Coordinated Convoy Protection Among Teams of Unmanned Aerial Vehicles

Magnus Egerstedt*, Amir Rahmani†, Ryan Young‡

Abstract

This chapter investigates how to control and coordinate teams of Unmanned Aerial Vehicles (UAVs) to provide protection to convoys of ground vehicles. This is a rich yet canonical problem when coordinating multiple UAVs in that coordinated movements, task assignments, and resource balancing must all be performed for a successful completion of the mission. Time-optimal paths for providing convoy protection to stationary ground vehicles are presented and these algorithms are extended to moving ground vehicles. The assignment problems, associated with dispatching UAVs from the convoy to inspect and clear potential threats, are moreover discussed.

1 Introduction

Airplanes can be used to provide close air support or large-scale area surveillance for ground convoys in unknown and potentially dangerous environments. Wide spread use of Unmanned Ground Vehicles (UGVs) to conduct tasks in these environments has necessitated the design of practical approaches to effectively control and coordinate multiple UAVs to provide coverage, surveillance, tracking and convoy protection for the UGVs (see, for example, [5, 10, 18, 19, 25, 41]). This chapter discusses this problem, which can be decomposed into two subproblems. The first problem involves the design of coordinated UAV maneuvers to ensure the ground convoy is protected. The second problem concerns how to assign and dispatch UAVs from the convoy to clear potential threats in such a way that the fuel consumption is balanced across the team of UAVs.

1.1 Problem 1: Time Optimal Protection Paths

UAVs often use on-board cameras in order to obtain information about surrounding areas. In many surveillance applications with small UAVs (for example [27, 30]), the motion of the camera is typically decoupled from that of the UAV, using a gyro stabilized camera platform that keeps the camera pointing in the same direction regardless of the motion of the UAV. This approach provides the UAV with stable images even under high frequency oscillations associated with the UAV itself. Hence, the assumption is made that the on-board camera always points down and, as such, the UAV monitors a circular disk on the ground.

To solve the problem of providing coordinated protection maneuvers, an optimal approach is adopted for path planning and coordination of multiple UAVs. The UAVs are modeled as Dubins vehicles flying at constant altitude. Limited ranges of sensors on board the UAVs, together with their kinematic constraints, might make it impossible to provide coverage to the ground vehicles with a single UAV. In this case, it is of interest to find the best path for an individual UAV so that it can monitor the ground vehicles for the longest time, and then coordinate the UAVs in such a way that the ground vehicles are visible to at least one UAV at any given time, as illustrated in Figure 1.

*magnus@gatech.edu, School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332

†a.rahmani@miami.edu, Department of Mechanical and Aerospace Engineering, University of Miami, Coral Gables, FL 33146

‡syoung@rockwellcollins.com, Advanced Technology Center, Rockwell Collins, Cedar Rapids, IA 52498

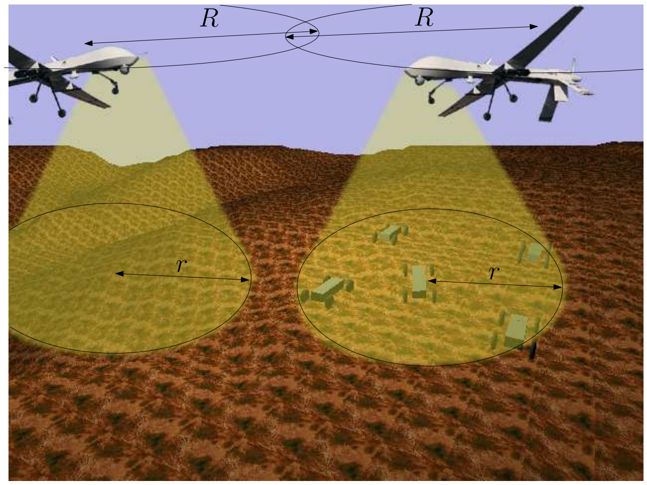


Figure 1: UAVs providing convoy protection to UGVs. The UAVs are assumed to be kinematically restricted by their minimum turning radius R. The sensors on board the UAVs also have limited range and are assumed to be able to observe a disk of radius r on the ground.

A Dubins vehicle is a planar vehicle with bounded turning radius and constant forward speed. L.E. Dubins was the first to give a characterization of time-optimal trajectories for such a vehicle using geometric methods [13]. Shortest-path problems for Dubins vehicles have been since studied extensively (see [1, 7, 14] for example). A Dubins vehicle that can move backwards was studied by Reeds and Shepp [35], and the shortest-path problem for a Reeds-Shepp vehicle was further studied and classified by Souères and Laumond [40]. Walsh *et al.* [42] found optimal paths using quadratic cost functions for a Dubins airplane in $SE(2)$, $SO(3)$ and $SE(3)$. McGee *et al.* [26] obtained time-optimal paths for Dubins vehicles in constant wind. Dubins vehicles have moreover been used as a simplified model to describe planar motions of UAVs in [37]. Chitsaz *et al.* [9] extend the Dubins' model from $SE(2)$ to $SE(2) \times \mathbb{R}$ to account for altitude changes and gave a characterization of the time-optimal trajectories for this model based on the final altitude.

Optimal trajectories of a Dubins vehicle are often constructed using motion primitives (see [6, 16, 20] for example). For point-to-point minimum-time transfer problems, it has been shown that the optimal solutions are curves consisting of only three motion primitives: line-segments and circular arcs turning maximally to the left and to the right (see [13, 24, 39]). The optimal paths for the point-to-point minimum-time transfer problem are characterized by sequences of these three motion primitives. This chapter extends this to the case of stationary convoy protection, where resulting time-optimal paths are characterized by sequence of only two motion primitives (maximally turning left and turning right) and do not include any line-segments. In fact, this chapter draws upon the results in [11]. Similar to many previous time-optimal path-planning work (e.g [7, 9, 11, 26, 40, 39]), it makes use of Pontryagin's Minimum Principle to derive optimal trajectories.

When addressing this problem, the first case under investigation is when the convoys are stationary, and global optimal paths are found for a single UAV that maximize the coverage time, i.e., the time the UAV spends with the convoy being visible. It is then shown how to coordinate a group of UAVs to provide continuous coverage of the convoys and the minimum number of UAVs required to achieve continuous convoy protection for all time, as well as how to extend these results to moving rather than stationary ground convoys.

Dubins vehicles flying at constant altitude with unit speed[1] and minimum turn radius of R, can be modeled as

$$\begin{cases} \dot{x} = \cos(\theta) \\ \dot{y} = \sin(\theta) \\ \dot{\theta} = \omega, \end{cases} \tag{1}$$

[1] The unit speed assumption is justified since the results describe paths which are invariant under different forward speeds.

where x and y are the position of the UAV in the x-y plane at the altitude the UAV is flying, and ω is the angular velocity of the vehicle. The angular velocity is bounded by the inverse of the minimum turn radius R of the vehicle, i.e., $\omega \in [-\frac{1}{R}, \frac{1}{R}]$. The state of the system be given by $q(t) = [x(t), y(t), \theta(t)]^T$.

As already stated, it is assumed that the cameras on board the UAVs can monitor a disk of radius r on the ground (see Figure 1 for an illustration of this scenario). That is, it is assumed that the on-board camera is attached to a gyro and is always looking down, regardless of the bank angle of the UAV. A gimballed camera system is commonly used in UAV surveillance applications to decouple camera from dynamics of the plane (see for example [27, 30]). In addition, to simplify the problem, the ground convoys are considered to be points located at the centroid of the convoys in the x-y plane. Hence, successful convoy protection is defined as being achieved when the centroid of the UGVs is visible to at least one of the UAVs at any time. Or, in other words, convoy protection is provided by the UAV if the distance between the UAV and the centroid of the UGVs are less than or equal to r.

The disk of observation and its radius r certainly depend on the altitude of the UAV, but to ensure quality of observation and successful protection, cameras or sensors on board the UAV have narrower field of view than the UAV's turning radius. This is especially true for cameras and sensors that carry out tasks using computer vision algorithms, which require certain level of image resolution. In these cases, $R > r$, and if the UGVs are stationary, then a single UAV is not capable of providing convoy protection to the ground vehicles indefinitely and a control strategy is needed to optimize the time in which convoy protection is achieved. This can be seen by drawing a circle of radius r using the position of the centroid of UGVs as the center. If there is only one UAV, then due to the fact that $R > r$, the UAV will eventually fly out of this circle no matter where it starts.

Note that in case of static convoys, if $R \leq r$, then the convoy protection problem is trivial since it can be solved by using a single UAV flying on a circular path of radius R with the center being the centroid of the ground vehicles. In this case, convoy protection is provided for all time using only one UAV.

In this chapter, the problem of controlling and coordinating UAVs is considered, with the task of the UAVs being that of providing convoy protection for both stationary UGVs and UGVs moving on a straight line. In both cases it is assumed that $R > r$. Let the *convoy circle* denote a disk of radius r centered at the centroid of the UGVs. Using the convoy circle, it can be seen that convoy protection is achieved if at least one UAV is present inside the convoy circle at any time. Because of the kinematic constraint (turning radius R of the UAV), the UAVs are required to be coordinated so that they collectively provide continuous convoy protection while individual UAVs enter and leave the convoy circle.

1.2 Problem 2: Assigning and Dispatching UAVs

In the convoy protection problem, being able to have the UAVs track the UGVs is not enough. Sometime UAVs need to be dispatched in order to clear potential threats. In fact, given a single UAV and N known threats that must be cleared, together with a cost associated with traversing between the threats, the problem of selecting the order in which the threats should be visited (and cleared) is a variant of the well-studied traveling sales person (TSP) problem [8, 17]. Unfortunately, TSP is a NP-hard problem, i.e., there are no polynomial (in the number of threats) time algorithms that can solve the problem. As a consequence, *any attempt at tackling the full-fledged TSP using limited computational resources is doomed to failure.* The reason for this computational intractability is that, in order to solve the TSP problem, one has to establish the order in which the threats are visited. If, on the other hand, this order is a priori given, the problem would already be solved.

If there are multiple UAVs (as is the case in the convoy protection scenario under consideration here), the complexity of the TSP problem is reduced if the number of UAVs is greater than or equal to the number of threats (including both known and pop-up threats). The reason for this is that one ends up with a so-called matching problem [31], where one simply has to decide which threat should be visited by which UAV. And, since there are no threats left unvisited after all UAVs have visited one threat, there is again no ordering of the threats that must be decided upon. The matching problem can be solved in cubic time (in the number of threats) using the Hungarian algorithm [23], as is done, for example, in [21, 28]. If there are fewer UAVs than threats then the NP-hardness result still applies [2, 32], and this is the situation (fewer UAVs than threats) that can be expect in real-world operations.

Numerous algorithms for approximating and addressing the TSP problem in a computationally tractable manner have been proposed. These algorithms typically fall under two categories, namely (1) greedy algorithms [4, 34, 36, 38], where each agent is trying to locally maximize and minimize a cost based solely on the instantaneous cost rather than over the whole time horizon, and (2) auction-based and game theoretic algorithms [3], where the agents get to "bid" on which tasks they want to be assigned to. In the greedy case, the solution is clearly suboptimal, even though it was shown in [2, 32] that asymptotically (as the number of threats approach infinity), the solution gets to within a constant factor of the optimal solution. Auction-based solutions, on the other hand, typically require a significant amount of information to be shared by the agents and, despite this increase in shared information and the required communication bandwidth, the inherent complexity of the problem remains the same. As such, auction-based methods may provide some guidance as to finding suboptimal solutions but they do not, unfortunately, overcome the complexity issue and they come with an increased communications overhead. Solutions that fall in-between these two approaches were presented in [33], where decentralized suboptimal solutions where obtained by focusing on the feasibility of the solution over limited time horizons rather than the minimization of any particular cost function, and in [29], where the assignment and allocation problems were addressed for more specialized surveillance missions. This last approach (as exemplified by [29, 33]) is one that will be followed in this chapter in that the special structure of the convoy protection problem will be leveraged to obtain good suboptimal solutions to an intractable problem.

The convoy protection problem is *not* a standard TSP (single or multi-UAV) problem even though it shares some of the same features. The two main differences are:

- Threat dynamics: The threats are not given all at once. Instead they show up throughout the mission, and as a consequence, the problem must be resolved whenever a new threat appears.
- Overall cost: The cost to be minimized is not the total distance traveled or the total fuel consumed by the team of UAVs. Instead it needs to reflect the fact that the fuel consumption should be balanced among the different UAVs, since success of convoy protection mission depends on participation of all UAVs in convoy protection effort. As such, it is a min-max cost in that what must be minimized is the fuel consumption of the UAV who has to expend the most fuel.

As a consequence of these considerations, the convoy protection problem is still NP-hard and in order to be able to still solve it in a resource constrained environment, some simplifying assumptions must be made. However, one can take the explicit *structure* of the convoy protection problem into account when making these assumptions.

A1 *Bounded number of threats:* At any given time, only a fixed number of threats will be considered (e.g., road-side bombs). Even though no a priori upper limit is placed on the number of threats that may appear, the algorithm will be restricted to only consider the closest M threats, where M is a fixed number. Without further assumptions, this will still make the complexity factorial in M, which may be a large number (depending on what M is), but at least it is bounded.

A2 *One-dimensional threat distribution:* As the convoy is moving along an established route, e.g., a road, only threats on (or close to) the road will be considered. As such, the threats will be placed along a one-dimensional path and they will be encountered sequentially. The result is that the order in which the threats are cleared is already given in that the closest threat is cleared first.

A3 *Not all threats must be cleared:* The objective of the convoy protection algorithm is to ensure that the convoy is not endangered by any threats. There are two ways in which this can be achieved. The first is by clearing threats, as previously discussed. However, one can also re-plan the path the convoy takes in order to produce a detour around a threat. This will be done if the threat cannot be cleared in time which allows for threats to be ignored, that would otherwise require too much fuel to be cleared.

These three assumptions will allows for a polynomial-time algorithm for solving the assignment aspect of the overall convoy protection problem.

2 Optimal Convoy Protection Maneuvers

2.1 Single UAV Time-Optimal Paths

The first problem to consider is the problem of using one UAV to provide convoy protection to some stationary UGVs for the maximum amount of time, which is equivalent of finding the longest feasible path inside the convoy circle. What needs to be determined is both the optimal path for a single UAV starting at a fixed initial condition, and the optimal path if the UAV is allowed to pick the initial condition (position and heading) when entering the convoy circle.

Fix the origin of the x-y plane at the centroid of the UGVs. A maximum-time, optimal control problem with state constraint $x^2 + y^2 - r^2 \leq 0$ and input constraint $|w| \leq \frac{1}{R}$, is then obtanied. Furthermore, it can be assumed that the UAV starts at a point on the state constraint boundary (convoy circle). This assumption does not limit the generality of the result, since, if the UAV starts inside the convoy circle, Bellman's Principle of Optimality can be used to obtain the optimal path for the UAV by integrating backwards in time.

To facilitate the analysis, it is useful to impose an extra terminal manifold constraint. Since the optimal solution always has the terminal state (henceforth denoted as the exit state) be on the boundary of the state constraint set (exiting the convoy circle), the terminal constraint of being on the convoy circle when exiting is enforced. To simplify the notation, the following notation is introduced: $q_T := q(T)$ and $[x_T, y_T, \theta_T] := [x(T), y(T), \theta(T)]$. The terminal manifold can then be defined as the set of states that satisfy:

$$M(q_T) = x_T^2 + y_T^2 - r^2 = 0. \tag{2}$$

The optimal control problem is given by:

Problem 2.1

$$\min_{\omega(t)} J = \int_0^T -1dt, \tag{3}$$

subject to the dynamics of (1) with a given initial condition $q(0)$, the input constraint

$$-\frac{1}{R} \leq \omega(t) \leq \frac{1}{R}, \tag{4}$$

the state constraint

$$x(t)^2 + y(t)^2 - r^2 \leq 0 \tag{5}$$

$$x(0)^2 + y(0)^2 - r^2 = 0 \tag{6}$$

and the terminal manifold constraint

$$M(q(T)) = x(T)^2 + y(T)^2 - r^2 = 0. \tag{7}$$

This problem is henceforth denoted as $\Pi_{q(0)}$.

When solving this problem, one immediately should exclude initial conditions that generate no paths entering the convoy circle. This occurs when the initial heading $\theta(0)$ points away from the convoy circle. The set of initial conditions Λ that are considered for the optimization problem are thus given by:

$$\begin{aligned} \Lambda \quad = \quad & \{q = [x, y, \theta]^T : x^2 + y^2 = r^2, \text{ and} \\ & -\frac{\pi}{2} < \theta - \text{atan2}(y, x) < \frac{\pi}{2}\}. \end{aligned} \tag{8}$$

The set Λ is called the feasible entry set. For simplicity of notation, it is assumed that all angles are taken modulo 2π.

$\Pi_{q(0)}$ is an optimal control problem with both input and state inequality constraints. Optimal control problem with state inequality constraints are usually hard or impossible to solve explicitly. Fortunately, in this problem, due to the special structure of the state inequality constraint (5), there are only 2 points on the state trajectory where the constraint is active. These two states correspond to when the UAV is entering and exiting the convoy circle. Due to this special structure, an auxiliary state is used to handle the state constraint. Define $\xi(x^2+y^2-r^2)$ as an inverted Heaviside function:

$$\xi(x^2+y^2-r^2)=\begin{cases} 0 : x^2+y^2-r^2\leq 0 \\ 1 : \text{otherwise.}\end{cases} \tag{9}$$

Define a new state τ as:

$$\dot{\tau}(t)=(x^2+y^2-r^2)^2\,\xi(x^2+y^2-r^2), \tag{10}$$

The state of the UAV is then augmented as $\bar{q}(t) = [q(t), \tau(t)]^T$. Assuming that $\tau(0) = 0$ and imposing the terminal constraint that $\tau(T) = 0$, it follows that $\dot{\tau}(t) = 0, \forall t \in [0, T]$, since $\dot{\tau}(t) \geq 0, \forall t \in [0, T]$. Hence,

$$\tau(t) = 0, \forall t \in [0, T]. \tag{11}$$

Note that the terminal constraint $\tau(T) = 0$ enforces the state inequality constraint (5). Using this auxiliary state τ, the state inequality constraint is transformed into an equivalent terminal constraint. When there is no ambiguity, it is assumed that the state constraint (5) is satisfied and $q(t)$ is still referred to as the state trajectory.

The Hamiltonian for this optimal control problem is:

$$\begin{aligned}\mathcal{H} &= -1+\lambda_1\cos\theta+\lambda_2\sin\theta+\lambda_3\omega+ \\ &\quad\ \lambda_4(x^2+y^2-r^2)^2\xi(x^2+y^2-r^2),\end{aligned} \tag{12}$$

where $\lambda = [\lambda_1, \cdots, \lambda_4]^T$ are the costates. The costates satisfy the following differential equations in the time interval $[0, T]$:

$$\begin{aligned}\dot{\lambda}_1 &= -\frac{\partial\mathcal{H}}{\partial x} = -2x\lambda_4(x^2+y^2-r^2)\xi(x^2+y^2-r^2),\\ \dot{\lambda}_2 &= -\frac{\partial\mathcal{H}}{\partial y} = -2y\lambda_4(x^2+y^2-r^2)\xi(x^2+y^2-r^2),\\ \dot{\lambda}_3 &= -\frac{\partial\mathcal{H}}{\partial \theta} = \lambda_1\sin\theta-\lambda_2\cos\theta\\ \dot{\lambda}_4 &= -\frac{\partial\mathcal{H}}{\partial \tau} = 0.\end{aligned}$$

When the state constraint (5) is satisfied, the last term in the Hamiltonian (12) does not contribute since $\lambda_4(x^2+y^2-r^2)^2\xi(x^2+y^2-r^2) = 0, \forall t \in [0, T]$, and thus the Hamiltonian becomes:

$$\mathcal{H} = -1+\lambda_1\cos\theta+\lambda_2\sin\theta+\lambda_3\omega. \tag{13}$$

The necessary optimality condition from the Pontryagin's Minimum Principle states that

$$\begin{aligned}&\mathcal{H}(\bar{q}^\star(t),\lambda^\star(t),\omega^\star(t),t)\leq\mathcal{H}(\bar{q}^\star(t),\lambda^\star(t),\omega(t),t),\\ &\qquad\qquad\forall\omega(t)\in\left[-\frac{1}{R},\frac{1}{R}\right],t\in[0,T],\end{aligned} \tag{14}$$

where $\bar{q}^\star(t)$ denotes the optimal augmented state trajectory, $\lambda^\star(t)$ denotes the optimal costate trajectory corresponding to $\bar{q}^\star(t)$, and $\omega^\star(t)$ is the optimal control.

Using the necessary optimality condition (14) on the Hamiltonian equation (13), one can see that the optimal control $\omega^\star(t)$ is a function of the costate $\lambda_3^\star(t)$:

$$\omega^\star(t)=\begin{cases}-\frac{1}{R}, & \text{if } \lambda_3^\star(t)>0\\ \frac{1}{R}, & \text{if } \lambda_3^\star(t)<0\\ \text{undetermined}, & \text{if } \lambda_3^\star(t)=0\end{cases} \tag{15}$$

Thus, it can be seen that when $\lambda_3^\star(t) > 0$, the optimal control is maximum turning right, and when $\lambda_3^\star(t) < 0$, the optimal control is maximum turning left. When $\lambda_3^\star(t) = 0$ for a finite time interval, then any control $\omega(t) \in [-\frac{1}{R}, \frac{1}{R}]$ satisfies the Minimum Principle, and the finite time interval when this case arises is called a singular interval (for discussions on singular intervals for optimal control problems, see any standard text on optimal control, such as [22]). Hence, the optimal control is in the form of bang-bang (if there is no singular interval) or bang-off-bang (if there are singular intervals). And, if there is a singular interval for $\Pi_{q(0)}$, then it is necessary that there exists a time interval $[t_1, t_2]$ such that, $\lambda_3(t) = 0$ and $\dot{\lambda}_3(t) = 0$ for all $t \in [t_1, t_2]$. For Dubins vehicles with dynamics specified in equation (1), singular intervals result in line segments as part of the optimal path and as a consequence, line segments are usually part of the optimal paths for shortest-path (or minimum-time) Dubins vehicle problems.

Definition 2.1 *For a state trajectory $q(t), t \in [0, T]$ satisfying the state constraint (5), if the costate trajectory $\lambda_3(t)$ and corresponding input $\omega(t)$ satisfies the control strategy (15), then $q(t)$ is referred to as a Candidate Optimal Trajectory (COT).*

Pontryagin's Minimum Principle states that being a COT is a necessary condition for being an optimal solution and, assuming that a trajectory $q(t)$ is a COT but only its terminal state q_T is given, the entire trajectory $q(t)$, its corresponding costate trajectory $\lambda(t)$ satisfying the necessary optimality condition (14) and the control $\omega(t)$ satisfying the optimal control strategy (15) can all be uniquely determined from the terminal state q_T, as is shown in [11, 12]. In fact, in [12], it was shown that an optimal trajectory can not contain both a circular arc and a line segment. In addition, the COT can be either a line through the origin, or a curve consisting of circular arcs of radius R. Hence, if the optimal control is not constant, then it can only change between $\omega = -\frac{1}{R}$ and $\omega = \frac{1}{R}$, since there is no singular interval in that case. Henceforth, the term *switching time* is used for the time instant when the control law switches between $\omega = -\frac{1}{R}$ and $\omega = \frac{1}{R}$. Furthermore, a switching point in a state trajectory is defined as the state when the controller switches.

For a number of terminal conditions q_T, the corresponding COTs are shown in Figure 2. For all of these terminal conditions, x-y coordinates of the terminal state are fixed at $[r, 0]^T$ (hence $\psi_T = 0$), but the exit angle θ_T is allowed to vary. The path in the x-y plane $[x(t), y(t)]$, costate $\lambda_3(t)$ and angle $\theta(t)$ are plotted from left to right.

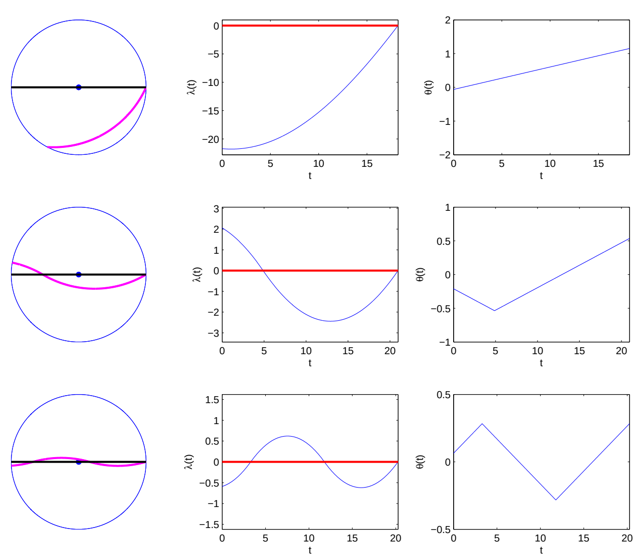


Figure 2: A number of COTs are plotted for different exit angles θ_T resulting in different number of switchings. The left plots are the COTs in x-y plane, the middle plots are the costates λ_3 and the right plots are the angles $\theta(t)$. The first, second and third row correspond to the cases when this is 0, 1 and 2 switchings, respectively.

An unusual aspect of this problem is that, unlike the Dubins vehicle shortest-path problem, the optimal paths for Problem 3.1 do not in fact contain line segments, as shown in [12], even though such maneuvers do constitute valid COTs. Additionally, it can be shown ([12]) that the optimal paths never switch more than once, which leads to the following key result when characterizing the best protection paths:

Theorem 2.1 *For any initial condition $q(0)$, the optimal solution to Problem 2.1 is a UAV trajectory that does not switch more than once and that does not contain any straight line segments.*

Based on this theorem, the set of optimal motion primitives does not contain the motion of going straight. As such, two motion primitives $\{\mathbf{L}, \mathbf{R}\}$ are defined, where $\mathbf{L}$ and $\mathbf{R}$ motion primitives turn the vehicle maximally to the left and right, respectively. Furthermore, since the optimal trajectory only switch once, there are only 4 possible sequences of the $\{\mathbf{L}, \mathbf{R}\}$ motion primitives, namely

$$\{\mathbf{L}, \mathbf{R}, \mathbf{LR}, \mathbf{RL}\}, \tag{16}$$

where $\mathbf{LR}$ stands for turning left then right and $\mathbf{RL}$ for turning right then left. The actual optimal paths can now be determined directly from the costate equations. In fact, since there are only four possibilities for the motion sequences in an optimal trajectory, it is easy to determine the global optimal path for any initial condition. A set of optimal paths for initial conditions with heading $\theta(0) = \frac{\pi}{2}$ are shown in Figure 3. Figure 3 also shows the optimal switching surface on which switchings are optimal and it can be observed that when a switching is needed for the optimal trajectory, the switching point, the origin and the exit point are on the same line.

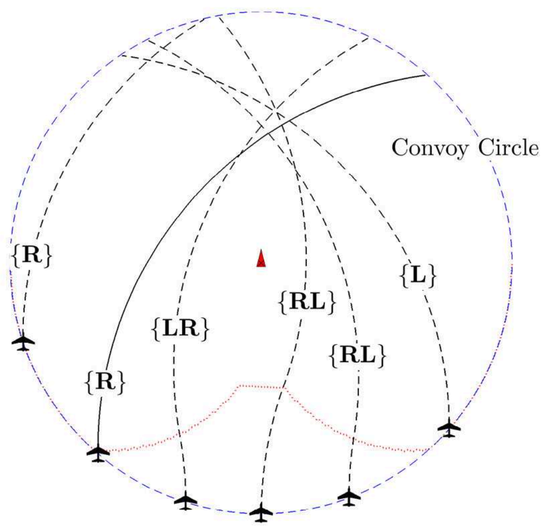


Figure 3: A number of optimal state trajectories with initial heading $\frac{\pi}{2}$. The optimal switching points are plotted together to form the optimal switching surface. In this case, $R = 1.5r$. The dashed paths corresponds to optimal paths. If $x(0) \in (-\frac{r^2}{R}, 0]$, then $\mathbf{LR}$ is optimal. If $x(0) \in [0, \frac{r^2}{R})$, then $\mathbf{RL}$ is optimal. If $x(0) \in [\frac{r^2}{R}, r)$, then $\mathbf{L}$ is optimal. Otherwise, $\mathbf{R}$ is optimal. The optimal switching points are plotted together to form the optimal switching surface. The solid path corresponds to the initial condition that $x(0) = -\frac{r^2}{R}$. This path and the path where $x(0) = \frac{r^2}{R}$ are equal in length and longer than all other optimal paths with the same initial heading.

By rotating the initial state until the initial heading is $\frac{\pi}{2}$, the switching surface in Figure 3 provides a control law which produces the optimal trajectory for any given initial condition. Now, this result is extended to address a perhaps more important problem: finding the optimal path inside the convoy circle with initial condition free to choose.

Problem 2.2

$$\min_{q(0)} J^\star(q(0)), \tag{17}$$

where $J^\star(q(0))$ is the solution of Problem 3.1 ($\Pi_{q(0)}$), with the initial state $q(0)$.

Let the optimal initial condition be denoted by $q^\star(0)$, hence:

$$q^\star(0) = \arg\min_{q(0)} J^\star(q(0)). \tag{18}$$

The optimal path with this initial condition will be referred to as a globally[2] optimal path. $q^\star(0)$ will be called an optimal entry state. It is apparent that any rotation of this state around the origin is also an optimal entry state. Hence the globally optimal paths and optimal entry states are not unique. The set of optimal entry states, denoted by $\mathbb{Q}^\star$, can be exactly determined by noting that optimal paths with optimal entry points do *not* switch. Instead they are given my maximal left/right inputs while the entry point, the center of the convoy circle, and the exit point located on the same line, as found in [12].

2.2 Multi-UAV Convoy Protection

Due to kinematic constraint of the UAVs ($r < R$), it is impossible for one UAV to provide complete convoy protection for a group of UGVs. In this situation, multi-UAV coordination is required in order to successful carry out convoy protection. The previous section laid out the groundwork to achieve optimal convoy protection by a group of UAVs. In particular, the set of optimal initial conditions that produces a set of globally time-optimal trajectory was characterized. It can moreover be shown that these optimal trajectories not only specify a path inside the convoy circle, but also a path for a single UAV to come back to the convoy circle without changing direction. As shown in Figure 4, this path constitutes a circle of radius R and part of the path is a globally optimal path inside the convoy circle.

Definition 2.2 *The circular paths of radius R entering and exiting the convoy circle at states located on the same line as the center of the convoy circle are referred to as the optimal convoy protection paths.*

Figure 4 shows three examples of optimal convoy protection paths and it is straight forward to show the following key result:

Theorem 2.2 *Over all simple closed paths of a single UAV, the optimal convoy protection paths each maximizes the ratio of the length inside the convoy circle over its total length.*

Intuitively, an optimal convoy protection path maximizes the coverage ratio because it is the quickest path to come back to the convoy circle, always reenters optimally and repeats as a limit-cycle. And, to ensure that all the UAVs maximize their time providing convoy protection, their paths should be set to the optimal convoy protection paths such as the ones shown in Figure 4. In order to achieve successful convoy protection, it is required that the UGVs are visible to at least one UAV at all time. Thus, one can establish a lower bound on the number of UAV required to provide successful convoy protection for all time based on the length of the optimal convoy protection paths.

Corollary 2.3 *Given the convoy circle of radius r for the UGVs and minimum turning radius R for the UAVs, the minimum number of UAVs needed to provide convoy protection for all time is:*

$$N = \left\lceil \frac{\pi}{\arcsin(\frac{r}{R})} \right\rceil, \tag{19}$$

where $\lceil\cdot\rceil$ denotes the ceiling function.

Assume that there is N UAVs and they can start at an optimal initial condition $q^\star(0) \in \mathbb{Q}^\star$, the UAVs need to space themselves evenly in terms of the time entering the convoy circle. This can be achieved by slowing down and speeding up with respect to the other UAVs so that the i-th UAV enters the convoy circle at time $\frac{2\pi R}{N} i$. This strategy is possible since the optimal paths derived for this problem remain the same for UAVs of any speed (instead of unit speed).

[2] To clarify, the word *globally* is used here not within the context of local or global optimality. The solutions obtained for Problem 3.1 are global optimal trajectories each corresponds to a fixed initial condition. A globally optimal path in this chapter is defined as the longest path over all feasible initial conditions.

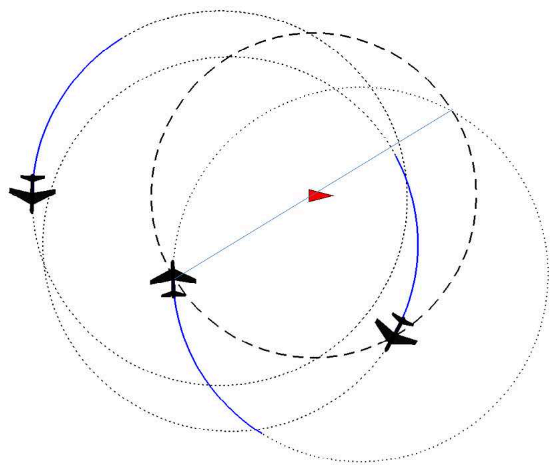


Figure 4: Three optimal convoy protection paths are shown. They maximize the time spent inside the convoy circle over the time outside of the convoy circle. The smaller dashed circle is the convoy circle, and the larger dotted circles are optimal convoy protection paths. The solid curve is the past trajectories of the UAV. One can see that the key to coordinate the UAVs for optimal continuous convoy protection is to fly individual UAVs on optimal convoy protection paths, and space them out so that there is at least one UAV inside the convoy circle for all time.

2.3 Moving Convoy Protection

In this section, the focus is shifted to a convoy protection strategy for moving UGVs. Again, it is assumed that the location of the UGVs are represented by their centroid as a point. Instead of being static, the case is considered where this point is moving in a constant direction with a constant and bounded speed.

Denote the speed of the UGVs as V_G. The UGVs are assumed to be moving in a constant heading of angle ϕ. The UAV speed is again normalized to 1. Hence the UAVs follow the dynamics in equation (1) and their states are denoted by $[x, y, \theta]^T$. The UAVs are assumed to be capable of flying with faster speed than the UGVs (this agrees with current state of technologies in terms of speed of ground robots versus UAVs). Hence, it is assumed that $V_G \leq 1$.

Now, a control strategy is proposed with a corresponding lower bound $V_G^\star$ so that if the speed of the UGVs is in this bound ($V_G \in [V_G^\star, 1]$), then one UAV is guaranteed to provide convoy protection for all time. Inspired by the motion primitives defined in the static convoy protection problem, the motion of the UAV is fixed to a sequence of maximally left and right turns, i.e., $\mathcal{M} = \{\mathbf{L}, \mathbf{R}, \mathbf{L}, \mathbf{R}, \cdots\}$ or $\mathcal{M} = \{\mathbf{R}, \mathbf{L}, \mathbf{R}, \mathbf{L}, \cdots\}$. It is assumed that the UAV and UGVs are initially on top of each other; i.e., the initial x-y coordinates of the UGVs is $[x(0), y(0)]^T$. Define the angle between the heading of the UGVs and initial heading of the UAV as β. Hence, $\beta = \phi - \theta(0)$. Again, to simplify notations, it is assumed that all angles are taken modulus 2π.

The motion switches between $\mathbf{L}$ and $\mathbf{R}$ every time the paths of UAV and UGVs intersect. With this control strategy, the path of the UAV and the UGVs intersect every time the UAV flies for a circular arc of angle 2β. An example of the trajectory of the UAV and the UGVs are shown in Figure 5. The initial motion primitive of the motion sequence $\mathcal{M}$ depends on β. If $\beta \in [0, \pi)$, then the path of the UGVs is to the left of the initial heading of the UAV and the first motion primitive is $\mathbf{L}$, otherwise, the first motion primitive is $\mathbf{R}$. The following discussion focuses on the case when $\beta \in [0, \pi)$ and the motion sequence is $\mathcal{M} = \{\mathbf{L}, \mathbf{R}, \mathbf{L}, \mathbf{R}, \cdots\}$, since, if $\beta \in (-\pi, 0]$, then the path of the UAV is symmetric to the path corresponding to the angle of $-\beta$.

It is desirable to control the UAV to meet the UGVs periodically. This goal can be achieved by carefully choosing the initial heading of the UAV based on the speed of the UGVs. If the UAV executes the proposed control strategy, then it flies for a circular arc of angle 2β for each motion primitive in $\mathcal{M}$. Assume that the UAV meet with the UGVs at the end of each motion primitive. For each motion primitive, the UAV travels for a distance of $2R\beta$ and the UGVs travel for a distance

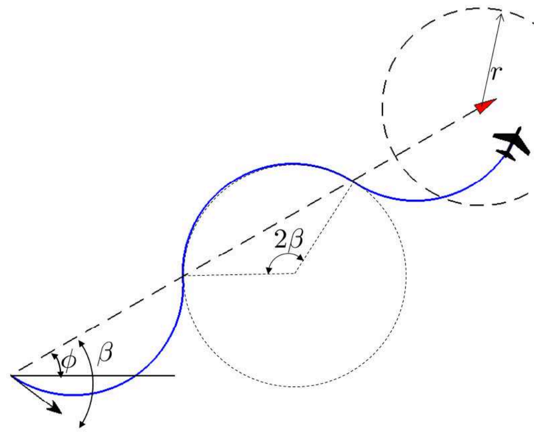


Figure 5: Example trajectory of a UAV providing convoy protection for the UGVs with the proposed control strategy. The solid curve is the path of the UAV. The dashed line is the path of the UGVs. The dashed circle is the convoy circle. The angle of the circular arc for each motion primitive is 2β. In this case, $\beta \in [0, \frac{\pi}{2}]$.

of $2R\sin(\beta)$. Since the UAV is unit speed, $V_G 2R\beta = 2R\sin(\beta)$, and therefore $V_G = \frac{\sin(\beta)}{\beta}$. In other words:

Lemma 2.4 *Assume that the UGVs move with constant speed V_G and heading ϕ, and the UAV starts at the same position as the UGVs with the initial heading $\theta(0)$. $\beta = \phi - \theta(0)$. Assume that $\beta \in [0, \pi)$ and hence $\mathcal{M} = \{\boldsymbol{L}, \boldsymbol{R}, \boldsymbol{L}, \boldsymbol{R}, \cdots\}$. Then if the UAV executes the proposed control strategy, and $V_G = \frac{\sin(\beta)}{\beta}$, then the UAV and the UGVs meet at the end of each motion primitive.*

Note that, if the UGVs travel with the same speed as the UAV, i.e. $V_G = 1$, then from Lemma 2.4, it follows that $\beta = 0$. In this case, the UAV will fly exactly on top of the UGVs. Using Lemma 2.4, one can thus obtain the lower bound for the speed of UGVs to achieve perpetual convoy protection with the proposed strategy.

Theorem 2.5 *Using the proposed control strategy, one UAV is sufficient to provide continuous convoy protection for all time, if V_G is bounded below by $V_G^\star$, where*

$$V_G^\star = \frac{\sqrt{2rR - r^2}}{R\arccos(1 - \frac{r}{R})}. \tag{20}$$

When $V_G < V_G^\star$, convoy protection cannot be provided with a single UAV and one needs to coordinate multiple UAVs to provide perpetual convoy protection. Using a similar approach as in the static convoys case, it is possible to determine the minimum number of UAVs required. In this case, on the path of each execution of one motion primitive, there are two segments of the path when the distance between the UAV and the UGVs is less than or equal to r. Positions of UAV and UGVs and hence their distance can be uniquely characterized by the arc angle γ as $d(\gamma)$. Convoy protection is provided by one UAV for two circular arcs of angle $\gamma^\star$ for each execution of one motion primitive, where $d(\gamma^\star) = r$. Refer to Figure 6 for an example.

Similar to the multi-UAV coordination approach in the previous section, one can use a timing strategy to schedule the UAVs such that, at any time, one of the UAVs is inside the convoy circle. First, note that the minimum number of UAVs required to provide continuous convoy protection can be obtained by the following corollary, which directly follows from the fact that, for each motion primitive, the length of the path in which one UAV stays inside the convoy circle is $2R\gamma^\star$, while the length of the entire path for the motion primitive is $2R\beta$.

Corollary 2.6 *Using the proposed control strategy, if $V_G < V_G^\star$, then the minimum number of UAVs needed to provide continuous convoy protection for all time is $N = \lceil \frac{\beta}{\gamma^\star} \rceil$, where $\lceil \cdot \rceil$ denotes the ceiling function. $\gamma^\star$ can be obtained by solving a non-linear equation $d(\gamma^\star) = r$ and β is obtained from V_G ($V_G = \frac{\sin(\beta)}{\beta}$).*

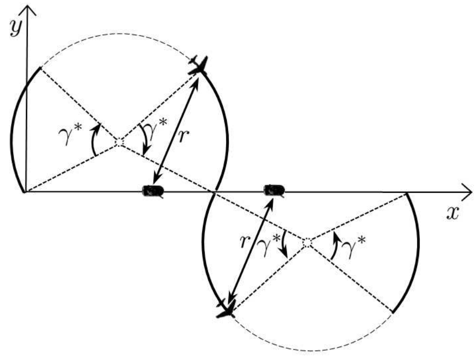


Figure 6: For each execution of one motion primitive, there are two segments of the path corresponding to two circular arcs of angle $\gamma^\star$, so that the distance between the UAV and the UGVs is less or equal to r when the UAV is on these segments. In this figure, the dashed curve is the path of the UAV, the solid curves are the segments of the path in which convoy protection is provided. The UAV and the convoys are drawn at the times when the UAV enters and exits these segments.

Figure 7 shows how one can schedule the UAVs to provide continuous convoy protection for all time. The key is to synchronize the position of the UGVs with individual UAVs at different times, so that when one UAV exits the convoy circle, there is at least one UAV inside the convoy circle and it is on the segment of its path in which the distance to the UGVs is less or equal to r.

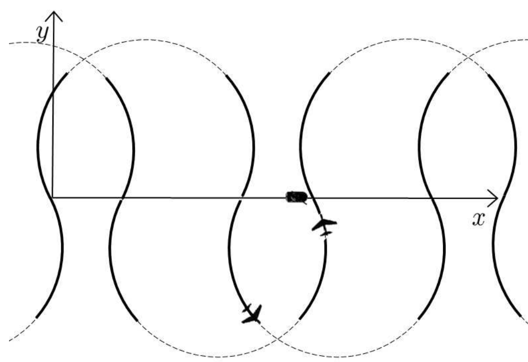


Figure 7: Example of using two UAVs to provide continuous convoy protection using the proposed strategy. In this figure, the dashed curves are the paths of the UAVs, the solid curves are the segments of the paths in which convoy protection is provided. In this case, every time one UAV exits the convoy circle, the other UAV is inside the convoy circle. This is always true if $N = \lceil \frac{\beta}{\gamma^\star} \rceil = 2$ and the times when the UAVs synchronize with the UGVs are spaced out.

3 The Assignment Problem

Now that suitable maneuvers have been obtained for making the UAV team fly over the ground convoy, the next issue concerns what UAVs should be dispatched from the convoy in order to investigate and potentially clear threats. To address this issue, one must solve the so-called assignment problem. The assignment problem is one of the fundamental problems in combinatorial optimization. In the setting of a UAV team providing ground convoy protection to a number of UGVs, it is important that the task is allocated to the right UAV and a certain cost objective is optimized. For the convoy protection mission scenario, the UGVs are traveling along a nominal path. The task of the UAVs is to provide protection to the ground vehicles, while clearing threats as they emerge in front of the path of the UGVs.

A natural choice for the cost objective is to balance the fuel consumptions of the UAVs. In a realistic convoy protection scenario, the fuel carried by each UAV is limited. By balancing the fuel consumption amongst the UAVs, the duration in which all UAVs remain operational is being maximized. Therefore, the maximum fuel consumption amongst the UAVs will be used as the cost function when designing the scheduling and task allocation algorithm for the UAVs.

To formulate the problem in a mathematical setting, one first needs to provide a model for the overall mission of the UGVs and the possible threats. Furthermore, since the convoy protection problem is highly related to the TSP problem, as already discussed, a number of simplifying assumptions are made in order to lower the complexity of the problem. This is of paramount importance as the algorithm must run repeatedly on a computationally limited resource in order to respond to dynamic threats.

In Section 1.2, a number of assumptions were discussed and here these assumptions are related to their impact on the formulation of the assignment problem. Assumption **A2**, simply says that the UGVs follow a set path. This is a realistic assumption as the UGV is expected to follow roads or other easily traversable structures in the environment. Along the nominal convoy path, there are a number of threats that the UAVs must survey and possibly clear. The threats are classified into two categories. One category of threats are *persistent*. These are threats that the UGVs must avoid since they cannot be cleared by the UAVs. And, as per Assumption **A3**, one also wants to allow for the possibility of threats not being cleared if they cannot be surveyed by a UAV in time. In fact, it is assumed that the UAV-UGV team can only detect the threats up to a distance D away from the location of the UGVs and, as a consequence of Assumption **A1**, the threat density is such that no more than a given number M of threats can appear within this distance at any given time. But, if a threat pops up in front of the UGVs within distance D and if all available UAVs are dispatched beyond the threat, clearing of the threat may not be possible. If this is the case, they will be treated as persistent threats. In both of these cases (inspected persistent threats and threats that can not be inspected in time) the path of the UGVs will be re-planned to avoid the threats.

The other category of threats are *non-persistent*, in which case the threats are removed after being checked by the UAVs. Regardless of the type, all threats must, if possible, be checked and surveyed by the UAVs and the threat category cannot be established without an UAV inspecting the location of the threat. If a UAV does not get to a threat in time, it will be treated as a persistent threat and, as such, force a re-planning of the UGV convoy.

In order to formulate the assignment problem, the UGVs are modeled as single point masses moving along a one-dimensional (not necessarily straight) path as shown in Figure 8. The UAVs are either flying alongside the UGVs (meaning they are at the same position along the path), or they can fly ahead of the UGVs. As they fly ahead of the UGVs, they do not have to follow the path. Instead, they are assumed to follow the Euclidean shortest distance to the target location. Hence, for this model, the motion of the UGVs is viewed as moving along a one-dimensional corridor which, as already discussed, allows one to establish an order among the threats which significantly cuts down on the complexity of the problem. At the same time, this is a realistic assumption that does not significantly limit the applicability of the proposed method.

With this model, the assignment algorithm does not consider the kinematic constraints of the UAVs and UGVs, and the UAVs are assumed to be able to change direction and turn around while checking threats. At time t, the position of the UGV team is denoted to be $x_g(t)$. The positions of the individual UAVs are denoted as $x_i(t)$, where $i \in \mathcal{I}$ denotes the index of the individual UAVs, and $\mathcal{I}$ is the set of all UAV indices.

It is assumed that the team can detect threats for a distance of D in front of the UGVs. Hence only threats contained in this window are known to the UGV-UAV team. At time t, it is assumed that there are $N(t)$ threats in this range. The location of threats are assumed to be fixed, and they are denoted as $\tau_j, j = 1, 2, ..., N(t)$. The sequence of the known threats at time t are denoted as $\bar{\tau}(t) = [\tau_1, \tau_2, ..., \tau_{N(t)}]$. This known threat sequence has time-varying length $N(t)$. $N(t)$ increases when a pop-up threat is detected, and decreases when a threat is cleared. Note that all threats (pop-up or not) are in front of the UGVs on the intended path.

Furthermore, let the total number of threats be denoted by N (where, for notational convenience, the explicit dependence on t is suppressed, as will be done in the remaining chapter), and the sequence of all threats is denoted as $\bar{\tau} = [\tau_1, \tau_2, ..., \tau_N]$. The overall mission of the UAVs is to clear all the threats. If any of the threats are hostile after being checked out by a UAV, then the path taken by the UGVs is re-planned, the problem is re-initialized, and the assignment algorithm is

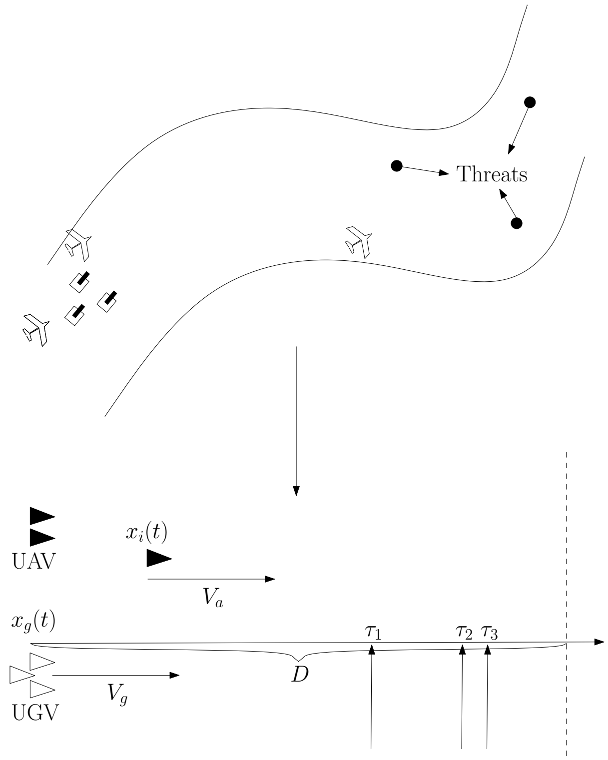


Figure 8: The assignment problem model

restarted.

Naturally, when the UAV is checking a threat, it should fly with a higher speed than when it is cruising along with the UGVs. The velocity of the UGV team is assumed to be V_g. It is assumed that when the UAVs are flying alongside the UGVs, they fly with the same speed V_g. However, when they are assigned to clear a threat or coming back after clearing a threat, they fly with the speed V_a. Throughout of this chapter it is assumed $V_a > V_g$.

Let the starting time of the overall mission be denoted by t_0, and the time when all N threats are cleared by T. It should be noted that since the aim is to develop a real-time algorithm to select and assign the UAVs to clear the threats, the only information available at time t is the $N(t)$ threats within range D, and the assignment algorithm only uses this information to make decisions.

The problem is to devise algorithms to dispatch the UAVs to clear all upcoming threats while balancing the fuel consumptions of the UAVs. The accumulated fuel consumption of i-th UAV at time t is denoted as $f_i(t)$. $f_i(t)$ is considered to be the state of the UAVs, and they are known. It should be noted that at the beginning of the mission, the accumulated fuel consumption of all the UAVs are 0, hence $f_i(t_0) = 0, \forall i$. With this information, one can formulate the UAV assignment problem.

Problem 3.1 *Design an algorithm that assigns UAVs such that the maximum fuel consumption amongst the UAVs is minimized when all threats $\bar{\tau}$ are cleared., i.e., solve the optimization problem:*

$$\min_{\Pi} \max_{j \in \mathcal{I}} \{f_j(T)\}, \tag{21}$$

where Π is a assignment algorithm that maps from time to $\mathcal{I}$.

3.1 Selection Policy and Task Allocation Algorithm

In this section, an algorithm is introduced that assigns the UAVs based on their current level of accumulated fuel consumption. The algorithm uses an optimization-based selection policy that selects (and assigns) one threat to one of the UAVs when an event triggers. Events that trigger the re-evaluation of the selection policy can be one of the following:

1. A threat is cleared by any of the assigned UAV.
2. A pop-up threat is detected.

In other words, the selection policy is only re-evaluated when the known threat sequence $\bar{\tau}(t)$ is modified. In order to allow real time execution of the algorithm, the optimization problem needed for the selection policy is chosen so that it can be solved quickly on-line. In order to reduce the problem from NP hard to polynomial complexity, only the closest threat is considered when an event triggers. This does not cause inefficiency in clearing multiple threats since each time a threat is cleared, the selection policy is re-evaluated. The position of the closest threat is denoted by $\tau \in \mathbb{R}^2$. Hence,

$$\tau = \min_{j \in \{1,2,\ldots,N(t)\}} \tau_j. \tag{22}$$

The selection policy is a function that maps time t to the set of the UAVs ($\mathcal{I}$), and it is the solution of an optimization problem. This optimization policy is based on computing the fuel consumption associated with letting a UAV clear the target threat (τ) and returning to the convoy. Even though this may actually not be what is done (as new threats are constantly reconsidered), the resulting algorithm minimizes the accumulated fuel consumption over all other assignments assuming that the assigned UAV will return to the convoy after the threat is inspected. Here, it is taken into account that the UGVs are moving constantly along the path with speed V_g, and that the closest threat τ may be a pop-up threat.

As fuel consumption is at the heart of the assignment costs, one needs to establish an appropriate model for this consumption. Although the developed algorithm must be quite general and should not rely on any particular choice of fuel model, the fuel consumption rate at any given moment is a nonlinear function of the UAV's speed, weight, and altitude. Therefore, the fuel consumption rate of the ith UAV at time t is given by

$$\frac{df_i(t)}{dt} = Q(v(t), w(t), h(t)), \tag{23}$$

where $v(t), w(t), h(t)$ are the UAVs speed, gross weight, and altitude at time t, respectively.

When an event triggers at time t, the following optimization problem is solved:

$$P_1(t) = \arg\min_{i\in\mathcal{I}} \left\{ \max_{j\in\mathcal{I}} \left\{ f_j(t) + \left\{ \begin{array}{l} Q(V_a, w^*, h^*)T_i(t), \text{ if } j = i \\ Q(V_a, w^*, h^*)S_j(t) + Q(V_g, w^*, h^*)(T_i(t) - S_j(t)), \text{ if } j \neq i \end{array} \right\} \right\} \right. . \tag{24}$$

The optimization problem is a min-max problem. $T_i(t)$ represents the time for i-th UAV to clear the threat and return to the UGVs (with speed V_a). If the j-th UAV is not assigned, $S_j(t)$ represents the time for it takes to return to the UGVs (with speed V_a). $T_i(t) - S_j(t)$ therefore represents the time for the j-th UAV to fly alongside the UGVs with speed V_g until the assigned UAV flies back. If the UAV j is flying alongside the UGV at time t, then $x_j(t) = x_g(t)$ and $S_j(t) = 0$. Hence, the quantity

$$f_j(t) + \left\{ \begin{array}{l} Q(V_a, w^*, h^*)T_i(t), \text{ if } j = i \\ Q(V_a, w^*, h^*)S_j(t) + Q(V_g, w^*, h^*)(T_i(t) - S_j(t)), \text{ if } j \neq i \end{array} \right. \tag{25}$$

represents the accumulated fuel consumption of the j-th UAV, assuming UAV i is assigned.

It is important to note here that the convoy protection problem formulation, along with the corresponding task assignment algorithm presented in this section, are independent of how the quantities $T_i(t)$ and $S_j(t)$ are computed for a specific operational scenario. In the experiments these quantities were computed assuming piecewise-linear convoy path defined by a series of waypoints. As shown in Figure 9, $P_1(t)$ assigns the UAV so that the UAV with most accumulated fuel consumed after the clearance of the threat is minimized.

Min-max problems are generally hard to solve, and the complexity of computing $P_1(t)$ is $O(|\mathcal{I}|^2)$, where $|\mathcal{I}|$ is the number of UAVs. However, under certain design choices of UAV speeds V_a and V_g, weight w^*, and altitude h^*, one can reduce the optimization problem to one that has $O(|\mathcal{I}|)$ (linear) complexity. The assumption needed for this is that it is more costly in terms of fuel consumption to be inspecting a threat than to remain with the convoy. And, under this assumption, one can rewrite the assignment problem in the following manner:

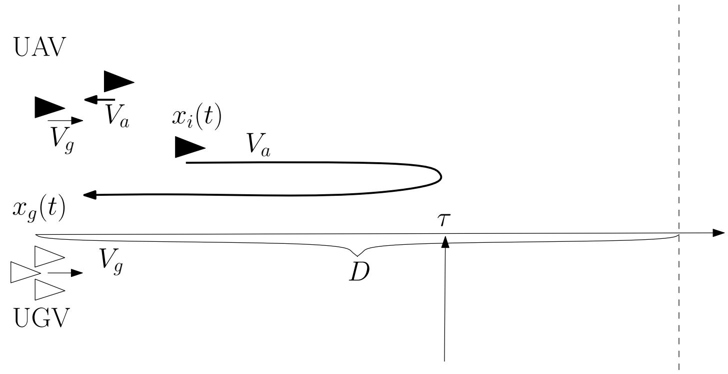


Figure 9: The operation of the selection policy. In this example, the threat is not a pop-up

Lemma 3.1 *If UAV speeds V_a and V_g, gross weight w^*, and altitude h^* are chosen such that*

$$Q(V_a, w^*, h^*) > Q(V_g, w^*, h^*), \tag{26}$$

then the optimization problem $P_1(t)$ can be solved by solving another optimization problem:

$$P(t) = \arg\min_{i \in \mathcal{I}} \{f_i(t) + Q(V_a, w^*, h^*)T_i(t)\}. \tag{27}$$

Hence, $P(t) = P_1(t)$.

This reformulation is useful in that it actually allows one to solve the assignment problem directly. To describe in words, $P(t)$ assigns the UAV so that, after the threat is cleared and returned to the UGVs, the total fuel consumed for the assigned UAV is minimum over all the UAVs. The projected time for the clearance of the threat is $T_i(t)$, where i denotes the assigned UAV. It should be noted that during this time, the UGVs has moved forward by $V_g T_i(t)$ distance.

Based on this reformulation, one can in fact formulate the corresponding task allocation algorithm which uses the above selection policy.

Algorithm: Task Allocation Algorithm
Initialize: Set t_0.

Iterate: until end of mission

If an event is triggered:

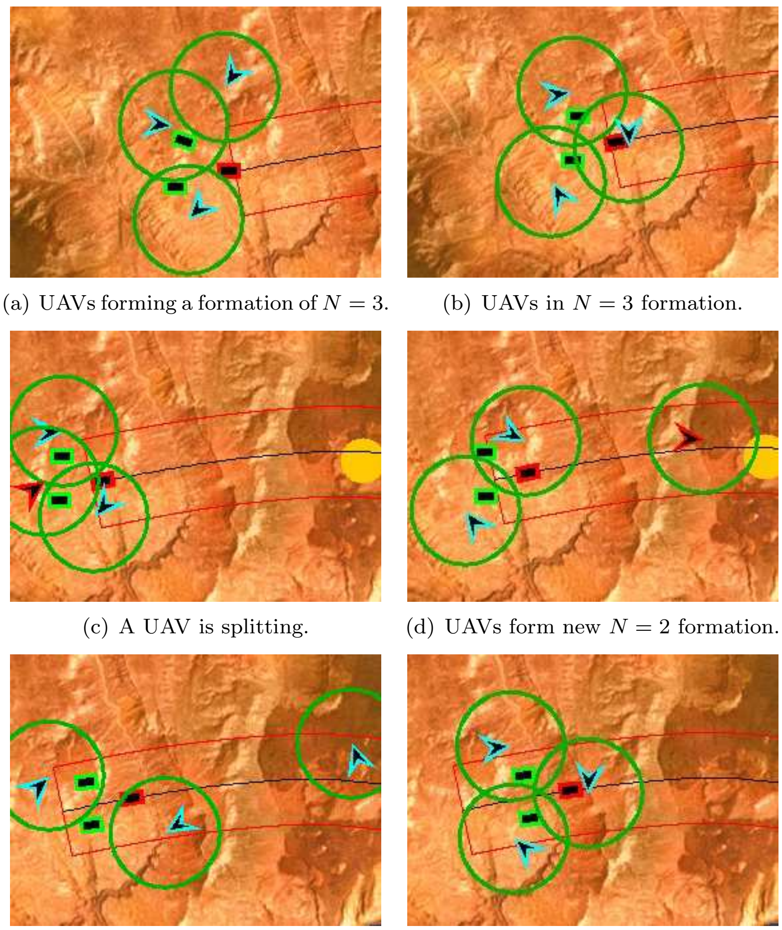
1. update $\bar{\tau}(t) = [\tau_1, \tau_2, ..., \tau_{N(t)}]$.
2. find $\tau = \min_{j \in \{1,2,\ldots,N(t)\}} \tau_j$.
3. Solve $i^\star = P(t)$ in Eq. (27) and use the solution $i^\star$ as the assignment for the i:th UAV to clear τ.

Wait until another event is triggered.

An illustration of the intended operation is depicted in Figures 10(a)-10(f). The UAVs will follow the ground vehicles in a set formation. If a threat is detected, a UAV will be assigned, that will balance fuel usage among the UAVs, to clear the threat and return to the formation.

4 Experimental Results

To demonstrate the practical aspects of the proposed algorithms as well as to show that they can indeed be successfully deployed in an operational environment characterized by numerous computational and communications limitations, the algorithms were implemented on a testbed consisting of both UAVs and UGVs.



(a) UAVs forming a formation of $N = 3$.
(b) UAVs in $N = 3$ formation.
(c) A UAV is splitting.
(d) UAVs form new $N = 2$ formation.
(e) A UAV is re-joining.
(f) UAVs return to a formation of $N = 3$.

Figure 10: Simulation of three UAVs executing the convoy protection algorithm. (N is the number of UAVs along the UGV convoy.)

The experiment consists of a mission that incorporates all details highlighted in this chapter. The mission is structured as follows:

1. The UGVs are given a set of waypoints which define a path to be taken through the (possibly hostile) environment. All UAVs are assigned to protect the convoy by flying over it while maintaining some spacing from one another.

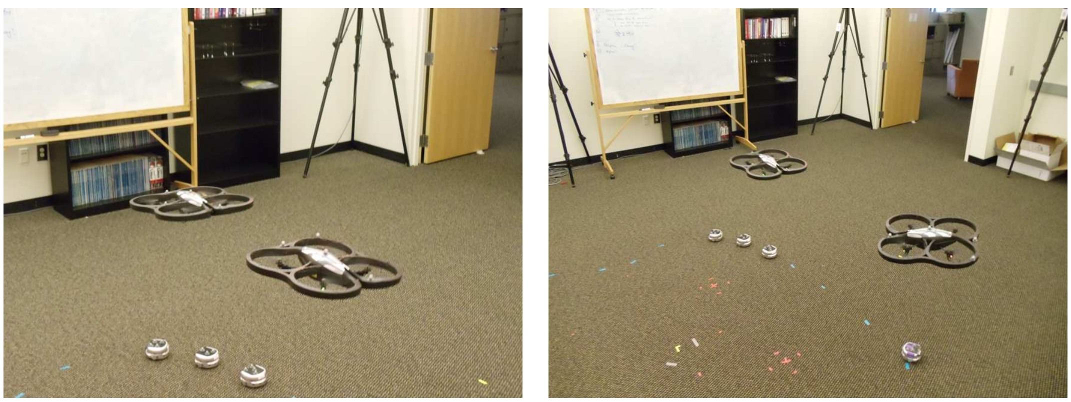
2. Once a possible threat has been identified, the projected fuel consumption for each UAV is computed, which in turn dictates which UAV to dispatch so as to balance fuel consumption.

3. The chosen UAV flies over to visit the threat, while the remaining UAVs remain with the convoy.

4. After the threat is neutralized (cleared), the assigned UAV returns to protect the convoy.

5. In the event that a threat cannot be neutralized (is persistent), the UAV signals the convoy to re-plan its path to avoid the threat.

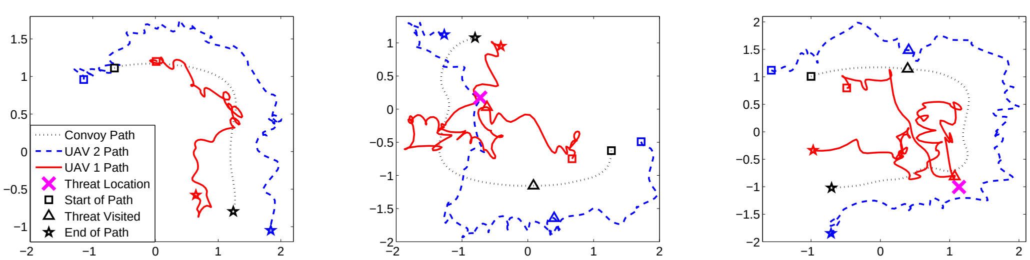
The plots in Figure 12 show the recorded trajectories of the UAVs and convoy during the experiment. In particular, Figure 12(a) shows both UAVs performing convoy protection by flying above the UGVs as they follow their intended path. Notice that the UAVs maintain a safe separation from one another. A photograph of this operation is shown in Figure 11(a).



(a) Two UAVs protect the convoy while maintaining spacing from one another.

(b) One UAV is dispatched to visit a threat, while the other remains with the convoy.

Figure 11: Photos showing the convoy protection and threat neutralization as carried out by the hardware platform.



(a) Two UAVs perform convoy protection on UGVs.

(b) A UAV is dispatched to clear a non-persistent threat.

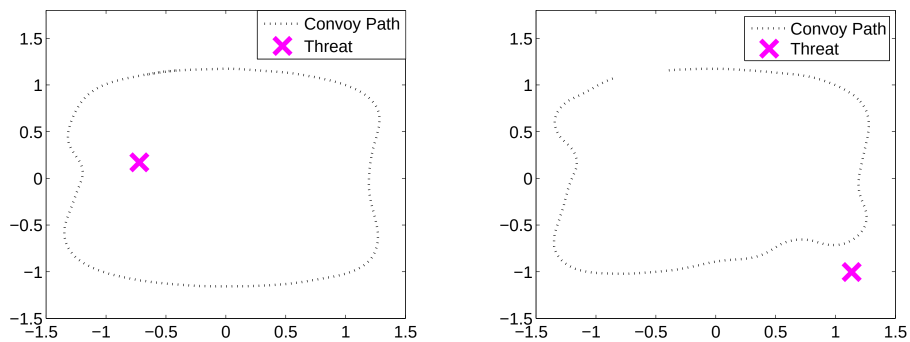
(c) A UAV identifies a persistent threat and the convoy re-plans its path.

Figure 12: Plots showing the trajectories of UAVs and the convoy which it protects as both a non-persistent and persistent threat are encountered.

Figure 12(c) shows the UAVs detecting a persistent threat which blocks the intended path of the convoy. The UAVs execute the task allocation algorithm to determine which UAV should be dispatched so as to balance fuel consumption. When the dispatched UAV determines that the threat is persistent, the UAV informs the convoy that it must replan its path so as to avoid the threat. Afterwards, the UAV returns to the convoy and the UGVs proceed in following the recomputed path to safely reach their destination.

To further illustrate effects of convoy path replanning, Figure 13(a) shows the originally intended path of the convoy corresponding to the trajectories in Figure 12(a) and 12(b). Meanwhile, Figure 13(b) shows the recomputed path taken by the convoy so as to avoid the persistent threat.

To validate that the task allocation algorithm performs as expected, a separate experiment was conducted in which 16 threats were presented to the convoy over an extended period of time. Figure 14 shows the fuel consumption of the two UAVs over this time and marks when a UAV is dispatched to clear a threat. As seen in the plot, the algorithm successfully determines which UAV to dispatch when a threat is encountered such that the fuel consumption is balanced amongst the two UAVs over time. This oftentimes results in the same UAV being dispatched multiple times consecutively so as to clear threats that are located within its vicinity.



(a) The original intended path taken by the convoy, corresponding to Figures 12(a) and 12(b).

(b) The recomputed path of the convoy upon being informed of a persistent threat, corresponding to Figure 12(c).

Figure 13: Plots showing the difference in path taken by the ground convoy before and after being informed about the presence of a persistent threat.

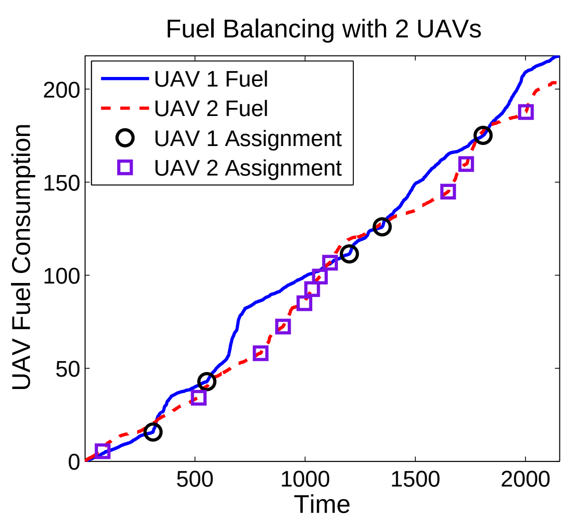


Figure 14: Plot showing the fuel consumption of the two UAVs while performing convoy protection.

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