Distributed Scheduling for Air Traffic Throughput Maximization During the Terminal Phase of Flight

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***Abstract*— FAA's NextGen program aims at increasing the capacity of the national airspace, while ensuring the safety of aircraft. This paper provides a distributed merging and spacing algorithm that maximizes the throughput at the terminal phase of flight using the information provided through the ADS-B framework. Using dual decomposition, aircraft negotiate with each other and reach an agreement on optimal merging times, with respect to an associated cost, that ensures proper inter-aircraft spacing. We provide a feasibility analysis that gives sufficient conditions to guarantee that proper spacing is achievable and derive maximum throughput controllers based on the air traffic characteristics of the merging flight paths.**

I. Introduction

Next Generation Air Transportation System (NextGen) is the FAA's vision to address the impact of air traffic growth by increasing the National Airspace System's capacity and efficiency, while improving the safety and reducing environmental impacts [12]. It is expected that under NextGen, the so called "performance based navigation" will allow aircraft to fly negotiated trajectories, changing the air traffic controller's tasks from clearance-based control to trajectory management. One of NextGen's goals is to explore improvements in terminal area operations, namely automatic merging and spacing of the incoming traffic paths, in order to increase the traffic capacity of the terminal phase and save fuel by reducing extraneous flight maneuvers such as holding patterns. Current systems completely rely on air traffic controllers to safely route aircraft. As a result, conflicts in merging routes are often identified too late and merging aircraft are asked to hold or redirect to wait for an opening, thus creating an excessive separation between the aircraft.

Safe and efficient merging and spacing in support of the FAA's NextGen is an active area of research and is the subject of a few large scale tests of systems developed based on the Automatic Dependent Surveillance-Broadcast (ADS-B) information. ADS-B, a crucial component of NextGen, relays highly accurate traffic information between equipped aircraft and a network of satellites and ground stations [13].

SafeRoute, which is implemented on UPS aircraft, is an example of a centralized and large scale ADS-B based technology. Air traffic controllers instruct the pilot to follow a particular aircraft, while an on-board system actively computes and displays a recommended aircraft velocity such that a safe distance is maintained with the leading aircraft and safe merging is guaranteed at the merge points [14]. In Point Merge, another centralized merging and spacing solution, aircraft approaching the terminal area achieve the desired separation by flying on one of the vertically spaced sequencing legs [2]. National Aeronautics and Space Administration (NASA) is also actively involved in air traffic management research [1]. NASA's Aviation Systems Division is focusing on hi-flow airports [10], high density en route operations, and automated separation assurance by using trajectory based tactical air traffic management [4].

A central theme in air traffic management research is the problem of conflict resolution amongst aircraft. Tomlin *et al.* use a game theoretic approach for conflict resolution of noncooperative aircraft [9]. Mao *et al.* provide sufficient conditions for stable conflict avoidance of two intersecting aircraft flows [3]. Rahmani *et al.* propose a decentralized deconfliction algorithm based on artificial potential functions [6]. Wollkind *et al.* use the bargaining technique of Monotonic Concession Protocol to detect and pseudo-optimally resolve conflicts [11]. Roy and Tomlin suggest a slot-based model where en-route traffic select an available slot and then maintain its positioning in the traffic flow, hence guaranteeing safety-of-flight [8].

In this paper we focus on merging and spacing for the terminal phase of flight. Spacing aircraft as closely as possible, while avoiding holding patterns, would increase the landing capacity of the airport, which is of great importance to the NextGen initiative. We stay true to safe-operation practices (flying between predefined waypoints) and utilize the inter-aircraft ADS-B protocol with the addition of a few extra negotiation parameters. In our suggested algorithm, aircraft use dual decomposition (e.g. [7]) to negotiate merging times and maintain proper spacing by minimizing an associated pairwise cost, while reaching agreement with the other parties on the arrival times. We provide a feasibility analysis that gives sufficient conditions for the attainability of safe separations based on the traffic characteristics of the merging flight paths. We demonstrate the viability of our algorithm and conditions using simulations.

II. Problem Statement

The goal of this work is to devise a merging and spacing procedure that increases air traffic throughput at the terminal phase of flight, while guaranteeing the safety of each aircraft. Specifically, we will address the scenario where two tracks of aircraft must merge into one track with a predetermined velocity and separation distance representing the terminal phase of flight. Furthermore, this two-track merging scheme

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can be generalized to binary tree structures such that any arbitrary number of tracks can be merged pairwise. This generalization will be further discussed in Section VI.

Referring to Figure 1, the goal is to merge the two legs of air traffic onto a terminal leg, while maintaining a spacing of at least Δ_{III} between all aircraft. The merging and spacing procedure is divided into three phases. In Phase I, the *Negotiation Phase*, aircraft approach waypoints WP1 and WP2 with a constant velocity V_{I}, spaced at least Δ_I apart on the same leg, where $\Delta_I \geq \Delta_{III}$. During this approach, aircraft on opposing legs will conduct pairwise negotiations to determine arrival times at WP3 and flight plans over Phase II so as to maintain a safe separation with other aircraft. In Phase II, the *Action Phase*, each aircraft executes the negotiated flight plan to travel from WP1/WP2 to WP3. As seen in Figure 1, without loss of generality we assume both WP1 and WP2 are a distance d from WP3 at an angle θ apart and we consider the two dimensional problem where tracks refer to the ground track of the aircraft.

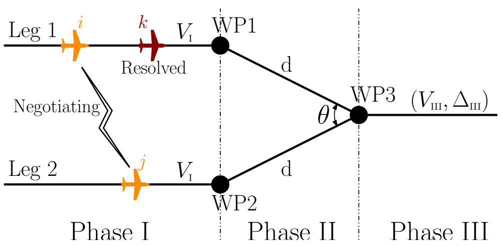


Fig. 1. Top view of two-track merging at the terminal phase of flight.

The flight plan constitutes a velocity $V_{\text{II}} \in [V_{\text{min}}, V_{\text{max}}]$ and a path deviation $h \in [0, h_{\text{max}}]$ from the straight line path between WP1/WP2 and WP3. As in [2] and [14], changing V_{II} and h modifies the arrival time at WP3, which we will use to space merging aircraft. This is illustrated in Figure 2.

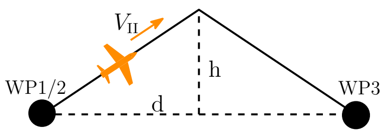


Fig. 2. Overhead view of velocity and path deviations during Phase II.

Path deviations occur in the direction opposite to the other leg of flight. Since the deviation h simply elongates the path flown by an aircraft, in practice it can be implemented as a constant curvature arc with curvature κ, given by

$$h^2 = \frac{1}{\kappa^2}\left(\sin^{-1}\left(\frac{d\kappa}{2}\right)\right)^2 - \frac{d^2}{4}, \text{ with} \tag{1}$$

$$\kappa_{\text{max}} \leq \frac{2}{d} \quad \text{requiring that} \quad h_{\text{max}} \leq \frac{d}{4}\sqrt{\pi^2 - 4}. \tag{2}$$

In Phase III, the *Terminal Approach Phase*, aircraft approach the terminal with constant velocity V_{III} and must have a minimum separation distance of Δ_{III} as shown in Figure 3.

Throughout the three phases, all aircraft have access to the following global information as labeled in Figures 1 - 3:

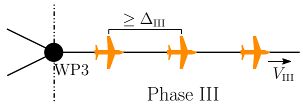


Fig. 3. Velocity and aircraft separation during Phase III.

V_{I} and Δ_I are the speed and minimum spacing that aircraft fly at in Phase I, d is the minimum distance to fly at Phase II, V_{min} and V_{max} are respectively the minimum and maximum velocities that aircraft can fly during Phase II, h_{max} is the maximum allowable path deviation in Phase II, and V_{III} and Δ_{III} are respectively the constant velocity of all aircraft and their minimum required separation during Phase III.

A. Negotiation and Local Information

Recall during the negotiation phase (Phase I), aircraft negotiate for arrival times at WP3 with aircraft on the opposite leg. After a pair has negotiated, the aircraft with the earliest arrival time will be assigned that arrival time and hence, be labeled a resolved aircraft. The other aircraft, still unresolved, will then conduct pairwise negotiations with the next unresolved aircraft from the opposite leg to determine an arrival time at WP3 after the most recently resolved aircraft's arrival time such that it is at least a minimum separation distance from this resolved aircraft. This pairwise negotiation is continued until all aircraft are assigned arrival times and will be discussed in greater detail in Section IV.

Let Aircraft i be the next unresolved aircraft on Leg 1 and Aircraft j be the next unresolved aircraft on the Leg 2, while Aircraft k is the last resolved aircraft. The following information is known to aircraft i: $t_i^{\text{WP1/2}}$ is Aircraft i's expected arrival time at WP1/WP2, $t_{i,0}^{\text{WP3}}$ is Aircraft i's Estimated Time of Arrival (ETA) at WP3 if choosing $V_{\text{II}} = V_{\text{I}}$ and $h = 0$ in Phase II, τ_i is the set of Aircraft i's *feasible* arrival times at WP3 while maintaining Δ_{III} separation from Aircraft k in Phase III, and t_k^{WP3} is Aircraft k's resolved time of arrival at WP3. Similarly, Aircraft j will know $t_j^{\text{WP1/2}}$, $t_{j,0}^{\text{WP3}}$, τ_j, and t_k^{WP3}. Aircraft i and j will also need to communicate additional information to each other throughout the negotiation process. These additional parameters are explained in detail in Section V as part of the proposed distributed solution.

B. Relating Velocity, Path Deviation, and Arrival Times

A fixed arrival time at WP3 for any Aircraft i on either leg leads to a corresponding set of possible (V_{II}, h) pairs that can be chosen for Phase II to meet the arrival time. Vice versa, bounds on V_{II} and h limit which arrival times at WP3 can be achieved. The fastest that Aircraft i can plan to arrive at WP3 is when it flies in a straight line using the maximum velocity, corresponding to $V_{\text{II}} = V_{\text{max}}$ and $h = 0$. Thus, T_{min}, the soonest that Aircraft i can reach WP3, is given by

$$T_{\text{min}} = t_i^{\text{WP1/2}} + \frac{d}{V_{\text{max}}}. \tag{3}$$

The slowest that Aircraft i can reach WP3 is by flying at the minimum velocity with the greatest path deviation, corresponding to $V_{\text{II}} = V_{\text{min}}$ and $h = h_{\text{max}}$. As a consequence,

T_{max}, the latest that Aircraft i can reach WP3, is

$$T_{\text{max}} = t_i^{\text{WP1/2}} + \frac{2}{V_{\text{min}}}\sqrt{h_{\text{max}}^2 + \frac{d^2}{4}}. \tag{4}$$

The set of *reachable arrival times* at WP3 for Aircraft i arriving at WP1/WP2 at time $t_i^{\text{WP1/2}}$ is therefore given by $R_i = [T_{\text{min}}, T_{\text{max}}]$. Suppose Aircraft k is the most recently resolved aircraft with arrival time t_k^{WP3} at WP3, then let the set of *feasible arrival times* at WP3 for Aircraft i be $\tau_i = R_i \cap [t_k^{\text{WP3}} + \frac{\Delta_{\text{III}}}{V_{\text{III}}}, \infty)$.

Assuming that WP1/WP2 is a distance d from WP3 and it is desired to reach WP3 at time $t_i^{\text{WP3}} = t_i^{\text{WP1/2}} + T$, then if $t_i^{\text{WP3}} \in R_i$, it is possible to do so with any choice of $(V_{\text{II}}, h) \in \mathbb{S}(d,T)$ where

$$\begin{aligned}\mathbb{S}(d,T) &= \{(V,h) : V \in [V_{\text{min}}, V_{\text{max}}], h \in [0, h_{\text{max}}],\\ &\qquad V = \frac{2}{T}\sqrt{h^2 + \frac{d^2}{4}}\}.\end{aligned}$$

The problem is to develop a distributed negotiation procedure, along with feasibility conditions, to determine terminal phase arrival times and flight plan parameters of each aircraft on Phase II that maintain a minimum inter-aircraft separation and minimize pairwise aircraft costs.

III. Feasibility

Before discussing the negotiation aspect of this framework, we will give sufficient conditions on initial aircraft spacings approaching WP1/WP2 to guarantee that arrival times exist for all aircraft that allow the minimum separation distance Δ_{III} to be maintained at all times. We will first show that conditions exist on the interval length and intersections of the reachable time sets R_i, for all Aircraft i, such that aircraft on opposite legs performing pairwise negotiation can agree on reachable arrival times at WP3 that guarantees a minimum separation between each other and also the previously resolved aircraft when in Phase III. This leads to conditions on the allowable choices of V_{min}, V_{max}, and h_{max} on Phase II, which in turn gives conditions on minimum aircraft spacing for each leg during Phase I. Let us define length of a reachable time set $R_i = [a_i, b_i]$ as $|R_i| = |b_i - a_i|$.

Proposition 1: If R_i, R_j, and R_{i+1} are such that $|R_x| \geq 2\frac{\Delta_{\text{III}}}{V_{\text{III}}}$, for $x \in \{i, j, i+1\}$, and $b_i \leq a_{i+1}$, then for all $c_i \in R_i$, there exists a $c_j \in R_j$ and $c_{i+1} \in R_{i+1}$ such that $|c_i - c_j| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$, $|c_i - c_{i+1}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$, and $|c_{i+1} - c_j| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$.

Proof: Choose

$$\begin{array}{llll} c_j = a_j & \text{and} & c_{i+1} = b_{i+1}, & \text{if } a_j \leq c_i - \frac{\Delta_{\text{III}}}{V_{\text{III}}} \\ c_j = c_i + \frac{\Delta_{\text{III}}}{V_{\text{III}}} & \text{and} & c_{i+1} = b_{i+1}, & \text{if } c_i + \frac{\Delta_{\text{III}}}{V_{\text{III}}} \in R_j \\ c_j = a_j & \text{and} & c_{i+1} = b_{i+1}, & \text{if } a_j \leq a_{i+1} \\ c_j = b_j & \text{and} & c_{i+1} = a_{i+1}, & \text{otherwise.} \end{array}$$

■

Suppose Aircraft i and $i+1$ are on one leg and Aircraft j is on the opposite leg. The above proposition says that as long as certain conditions on the feasible time sets are met, any choice of arrival time at WP3 by Aircraft i has corresponding choices of arrival times at WP3 for Aircraft $i+1$ and j such that the three maintain a minimum spatial separation of Δ_{III} from each other in Phase III. This result can be used to show that the proposed pairwise negotiation algorithm is guaranteed to result in arrival times for each aircraft that satisfy the minimum separation in Phase III.

Theorem 3.1: If the following conditions are satisfied for every Aircraft i and $i+1$ following behind it on the same leg:

R1 $|R_i| \geq 2\frac{\Delta_{\text{III}}}{V_{\text{III}}}$, where $|R_i| = \frac{2}{V_{\text{min}}}\sqrt{h_{\text{max}}^2 + \frac{d^2}{4}} - \frac{d}{V_{\text{max}}}$,
R2 $b_i \leq a_{i+1}$, for $R_i = [a_i, b_i]$ and $R_{i+1} = [a_{i+1}, b_{i+1}]$,

then the proposed pairwise negotiation scheme will allow all aircraft to agree on arrival times at WP3 that guarantee a minimum inter-aircraft separation of Δ_{III} in Phase III.

Proof: Suppose some Aircraft $i+1$ and j are engaging in pairwise negotiation, with a previously resolved Aircraft i (if one exists). Proposition 1 guarantees that independent of what arrival time t_i^{WP3} Aircraft i chose, there is a set of $(t_{i+1}^{\text{WP3}}, t_j^{\text{WP3}})$ pairs that allow all three aircraft to maintain a minimum separation in Phase III. Pairwise negotiation chooses a pair of arrival times for Aircraft $i+1$ and j within that set that occur after t_i^{WP3}. Without loss of generality, assume that $t_j^{\text{WP3}} < t_{i+1}^{\text{WP3}}$. Then Aircraft j becomes the next resolved aircraft where t_j^{WP3} is chosen such that Aircraft j is guaranteed a minimum separation from all other previously resolved aircraft in Phase III. This process then continues inductively where Aircraft $i+1$ and $j+1$ must perform pairwise negotiation to determine a $(t_{i+1}^{\text{WP3}}, t_{j+1}^{\text{WP3}})$ pair until all aircraft have negotiated arrival times that guarantee the minimum separation requirement is met in Phase III. separation requirement is met in Phase III. ■

Condition R2 requires aircraft on the same leg in Phase I to have reachable arrival time sets that overlap at most only at the boundary of the intervals. This condition can be transformed to equivalent conditions on spacing for incoming aircraft on Legs 1 and 2.

Theorem 3.2: Condition R2 mentioned in Theorem 3.1 is equivalent to the distance between any two aircraft on the same leg during Phase I, Δ_I, being greater than or equal to $V_{\text{I}}\left(\frac{2}{V_{\text{min}}}\sqrt{h_{\text{max}}^2 + \frac{d^2}{4}} - \frac{d}{V_{\text{max}}}\right)$.

Proof: Assume at time t_0, Aircraft i is a distance $x_{\text{WP1/2}} - x_i$ from WP1/WP2 and Aircraft $i+1$ is following behind at a distance $x_{\text{WP1/2}} - x_{i+1}$ from WP1/WP2. Therefore, the arrival times at WP1/WP2 are

$$\begin{aligned} t_i^{\text{WP1/2}} &= t_0 + \frac{x_{\text{WP1/2}} - x_i}{V_{\text{I}}} \\ t_{i+1}^{\text{WP1/2}} &= t_0 + \frac{x_{\text{WP1/2}} - x_{i+1}}{V_{\text{I}}}. \end{aligned}$$

From Equations (3) and (4), we get that

$$b_i = t_i^{\text{WP1/2}} + \frac{2}{V_{\text{min}}}\sqrt{h_{\text{max}}^2 + \frac{d^2}{4}} \quad \text{and} \quad a_{i+1} = t_{i+1}^{\text{WP1/2}} + \frac{d}{V_{\text{max}}}.$$

Substituting into Condition R2 results in

$$x_i - x_{i+1} \geq V_{\mathrm{I}} \left(\frac{2}{V_{\mathrm{min}}} \sqrt{h_{\mathrm{max}}^2 + \frac{d^2}{4}} - \frac{d}{V_{\mathrm{max}}} \right).$$

■

Sufficient conditions also exist that ensure aircraft on Phase II cannot violate the minimum separation requirement.

Theorem 3.3: Assuming conditions R1 and R2 are met, a sufficient condition on θ, the angle between Leg 1 and Leg 2 in Phase II, which guarantees that aircraft on Phase II maintain the required minimum separation is given by

$$V_{\mathrm{min}} \geq V_{\mathrm{III}} \quad \text{and } \pi \geq \theta \geq \max\{\theta', \theta^*\},$$

where θ' and θ^* are given by

$$\begin{aligned} \alpha_1 \cos^2 \theta^* + \alpha_2 \cos \theta^* + \alpha_3 &\geq 0 \\ d \sin(\theta'/2) &\geq \frac{\Delta_{\mathrm{III}}}{2}, \end{aligned}$$

with α_1, α_2, α_3 defined in (6), (7), (8) respectively.

Proof: Assume one aircraft reaches WP3 first, the other aircraft must be at least $\frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}} V_{\mathrm{min}}$ behind, requiring $V_{\mathrm{min}} \geq V_{\mathrm{III}}$ to maintain our minimum distance requirement. Tracing aircraft trajectories backward in time by defining $s = -t$, one can see that minimum distance between aircraft occurs when they do not deviate from straight path while the first aircraft travels at V_{max} and the other one travels at V_{min} in Phase II. As shown

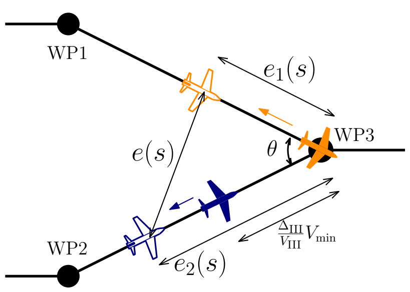


Fig. 4. Diagram of Phase II used for proof of Theorem 3.3

in Figure 4, distances left to WP3 are $e_1(s) = V_{\mathrm{max}} s$ and $e_2(s) = V_{\mathrm{min}}(s + \frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}})$, while the distance between the aircraft $e(s)$ can be computed from the law of cosines. Solving for the time s^* when the minimum distance is achieved and in return making sure that $e(s^*) \geq \Delta_{\mathrm{III}}$ will give a condition on the minimum allowed inter-leg angle θ^*, such that

$$\alpha_1 \cos^2 \theta^* + \alpha_2 \cos \theta^* + \alpha_3 \geq 0 \tag{5}$$

with

$$\alpha_1 = -V_{\mathrm{max}}^2 V_{\mathrm{min}}^2 \frac{\Delta_{\mathrm{III}}^2}{V_{\mathrm{III}}^2}, \tag{6}$$

$$\alpha_2 = 2 V_{\mathrm{max}} V_{\mathrm{min}} \Delta_{\mathrm{III}}^2, \tag{7}$$

$$\alpha_3 = \Delta_{\mathrm{III}}^2 \left(\frac{V_{\mathrm{max}}^2 V_{\mathrm{min}}^2}{V_{\mathrm{III}}^2} - V_{\mathrm{max}}^2 - V_{\mathrm{min}}^2 \right). \tag{8}$$

In addition, Legs 1 and 2 must be at least Δ_{III} apart, meaning that $d \sin(\theta'/2) \geq \frac{\Delta_{\mathrm{III}}}{2}$. Hence, angle $\theta \geq \max\{\theta', \theta^*\}$. ■

In summary, the following conditions are sufficient to guarantee complete feasibility:

C1 $\Delta_I \geq V_{\mathrm{I}} |R_i| \geq 2 \frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}}$.
C2 $V_{\mathrm{min}} \geq V_{\mathrm{III}}$.
C3 $\pi \geq \theta \geq \max\{\theta', \theta^*\}$.

Now that we can guarantee when pairwise negotiations will result in arrival times such that every aircraft maintains a minimum separation distance throughout all three phases, we discuss how to determine these arrival times such that each aircraft can attempt to minimize its fuel consumption and deviation from its initial time of arrival.

IV. Pairwise Optimization Problem

The pairwise negotiations for arrival times at WP3 will minimize a pairwise cost for both aircraft, consisting of the sum of Maneuvering and Delay costs for each aircraft and a joint Separation Cost. For an Aircraft i moving into Phase II, its Estimated Time of Arrival (ETA) at WP3, which we call $t_{i,0}^{\mathrm{WP3}}$, is the time it takes to fly a straight line from WP1/WP2 to WP3 using the same velocity as in Phase I. Any additional deviation in the path or change in velocity corresponds to an increase in fuel consumption and is penalized.

Given an arrival time at WP3, the associated Maneuvering and Arrival Delay cost for an Aircraft i is given by

$$J_i(t_i^{\mathrm{WP3}}) = \min_{(V_{\mathrm{II}}, h)} (k_{1,i} h^2 + k_{2,i}(V_{\mathrm{II}} - V_{\mathrm{I}})^2) + k_{3,i}(t_i^{\mathrm{WP3}} - t_{i,0}^{\mathrm{WP3}})^2,$$

such that $(V_{\mathrm{II}}, h) \in \mathbb{S}(d, t_i^{\mathrm{WP3}} - t_i^{\mathrm{WP1/2}})$. The weights $k_{1,i}$, $k_{2,i}$, $k_{3,i} \in \mathbb{R}_+$ may be chosen differently for each aircraft. The minimum term chooses the best V_{II} and h pair to arrive at WP3 at time t_i^{WP3}, which minimizes the penalty on deviations in path and velocity.

The Separation Cost penalizes a proposed pair of arrival times if they lead to aircraft having a separation greater than Δ_{III} in Phase III. We will refer to this cost as being a joint cost since it relies on both t_i^{WP3} and t_j^{WP3}.

$$J_{ij}\left(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}\right) = \gamma_{ij} (|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| - \frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}})^2, \ \gamma_{ij} > 0.$$

There are two constraints on allowable choices of WP3 arrival times. The first is that they must be feasible for the aircraft and so we require $t_i^{\mathrm{WP3}} \in \tau_i$, $t_j^{\mathrm{WP3}} \in \tau_j$. The negotiated arrival times must ensure that a minimum separation of Δ_{III} is achieved in Phase III, which is accomplished by the constraint $|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| \geq \frac{\Delta_{\mathrm{III}}}{V_{\mathrm{III}}}$.

Letting each aircraft i and j be responsible for its own Maneuvering and Arrival Delay cost as well as half of the Separation Cost, the individual costs are

$$U_i(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_i(t_i^{\mathrm{WP3}}) + \frac{1}{2} J_{ij}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}),$$

$$U_j(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_j(t_j^{\mathrm{WP3}}) + \frac{1}{2} J_{ij}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}).$$

These costs can be combined to create the pairwise cost, and hence the following pairwise optimization problem:

Problem 4.1:

$$\min_{t_i^{\text{WP3}} \in \tau_i, t_j^{\text{WP3}} \in \tau_j} (U_i(t_i^{\text{WP3}}, t_j^{\text{WP3}}) + U_j(t_i^{\text{WP3}}, t_j^{\text{WP3}})),$$

such that $|t_j^{\text{WP3}} - t_i^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$.

We use distributed pairwise negotiation to solve this problem.

V. Distributed Solution

Dual decomposition is proposed to reach agreement (as seen in [7]) among a pair of aircraft on which arrival times at WP3 minimizes the pairwise cost between them, while satisfying the separation constraint. First, we introduce the notion of Aircraft i's estimate of what Aircraft j's arrival time should be, given by t_{ij}^{WP3}. The dual optimization problem to the primal Problem 4.1 is now written as

$$\max_{\lambda_1, \lambda_2} \min_{t^{\text{WP3}}} U_i(t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}) + U_j(t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}})$$
$$+\lambda_1(t_{ii}^{\text{WP3}} - t_{ji}^{\text{WP3}}) + \lambda_2(t_{jj}^{\text{WP3}} - t_{ij}^{\text{WP3}}),$$

such that $t_{ii}^{\text{WP3}}, t_{ji}^{\text{WP3}} \in \tau_i$ and $t_{jj}^{\text{WP3}}, t_{ij}^{\text{WP3}} \in \tau_j$, with constraint:

$$|t_{ij}^{\text{WP3}} - t_{ii}^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}} \text{ and } |t_{jj}^{\text{WP3}} - t_{ji}^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}.$$

The primal problem has a bounded non-convex cost, meaning the dual problem has weak duality and so its solution cannot be guaranteed to result in a global minimum. We therefore seek arrival times that achieve local minima for the pairwise constrained optimization problem.

A. Dual Decomposition Solution

In [5] and [7], methods are presented for decomposing this dual optimization problem into subproblems that each aircraft can solve. As a result, the negotiation is broken down into steps. First, each Aircraft solves a minimization problem based on its own arrival time estimates and given λ values. Then, arrival time estimates are communicated between the aircraft and each aircraft takes a gradient step to update its value of λ. Finally, the updated λ values are communicated to the other aircraft and the cycle begins again. These steps repeat until the other aircraft's arrival time estimates agree with the aircraft's own calculated arrival time. The following describes the subproblems of the dual problem that are solved at each of these steps. Aircraft i solves

$$\min_{t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}} U_i(t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}) + \lambda_1 t_{ii}^{\text{WP3}} - \lambda_2 t_{ij}^{\text{WP3}} \tag{9}$$

such that $t_{ii}^{\text{WP3}} \in \tau_i$, $t_{ij}^{\text{WP3}} \in \tau_j$, and $|t_{ij}^{\text{WP3}} - t_{ii}^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$.

Aircraft j solves

$$\min_{t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}}} U_j(t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}}) - \lambda_1 t_{ji}^{\text{WP3}} + \lambda_2 t_{jj}^{\text{WP3}} \tag{10}$$

such that $t_{ji}^{\text{WP3}} \in \tau_i$, $t_{jj}^{\text{WP3}} \in \tau_j$, and $|t_{jj}^{\text{WP3}} - t_{ji}^{\text{WP3}}| \geq \frac{\Delta_{\text{III}}}{V_{\text{III}}}$.

Next, Aircraft i and j take the gradient steps

$$\lambda_1^+ = \lambda_1 + t_{ii}^{\text{WP3}} - t_{ji}^{\text{WP3}}, \text{ and } \lambda_2^+ = \lambda_2 + t_{jj}^{\text{WP3}} - t_{ij}^{\text{WP3}}. \tag{11}$$

In order to solve these problems, Aircraft i must communicate t_{ij}^{WP3} and λ_1 to Aircraft j, while Aircraft j must communicate t_{ji}^{WP3} and λ_2 to Aircraft i. Each aircraft must solve the minimization problem once for when Aircraft i arrives first, and again for when Aircraft j arrives first, and choose the best scenario of the two to execute.

VI. Binary Tree Generalization

The proposed merging procedure can be generalized to any number of incoming tracks by forming a binary tree structure as shown in Figure 5. The choice of velocity and separation on a leg will determine the required velocity and separations of the two legs that merge into it. Therefore, the conditions chosen for the terminal leg will set constraints on Leg 7 and Leg 8, which will in turn give conditions on Legs 1, 2, 5, and 6, and finally for Legs 3 and 4.

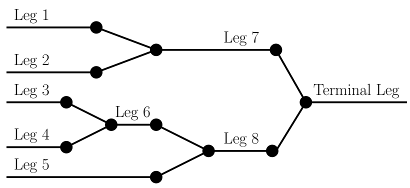
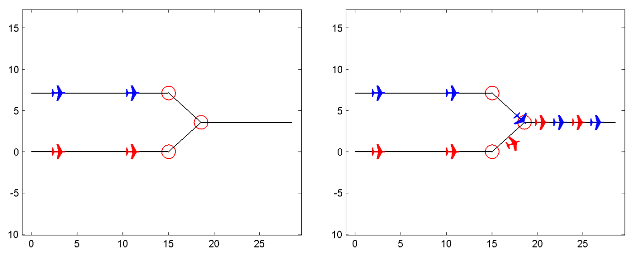


Fig. 5. Binary tree structure for merging multiple tracks.

VII. Simulation Results

We demonstrate the viability of this approach with a simulation run using the following parameters: $V_{\text{I}} = 1$, $\Delta_{\text{I}} = 8.1$, $V_{\text{III}} = 0.5$, $\Delta_{\text{III}} = 2$, $d = 5$, $V_{\text{max}} = 1.81$, $V_{\text{min}} = 0.5$, $h_{\text{max}} = 1$, and $\theta = \pi/2$. The parameters were chosen such that incoming aircraft on Legs 1 and 2 have spacing close to the minimum separation distance as allowed by the feasibility conditions. Figure 6 shows simulation results of aircraft approaching a merge point on both legs and then successfully merging onto the terminal leg, while spaced at least Δ_{III} apart.



(a) Incoming air traffic (b) Merged air traffic

Fig. 6. Simulation results for merging and spacing air traffic on two legs.

Plots of the separations at WP3 when the incoming separation on Legs 1 and 2 is exactly Δ_{I} is shown in Figure 8(a). Since Δ_{I} is extremely close to violating the feasibility conditions, the arrivals at WP3 are spaced almost exactly Δ_{III} apart. Figure 8(b) shows the case when the incoming flights on Legs 1 and 2 are randomly generated such that consecutive aircraft on each leg are spaced at least Δ_{I} apart.

As an example, we show a particular pairwise negotiation between Aircraft 1 on Leg 1 and Aircraft 2 on Leg 2, with no other aircraft preceding them. Aircraft 1's arrival time at WP1 (t_1^{WP1}) is 12 and Aircraft 2's arrival time at WP2,

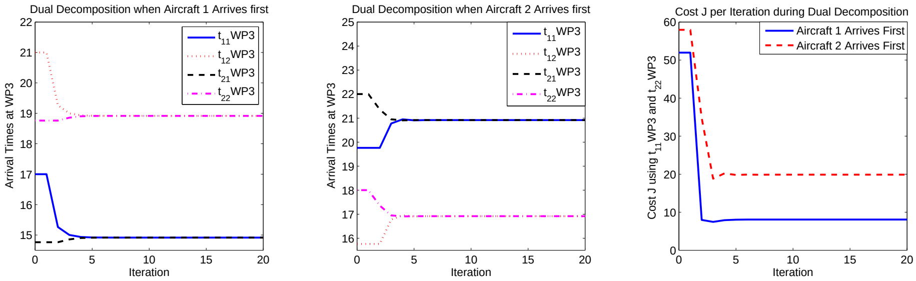


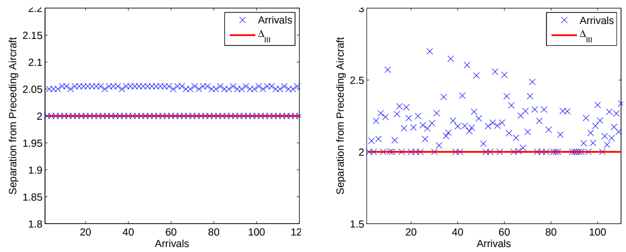
Fig. 7. Arrival time agreement (left and center) and associated cost using negotiated arrival times (right).

(t_2^{WP2}) is 13. Notice that if the two aircraft had not negotiated and just proceeded in Phase II with $V_{\text{II}} = V_{\text{I}}$ and $h = 0$, then they would reach WP3 one time unit apart and thus lose separation. Therefore, both aircraft negotiate for a time (and order) to arrive at WP3 given the feasible time sets $\tau_1 = [14.76\ 22.77]$ and $\tau_2 = [15.76\ 23.77]$.

Since $\Delta_{\text{III}} = 2$ and $V_{\text{III}} = 0.5$, any arrival time that the two negotiate must be $\frac{\Delta_{\text{III}}}{V_{\text{III}}} = 4$ apart from each other at all times. The cost weights for Aircraft 1 were chosen to be $k_1 = 10$, $k_2 = 2$, $k_3 = 1$, while the cost weights for Aircraft 2 were chosen to be $k_1 = 3$, $k_2 = 8$, $k_3 = 3$. Thus, Aircraft 1 penalizes deviations in the trajectory more than it does changes in its velocity and so would prefer to speed up or slow down rather than deviate from the straight-line path. Aircraft 2 is opposite in that its cost weights make it more willing to deviate its path than to change its speed.

Aircraft 2 weighs its ETA more than Aircraft 1 through k_3, so Aircraft 2 would like to arrive at WP3 at the same time it would have arrived if it had chosen $V_{\text{II}} = V_{\text{I}}$ and $h = 0$. Finally, γ was chosen to be 10 in the joint cost to give some preference towards the two negotiated arrival times at WP3 being close but never less than $\frac{\Delta_{\text{III}}}{V_{\text{III}}}$ apart.

The converging trajectories of t_{11}^{WP3}, t_{12}^{WP3}, t_{21}^{WP3}, and t_{22}^{WP3} during the dual decomposition are shown in Figure 7 (left and center). In Figure 7 (left) when Aircraft 1 arrives first, the final arrival times are $t_{11}^{\text{WP3}} = 14.92$ and $t_{22}^{\text{WP3}} = 18.92$. In Figure 7 (center) when Aircraft 2 arrives first, the final arrival times are $t_{11}^{\text{WP3}} = 20.92$ and $t_{22}^{\text{WP3}} = 16.92$. The associated costs in Figure 7 (right), computed using planned arrival times t_{11}^{WP3} and t_{22}^{WP3}, are $J = 8.074$ when Aircraft 1 arrives first, and $J = 19.86$ when Aircraft 2 arrives first. Note that the associated costs do not have to be monotonically decreasing since forcing t_{11}^{WP3} and t_{22}^{WP3} to satisfy the minimum separation constraint may increase the associated cost. Comparing the final associated costs, we see that Aircraft 1 "wins" and therefore plans to arrive at WP3 at time $t_1^{\text{WP3}} = t_{11}^{\text{WP3}} = 14.92$ and chooses $h = 0$ and $V_{\text{II}} = 1.7123$ as expected; recall Aircraft 1 heavily penalizes deviations in its path. Aircraft 2 "lost" and must negotiate with the next unresolved aircraft.



(a) Arrival separation for minimally spaced incoming aircraft.

(b) Arrival separation for randomly and safely spaced incoming aircraft.

Fig. 8. Separation distances for consecutive aircraft arrivals.

VIII. Conclusions

In this work, sufficient conditions on the incoming aircraft spacing as well as on the set of reachable arrival times were given to guarantee feasible arrival times at a merge point, which guarantees minimum aircraft separation on the terminal phase of flight. Distributed methods were also presented that allow pairwise negotiations over opposite legs to determine arrival times that guarantee satisfying a minimum spacing requirement between aircraft and seeks to minimize fuel consumption and changes in the ETA. Simulations were performed that demonstrate the viability of this approach.

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