subseteq \mathcal{S}$, then any $m \in M$ is also in $\mathcal{S}$, which means that

$$\limsup_{t\to\infty} \inf_{m \in \mathcal{S}} \|x(t) - m\| = 0$$
$$\limsup_{t\to\infty} d(x(t), \mathcal{S}) = 0,$$

which satisfies our definition of feasibility. ∎

IV. Examples

In this section, we will provide several examples of HSI control structures imposed on multi-robot systems for which we can find CLFs and show that a user can achieve a particular geometric configuration with a swarm of robots. First, we will revisit our previous example of a single-leader network, where the user guides the swarm of robots to a common rendezvous location. Then, we will revisit a broadcast control HSI control structure proposed in a previous paper [13] in the context of the approach established in this paper.

A. Rendezvous with Single-Leader Networks

Rendezvous is similar to consensus in that all robots meet up at the same location; however, let us suppose rendezvous captures the additional constraint that all robots should meet up at a particular location. The specification set that encodes this objective is $\mathcal{S} = \{x \in \mathbf{R}^n \mid x_i = \alpha,\ \alpha \in \mathbf{R},\ i = \{1, 2, \ldots, n\}\}$, or more concisely, $\mathcal{S} = \alpha\mathbf{1}$, where α is the rendezvous location.

We chose a candidate CLF [18] given by

$$V(x) = \frac{1}{2}\|x - \alpha\mathbf{1}\|^2, \quad (15)$$

which captures the disagreement between the current state of the robotic swarm and the rendezvous location. $V(x)$ is positive definite everywhere except at the desired equilibrium point $x = \alpha\mathbf{1}$ and is radially unbounded ($\|x\| \to \infty \Rightarrow V(x) \to \infty$).

Next, we need to compute $\dot{V}(x, v)$, which is defined by

$$\begin{aligned}\dot{V}(x, v) &= \nabla V(x) \cdot f_H(x, v) \\ &= (x - \alpha\mathbf{1})^T(-L_f x + l v) \\ &= -(x - \alpha\mathbf{1})^T L_f x - (\alpha - x_n)v.\end{aligned} \tag{16}$$

If $V(x)$ is a CLF, then it must be true that for each $x \in \mathbf{R}^n$, there exists $v \in \mathcal{V}, \mathcal{V} = \mathbf{R}$ such that $\dot{V}(x, v) < 0$ when $x \neq \alpha\mathbf{1}$ and $\dot{V}(x, v) = 0$ when $x = \alpha\mathbf{1}$. In Equation (16), we can see that even if $-(x - \alpha\mathbf{1})^T L_f x$ is positive, we can always chose $v \in \mathbf{R}$, such that $\dot{V}(x, v) < 0$. Therefore, $V(x)$ is a CLF that guarantees that there exists $v(t) \in \mathcal{V}$, such that the user can guide the swarm of robots from $x(t_0) \in \mathbf{R}^n$ to $x = \alpha\mathbf{1}$ as $t \to \infty$.

Figure 1 is a demonstration of rendezvous with a single-leader network. To aid in the visualization, the above candidate CLF and the single-leader network system have been extended to $\mathbf{R}^2$. Since the robots are single integrators, the dynamics along each dimension, x and y, are decoupled. The user applies a constant control input $v \in \mathbf{R}^2$ to guide the leader robot to the origin.

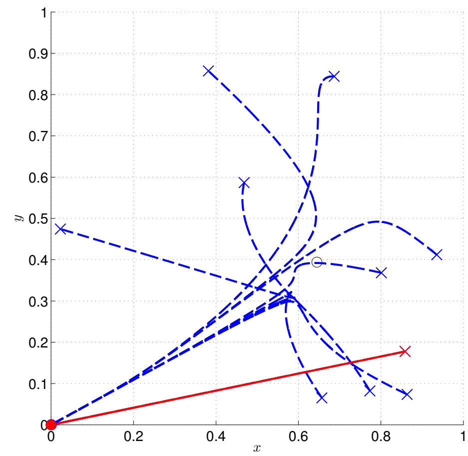
Figure 1a shows the trajectories of ten robots that are organized over an arbitrary connected, static network topology. The solid, red trajectory belongs to the leader robot that is controlled by the user, while the dashed, blue trajectories belong to all other robots in the swarm. $\times$ denotes their starting location, while $\circ$ denotes the rendezvous location if $v(t) = 0, \forall t$, and $\bullet$ illustrates the robots' actual final position.

We see in Figure 1a that by guiding the leader robot to the origin, the user can change the rendezvous location of the swarm of robots to the origin. Figure 1b shows that the CLF $V(x, y)$ is positive, but "energy" dissipates as robots converge on the rendezvous location, while Figure 1c shows that $\dot{V}(x, y, v)$ remains negative during the interaction. Consequently, it is feasible for the user to use this HSI control structure to chose the rendezvous location of a swarm of robots. Similarly, this combination of multi-robot system and HSI control structure would be effective in setting the flocking direction if the state x were the orientation θ of each robot, rather than its position.

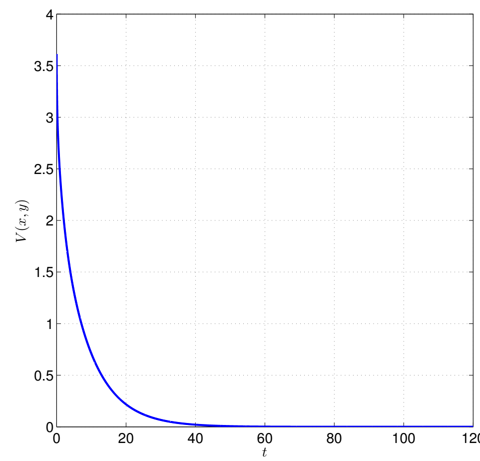
B. Separation with a Broadcast Signal

In a previous paper [13], we showed that it is possible to use a broadcast signal to separate a swarm of heterogeneous robots, but let us revisit this problem in the context of this paper. Suppose that broadcasting an input signal is a HSI for a swarm of two types of robots, and we would like to know if it is feasible to separate the two types of robots by a distance of Δ by broadcasting an input signal. Each robot $i \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, n\}$ is the set of all robots, belongs to one of two classes in $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2\}$. A class membership function $\pi : \mathcal{N} \to \mathcal{C}$ maps each robot i into one of the two classes. The dynamics of each robot are

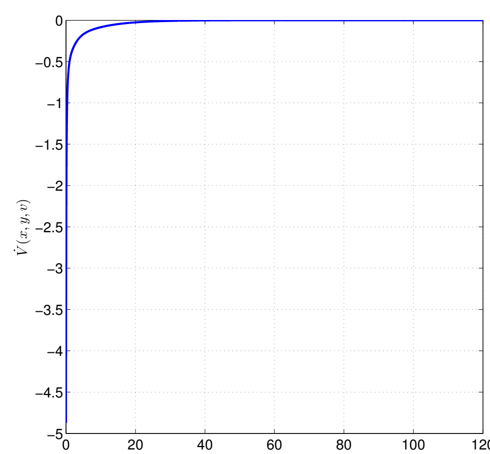
$$\begin{aligned}\dot{x}_i(t) &= u_i(t) \\ &= \gamma_{\pi(i)}\left(\sum\nolimits_{j \in N(i)} (x_j(t) - x_i(t)) + v(t)\right),\end{aligned} \tag{17}$$



(a) Trajectories in $\mathbf{R}^2$



(b) $V(x, y)$



(c) $\dot{V}(x, y, v)$

Fig. 1: A user is guiding a swarm of 10 robots to rendezvous at the origin by interacting with the leader robot.

where $\gamma_{\pi(i)}$ is the "weight" of robot i as a function of its class, $j \in N(i)$ if robot i and robot j are separated by a distance less than Δ and $v(t) \in \mathbf{R}_+ \cup \{0\}$ is the broadcast input signal. If we use the initial conditions $x_i(t_0) = x_j(t_0),\ \forall i,j \in \mathcal{C}_1$ and $x_i(t_0) = x_j(t_0),\ \forall i,j \in \mathcal{C}_2$, which corresponds all robots of same type starting together, then we can simplify the dynamics (as shown in the paper) to

$$\begin{aligned} \dot{\chi}_1 &= -\gamma_1(N_2(\chi_1 - \chi_2) - v) = f_{H,1}(\chi, v) \\ \dot{\chi}_2 &= \gamma_2(N_1(\chi_1 - \chi_2) + v) = f_{H,2}(\chi, v), \end{aligned} \tag{18}$$

where $\chi_i \in \mathbf{R}$ represents the shared position of all robots of type $\mathcal{C}_i$, and $\chi = [\chi_1, \chi_2]^T$.

A specification set that encodes a separation distance of Δ between the two types of robots is $\mathcal{S} = \{x \in \mathbf{R} \mid \|x_i - x_j\| = \Delta,\ \forall i,j,\ i \in \mathcal{C}_1, j \in \mathcal{C}_2\}$. Consequently, we pick a candidate CLF [22]

$$V(\chi) = \frac{1}{4}\left(\|\chi_1 - \chi_2\|^2 - \Delta^2\right)^2, \tag{19}$$

which is positive definite everywhere except at the quasi-static equilibrium points, where $\|\chi_1 - \chi_2\| = \Delta$. Next, we need to show that

$$\dot{V}(\chi, v) = \begin{bmatrix} \frac{\partial V}{\partial \chi_1} \\ \frac{\partial V}{\partial \chi_2} \end{bmatrix}^T \begin{bmatrix} f_{H,1}(\chi, v) \\ f_{H,2}(\chi, v) \end{bmatrix} < 0 \tag{20}$$

Suppose that in this example the domain is $D = \left\{\chi \in \mathbf{R}^2 \,\middle|\, 0 \le \|\chi_1 - \chi_2\| \le \Delta, \chi_1 \le \chi_2\right\}$, that all robots of the same type start at the same location $[\chi_1(t_0), \chi_2(t_0)]^T \in D$, that the "weights" of the the two types of robots are ordered $0 < \gamma_1 < \gamma_2$, and that

$$\begin{aligned} \dot{V}(\chi, v) = &-(\gamma_1 N_2 + \gamma_2 N_1)(\chi_1 - \chi_2)^2(\|\chi_1 - \chi_2\|^2 - \Delta^2) \\ &- (\gamma_2 - \gamma_1)(\chi_1 - \chi_2)(\|\chi_1 - \chi_2\|^2 - \Delta^2)v, \end{aligned} \tag{21}$$

then for every $\chi \in D$,

$$v \ge \frac{\gamma_1 N_2 + \gamma_2 N_1}{\gamma_2 - \gamma_1}(\chi_2 - \chi_1) \tag{22}$$

will ensure that $\dot{V}(\chi, v) \le 0$, where $\dot{V}(\chi, v) = 0$ only whenever $\|\chi_1 - \chi_2\| = \Delta$ or $\chi_1 = \chi_2$. By LaSalle's invariance principle, this system will converge to the largest invariant set M in $\left\{\chi \in \Omega \,\middle|\, \dot{V}(\chi, v) = 0\right\}$ as $t \to \infty$, where Ω is the compact subset

$$\left\{\chi \in \mathbf{R}^2 \,\middle|\, V(\chi) \le \frac{1}{4}\Delta^4 - \epsilon, \epsilon > 0\right\} \subset D. \tag{23}$$

The largest invariant set M is

$$\left\{\chi \in \Omega \,\middle|\, \dot{V}(\chi, v) = 0,\ \|\chi_1 - \chi_2\| = \Delta, \right. \\ \left. v = \frac{\gamma_1 N_2 + \gamma_2 N_1}{\gamma_2 - \gamma_1}\Delta\right\}, \tag{24}$$

because for this particular $v \in \mathcal{V}$, $\dot{\chi}_2 - \dot{\chi}_1 = 0$, such that $\|\chi_1 - \chi_2\| = \Delta$ will always hold and thus $\dot{V}(\chi, v) = 0$ and $V(\chi) = 0$. $M \subseteq S$; therefore, it is feasible for the user to use this broadcast control HSI control structure to separate the two types of robots by a distance Δ if the system starts at $\chi(t_0)$ in Ω.

Figure 2 illustrates separation of a swarm of ten robots of $\mathcal{C}_1$ and five robots of $\mathcal{C}_2$ by a distance $\Delta = 0.4$. The user is applying a constant, positive broadcast signal

$$v = \frac{(\gamma_1 N_2 + \gamma_1 N_1)}{\gamma_2 - \gamma_1}\Delta,$$

analogous to using a wind tunnel to move robots (on a rail) with mass inversely proportional to γ_i. Figure 2a indicates the starting location of the $\mathcal{C}_1$ (blue) robots and $\mathcal{C}_2$ (red) robots by $\times$ and their final positions by $\bullet$. Initially, the separation between the two types of robots is less than Δ, but eventually, their separation equals Δ. This plot is confirmed by Figure 2b which shows that the CLF $V(\chi)$ is positive, but "energy" dissipates as the desired separation distance is achieved, while Figure 2c shows that $\dot{V}(\chi, v)$ remains negative during the interaction. Consequently, it is feasible for the user to use a strong enough broadcast signal to separate the two types of robots in the example by a distance of Δ.

C. Remarks

The two examples in the section show that a CLF approach is useful to show convergence of the HSI control structured multi-robot system to a specification set. In fact, our definition of a HSI control structure allows us to use CLFs directly, and the CLFs themselves can typically be constructed by inspecting the specification set.

V. Conclusion

In this paper, we have provided a precise definition for what it means to impose a human-swarm interaction (HSI) structure on a multi-robot system and to achieve a geometric configuration with a swarm of robots. With these two definitions in hand, we defined that feasibility in this context implies that a user can successfully guide a swarm of robots into some desired geometric configuration. We have also shown that finding a control Lyapunov function (CLF) implies feasibility, such that CLFs can be used to show that a particular combination of multi-robot system and HSI control structure is appropriate for achieving a particular geometric configuration or set of configurations as demonstrated by the included examples.

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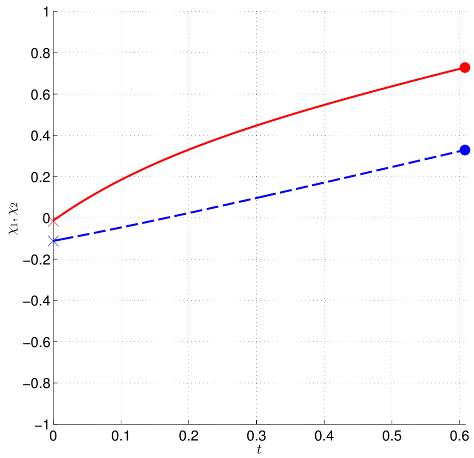
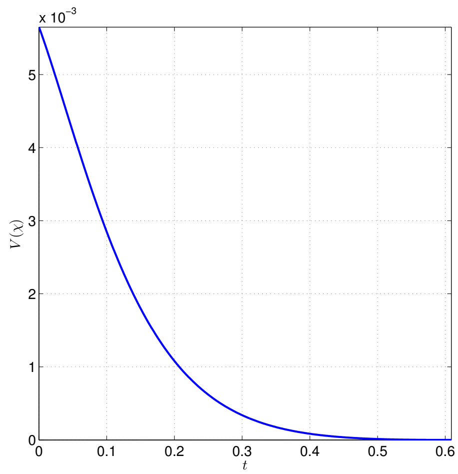
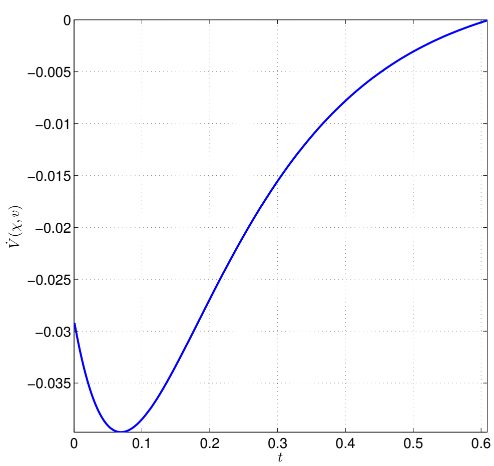


(a) Trajectories in $\mathbf{R}$



(b) $V(\chi)$



(c) $\dot{V}(\chi, v)$

Fig. 2: A user is separating a swarm of ten robots of $\mathcal{C}_1$ ($\gamma_1 = 0.1$) and five robots of $\mathcal{C}_2$ ($\gamma_2 = 0.5$) by a distance $\Delta = 0.4$ with a broadcast signal v.

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