Pair-wise Agreement Using Set-Valued Sensors ⋆

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Abstract: In this paper, we study the effect that set-valued sensors have on coordination algorithms. In particular, we investigate the two-agent rendezvous problem with severely limited sensing. We characterize conditions under which a agent can locate a stationary target when the sensing modality is probabilistic and set-valued. We generalize this to the case where a mobile agent is locating the target and show that the location of the stationary agent can be approximated up to a set of measure zero.

1. INTRODUCTION

A typical example of a large scale multi-agent network is a naval surveillance network comprised of ground, underwater, surface and aerial unmanned vehicles which are required to coordinate to achieve different tasks like reaching agreements, achieving formations and area coverage. These tasks have been extensively studied by the multi-agent community (e.g Tanner et al. [2007], McNew et al. [2007], Lawton et al. [2003], Shucker et al. [2006] and Howard et al. [2002]).

Traditional coordination algorithms involve manipulation of states defined at different nodes in the network and requires measurements which would allow individuals in the network to estimate the state. For example, consider the problem where a collection of l agents with states $(x_1, x_2 \dots x_l)$ (which can be positions, orientation etc) are trying to reach an agreement. Collecting the states $(x_1, x_2 \dots x_l)$ into a single state vector $x \in \mathbb{R}^l$, agreement can be achieved by executing the agreement protocol given by:

$$\dot{x} = -Lx \tag{1}$$

where L is the graph Laplacian for some interaction graph G which captures how the agents influence each other. The agreement protocol forces the state vector x to converge to the average of the initial state vector $x(0)$. This coordination algorithm has been extensively analyzed by the multi-agent controls community (eg Olfati-Saber et al. [2007], Jadbabaie et al. [2003]). To execute the agreement protocol, the most important piece of data required by an agent i is the relative state displacement $(x_i - x_j)$ with respect to its neighbour j. It is, in general, assumed that every agent in the network is able to obtain this information via communication with neighbouring agents or using sensor measurements.

But as we push to reduce the size of robots to micro and nanoscales, it is not possible to equip them with high-precision sensors due to size and weight constraints. One natural consequence is that sensors operating at such a scale are less reliable and tend to drift even in short time scales as they are highly susceptible to noise (see Dahlin [2012]). As a result, repeated measurements under the same conditions produce different results. The amount of reliable information that can be extracted from such measurements are severely limited. For instance, an agent located in an environment $W \subset R^k$ might be able to locate a subset of W in which its neighbour might be residing instead of a single point $p \in R^k$ relative to its own location. This paper investigates coordination algorithms in the context of such limited sensing.

This work is also motivated by the need to understand the minimum amount of information required to achieve specific tasks so that we can equip the agents with simplest and cheapest sensors which will allow a multiagent network(of any scale) to function properly. We model the uncertainity associated with such limited sensing hardware by developing a set representation of potential measurements that can be made and develop motion strategies that operate on these sets instead of points. The impact of limited sensing modalities has already been investigated in the context of robot localization problem in O'Kane and LaValle [2005], Erickson et al. [2008] and O'Kane and LaValle [2007].

As a first choice, we consider the two-agent rendezvous which involves two agents trying to rendezvous at some arbitrary point in the environment in which they are deployed. One of them is equipped with no sensors and is stationary and the other one is equipped with a set-valued sensors which returns sets when the agent attempt to make a measurement instead of scalar values like relative displacement. A similarly themed work is Fagiolini et al. [2011] which studies the impact of set-valued information exchange on the consensus problem via set-valued difference equations. The key difference between our work and theirs is that we focus on set-valued sensing and assume that the agents involved cannot communicate with each other.

The rest of the paper is organized as follows. In section 2,we describe set-valued sensors. In section 3, we study characterize the conditions under which the agent would be able to locate the other agent without moving. In

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section 4, we devise a motion strategy and prove that the strategy enables the agent to approximate the location of the other agent upto a zero measure set. In section 5, we repurpose the motion strategy devised in section 4 so that it can be implemented in hardware. Section 6 provides experimental results followed by the conclusion.

2. SET-VALUED SENSORS

Consider two agents, named S and T (searcher and target), deployed in a compact and connected environment $W \subset \mathbb{R}^k$. Agent S is equipped with a probabilistic set-valued sensor and is capable of moving in any direction. The agent is also aware of the topography of the environment and can plan its path between any 2 points in the environment. We would like to characterize the conditions under which the searcher S can locate the stationary target T situated at a unknown location x_T based on the uncertain measurements obtained from the sensor.

Since the environment W is a subset of $\mathbb{R}^k$, it is possible to assign volumes to subsets of W by using the standard Lebesgue measure defined on $\mathbb{R}^k$.

***Definition** 1.* Let (W, Σ_W, λ) be a measure space, where Σ_W denotes the Lebesgue measurable sets contained in W and λ is the normalized Lebesgue measure i.e ($\lambda(W) = 1$).

The sensor model is defined as follows. The potential measurements which can be made by the searcher S is a subset of the measurable sets (those which contain the target position x_T) given by

$$\mathcal{M} = \{M \mid x_T \in M, M \in \Sigma_W\}. \qquad (2)$$

A **measurement** made by the searcher S is simply a element of $\mathcal{M}$ sampled according to an arbitrary probability distribution imposed on $\mathcal{M}$ (see Fig 1). It is important to note that the searcher S has no control over this distribution. The probability distribution on $\mathcal{M}$ is chosen to reflect the performance and efficiency of the sensor. A measurement made with a good sensor would return a small open ball centered at x_T with high probability. One example of a "poor" sensor would be a camera. In that case, the searcher S would be able to determine that the target T is not in the immediate vicinity, which can be described by a set U, by processing the image returned by the camera. Then it can be inferred that the target lies in the complement of U. We can also consider sensors which can detect the general direction in which the target lies and thus allow us to determine a half-space in which the target lies.

In the following sections, we characterize the properties which the probability distribution on $\mathcal{M}$ must possess in order for the searcher S to be able to locate the target without moving. We also provide a motion strategy which can be employed by the searcher to recover the location of the target independent of the probability distribution on $\mathcal{M}$.

3. STATIONARY SEARCHER, STATIONARY TARGET

In this section, we assume that the searcher S and target T are located at x_S and x_T and do not change their locations.

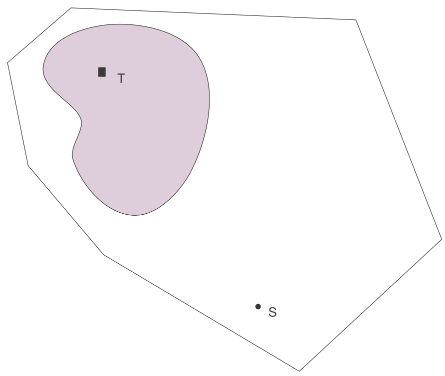


Fig. 1. S is the searcher and T is the target. The polygon represents the world W and the colored set represents the measurement

The stationary agent, stationary target is worthy of study as detecting another agent is usually considered to be a passive operation. Therefore, it is important to determine whether the searcher can solve the problem without taking any action. Also, making repeated measurements is a cheaper operation than moving around and therefore much preferable.

Let $\{M_i\}_{i=1}^{\infty}$ denote an infinite sequence of measurements made by the searcher S. The interpretation is that the measurement M_n is made before the measurement M_{n+1}. After making n measurements, S can improve its estimate of the location of the target T by simply combining its previous measurements by taking intersections. Define $C_n = \bigcap_{i=1}^{n} M_i$. The set C_n can be interpreted as the smallest region of the map W_m which contains the agent T given measurements $M_1, M_2, ...M_n$. The set C_n is also measurable for every n as it is constructed by taking countable intersections of measurable sets. Then, the volume of the set C_n given by $\lambda(C_n)$ can be regarded as a measure of uncertainity. Note that C_{i+1} is not necessarily a proper subset of C_i as it is possible to make measurements which do not yield any new information (i.e $C_i \subseteq M_{i+1}$). The following properties of the sequence $(\lambda(C_1), \lambda(C_2), \lambda(C_3)....)$ can be quickly established.

1) $\lambda(C_i) \leq 1$ (As $\lambda(W) = 1$ and $C_i \subset W$)
2) $\lambda(C_i) \geq 0$ (Non-negativity of measure)
3) $\lambda(C_{i+1}) \leq \lambda(C_i)$ (Follows from the recursive relation $C_{n+1} = C_n \cap M_{n+1}$)

The third point can be interpreted as "The uncertainity associated with the position of the target T can only decrease". More compactly, the sequence $(\lambda(C_1), \lambda(C_2), ...)$ is a monotone non-increasing sequence bounded below by 0 and bounded above by 1. If the sequence $(\lambda(C_1), \lambda(C_2), ...)$ converges to 0, then it means that the searcher S can approximate the position x_T of the agent T upto a set of measure 0.

We can identify the conditions under which the sequence $(\lambda(C_1), \lambda(C_2), ...)$ converges to 0 by studying the set of monotone non-increasing sequences which are bounded above by 1 and below by 0. This in turn can be used to determine conditions on the distribution on set of potential measurements $\mathcal{M}$.

Let the set Λ denote the set of all monotone infinite non-increasing sequences which are bounded below by 0 and bounded above by 1. Every element of the set Λ can be thought of as a potential trajectory taken by the sequence of measurements $\{\lambda(C_n)\}$.

Definition 2. Let $\omega = (\omega_1, \omega_2, ...) \in \Lambda$. Then the **nth projection** $V_n : \Lambda \to [0, 1]$ is defined as $V_n(\omega) = \omega_n$.

Let $\mathcal{F}$ denotes the smallest σ-algebra on Λ such that V_i **is a random variable for every** i. Let $P : \mathcal{F} \to [0, 1]$ represent a probability measure on $\mathcal{F}$ which is chosen as to represent the distribution from which the measurements are drawn. Then $(\Lambda, \mathcal{F}, P)$ represents a probability space. Note that $V_{i+1}(\omega) \leq V_i(\omega)$ for every ω and i. It follows that $\Delta V_i(\omega) = V_{i+1}(\omega) - V_i(\omega) \leq 0$. Let $\mathbb{E}$ denote the expectation functional associated with $(\Lambda, \mathcal{F}, P)$. The indicator function is $1_\Lambda(\omega) : \Lambda \to [0, 1]$ is such that $1_\Lambda(\omega) = 1$ and its expectation is $\mathbb{E}(1_\Lambda) = P(\Lambda) = 1$. Since $V_i(\omega) \leq 1_\Lambda(\omega)$, $\mathbb{E}(V_i) \leq \mathbb{E}(1_\Lambda) = 1$.

Definition 3. Let $\omega = (\omega_1, \omega_2, ...) \in \Lambda$. Let V_n denote the nth projection. Then $V_*(\omega) = \lim_{n\to\infty} V_n(\omega)$.

V_* is well defined as $V_*(\omega)$ is the limit of of the sequence ω which always exist. V_* is also a random variable as it is the pointwise limit of the random variables V_i. If we interpret $V_i(\omega)$ as the amount of uncertainity remaining after the ith measurement, Then $V_*(\omega)$ can be interpreted as the total uncertainity associated with the position of the target after all the measurements have been made. The following theorem the provides us with the conditions under which this uncertainity goes to 0.

Theorem 1. Let V_n denote the nth projection. Then $V_* = 0$ almost surely if and only if $\lim_{n\to\infty} \mathbb{E}(V_n) = 0$.

Proof.

Let 1_Λ denote the indicator function for Λ.

Assume $\lim_{n\to\infty} V_n = V_* = 0$ almost surely.
Since $\lim_{n\to\infty} V_n = V_*$ and $|V_i| \leq 1_\Lambda$, by the dominated convergence theorem, we have $\lim_{n\to\infty} \mathbb{E}(V_n) = \mathbb{E}(V_*) = 0$ as $V_* = 0$ almost surely.

Now, assume $\lim_{n\to\infty} \mathbb{E}(V_n) = 0$.
Since $|V_i| \leq 1_\Lambda$ for all i and $V_*(\omega) = \lim_{n\to\infty} V_i(\omega)$, by the dominated convergence theorem it follows that $\mathbb{E}(V_*) = \lim_{n\to\infty} \mathbb{E}(V_n) = 0$.
Then, we can express

$$E(V_*) = \int_\Lambda V_* dP = \int_{V_*=0} V_* dP + \int_{V_*>0} V_* dP = 0. \quad (3)$$

Then $\int_{V_*>0} V_* dP = 0$ which is true only if $P(V_* > 0) = 0$. Therefore $V_* = 0$ almost surely.

□

Suppose the P is such that the above theorem holds,then it follows that $P(V_* = 0) = P(\{\omega \mid \lim_{n\to\infty} \omega_n = 0 \text{ and } \omega \in \Lambda\}) = 1$. This means almost every sequence ω in Λ converges to 0 provided $\lim_{n\to\infty} \mathbb{E}(V_n) = 0$.

Relating the sequences in Λ to sequences of measurements $\{\lambda(C_n)\}$, this implies that almost every seqeunce of measurement allows us to approximate the location of the target T upto a set of measure 0. This does not necessarily imply that we would be able to recover the location T as

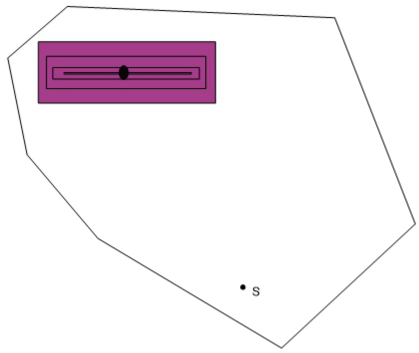


Fig. 2. The label S identifies the location of the searcher. The colored rectangles which contain T represent the measurements made by the searcher. It can be seen that the intersection of the rectangles forms a line which has a Lebesgue measure of 0

measure zero sets are not necessarily singletons (for e.g an infinite set of isolated points has measure 0). Things can be improved a little bit by imposing more restrictions on the set $\mathcal{M}$. For instance, we could require that all the sets in $\mathcal{M}$ be convex. This improves things as $\bigcap_{i=1}^{\infty} M_i$, for $M_i \in \mathcal{M}$ will now be convex and would have measure zero. But this still does not let us figure out T as the result of our measurements could be a line(e.g see Fig 2) which again has a zero measure.

Also, since we have no control over the distribution itself, there is no gurantee that the above theorem will hold. But the above analysis is done under the assumption that the agent located at x_S does not move during the measurement process. But since the measurements drawn from $\mathcal{M}$ are not influenced by the agent's location, there is nothing to be gained from mobility. This observation suggests a modification to the above analysis. If we allow the position of the agent to influence potential measurements, can we improve the above situation and devise a motion strategy for locating the target which is independent of the probability distribution? The question is answered in the affirmative in the following section.

4. MOVING SEARCHER, STATIONARY TARGET

In this section, we again assume that the target T is located at x_T. Now we assume that the searcher S is allowed to move and makes measurements sequentially from different positions and we let x_i denote the position of the agent when it makes the ith measurement. Let (W, Σ_W, λ) be the measure space given by Definition 1. We also assume that W is a **convex set**.

Let the set $\mathcal{H}_x$ denote the set of all closed half-spaces which contains x in their boundary. The set

$$\mathcal{M}_x = \{B \mid B = S \cap H_x, T \in H_x, H_x \in \mathcal{H}_x\} \quad (4)$$

denotes the set of all potential measurements which can be made by an agent located at $x \in S$. Every set in $\mathcal{M}_x$ is measurable as they are closed because Σ_s contains all the closed sets and is convex as S and closed halfspaces are convex. Since the measurements made by agent are forced to contain the agent in its boundary, the agent

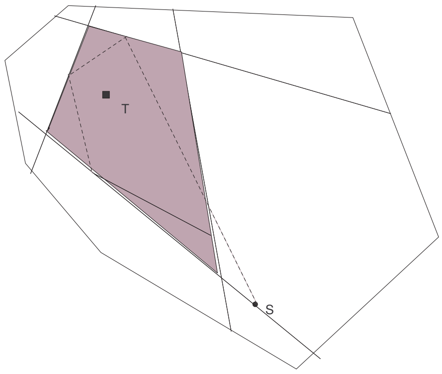


Fig. 3. The dotted lines represent the path taken by the searcher S. The points where the dotted lines meet the solid lines represent the position from which measurements where made. The colored set again represents the intersection of the measurements made upto that point

can influence the measurements by moving around(see Fig 3). We make the following definition which allows us to prove an lemma regarding the influence of position on uncertainity.

***Definition** 4.* Let $X \subset S$ be a set with non-empty interior. A point $x \in X$ is said to be **of at least depth** d **in** X if there exists a open ball $B_r(x) \subset X$ such that $r \geq d$.

One way in which the searcher S can control uncertainty is by being able to reduce uncertainty "locally" by driving to different depths as shown by the lemma given below.

Lemma 1. Let C be a measurable set which contains the target T. Let $x \in C$ be a point of at least depth d in C. Then for every $M \in \mathcal{M}_x$, the following holds

$$\lambda(M \cap C) \leq \lambda(C) - \tfrac{1}{2}gd^n \quad (5)$$

where g is the Lebesgue measure of the unit ball and n is the dimension of the euclidean space of which the environemnt W is a subset.

Proof. Let $g = \lambda(B_0(1))$. Let $M \in \mathcal{M}_x$. Since x is of at least depth d in C, $B_d(x) \subset C$. Let $C_a = C \setminus B_d(x)$ and $C_b = B_d(x)$. So $C = C_a \cup C_b$ and $C_a \cap C_b = \emptyset$ where $\emptyset$ is the empty set. Note that $\lambda(C_a) = \lambda(C) - gd^n$ and $\lambda(C_b) = gd^n$. Then $\lambda(C \cap M) = \lambda(C_a \cap M) + \lambda(C_b \cap M)$ as C_a and C_b are disjoint. Since every half plane centered at x bisects every open ball centered at x, $\lambda(C_b \cap M) = \frac{1}{2}gd^n$. We note that $\lambda(C_a \cap M) \leq \lambda(C_a) = \lambda(C) - gd^n$ and obtain the required inequality as follows:

$$\lambda(C \cap M) \leq \lambda(C) - gd^n + \tfrac{1}{2}gd^n = \lambda(C) - \tfrac{1}{2}gd^n \quad (6)$$

□

The above lemma states that if the searcher S drives to a depth d into a set C and makes a measurement from that point, then it is guranteed to reduce the uncertainty atleast by $\frac{1}{2}gd^n$. We can use this to devise a motion strategy.

***Definition** 5.* Let $X \subset W$ be a convex set. Then the **depth of X** is $d_X = sup\{r \mid B_r(x) \subset X, x \in X\}$

Note that if $d_X = 0$ for a set $X \subset W$, $\lambda(X) = 0$. This follows from the observation that if $d_X = 0$, then X has empty interior and therefore $X = \partial X$. Then it follows from the fact that the boundary of a bounded convex set has zero measure that $\lambda(X) = 0$.

***Definition** 6.* Let $C_0 = W$ denote the searcher S's initial estimate of T before it makes any measurements. The **motion strategy** for the searcher S is as follows. Let $\alpha \in [0, 1]$. Let $x_i \in C_i$ be of at least depth αd_{C_i} in the set C_i (i.e the agent moves to a fraction α of the depth d_{C_i} of its current estimate C_i to make its next measurement) and let $C_{i+1} = C_i \cap M$ where $M \in M_{x_i}$.

Now, the claim is that d_{C_i} converges to 0.

Theorem 2. Let C_i, x_i and α be as defined by Definition 6 . Then $lim_{n\to\infty} d_{C_n} = 0$.

Proof. We begin by noting that since $C_{i+1} \subseteq C_i$, $d_{C_{i+1}} \leq d_{C_i}$ as shrinking the set reduces its depth. Then $\{d_{C_i}\}_{i=1}^{\infty}$ is a monotone non-increasing sequence and therefore converges to something.
Now suppose $lim_{n\to\infty} d_{C_n} = c > 0$ and obtain a contradiction as follows.
Since $c \leq d_{C_i}$ for every i, $\alpha c \leq \alpha d_{C_i}$ for every i. Since x_i is of depth αd_{C_i} in C_i for every i, it is also of depth αc in C_i for every i. We show that this contradicts the non-negativity of the measure λ by an inductive argument.
We know that $\lambda(C_1) \leq \lambda(C_0) - \frac{1}{2}g\alpha^n c^n$ by Lemma 1.
Now assume $\lambda(C_k) \leq \lambda(C_0) - \frac{n}{2}g\alpha^n c^n$.
Then $\lambda(C_{k+1}) \leq \lambda(C_k) - \frac{1}{2}g\alpha^n c^n$ (again by Lemma 1).
Combining the above inequality with the assumption made in the inductive step, we obtain $\lambda(C_{k+1}) \leq \lambda(C_0) - \frac{k+1}{2}g\alpha^n c^n$. Since $C_0 = S$, $\lambda(C_0) = 1$. Then $\lambda(C_k) \leq 1 - \frac{k}{2}g\alpha^n c^n$. When $k > \frac{2}{g\alpha^n c^n}$, we obtain $\lambda(C_k) < 0$ which contradicts the non-negativity of the measure λ. Therefore we can conclude that $lim_{n\to\infty} d_{C_n} = 0$.

□

We can deduce a few facts about $C_\infty = \bigcap_{i=1}^{\infty} C_i$, which denotes the best estimate of the location of the target T that can be made by the searcher S, from the above theorem. Since $C_\infty \subseteq C_i$, $d_{C_\infty} \leq \inf\{d_{C_i} \mid i \in \mathbb{N}\} = 0$. Since the depth of a set cannot be less than 0, we obtain $d_{C_\infty} = 0$ implying that C_∞ has a empty interior. Also C_i is convex for every i as $M \in \mathcal{M}_{x_i}$ is convex for every i. Therefore C_∞ is convex. But since C_∞ has a empty interior $\partial(C_\infty) = C_\infty$ and boundary of any convex set has a measure of 0. Therefore $\lambda(C_\infty) = 0$. This implies that the above motion strategy allows us to approximate the location of the target T upto a set of measure 0.

5. ALTERNATE STRATEGY

The motion strategy devised in the previous section assumes that the agent is able to compute the depth d_{C_i} of the set C_i (in order to calculate αC_i). This computation is equivalent to computing the chebychev center of a polytope and is computationally intensive. This can be circumvented as described below by making a relatively reasonable assumption. We assume that given a measurable set C and small positive number $r \in \mathbb{R}$, the searcher S would be able to find a point p in C such that p is of at least depth r in C (assuming such a point exists).

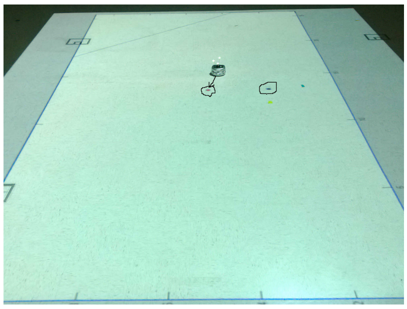
If the choice of r is small relative d_C, a simple strategy to find such points would be to sample a huge number of points from a uniform distribution imposed on C. Let the set $X_r = \{p \mid p \text{ is at least of depth } r \text{ in } C\}$. Assuming uniform distribution on C, the probability that the point p of depth r in C is given by $\frac{\lambda(X_r)}{\lambda(C)}$. If we choose $r = 0$, the probability $\frac{\lambda(X_r)}{\lambda(C)} = 1$ as every X_r is equal to the interior of C. Similarly, it is equal to 0, when $r = d_C$ and varies continuously when $r \in (0, d_C)$. So for a sufficiently small non-zero r, the probability $\frac{\lambda(X_r)}{\lambda(C)}$ can be made as close to 1 as possible which ensures that a large fraction of the points generated by sampling is of at least depth r in C.

The above observation ensures that sampling is a reasonable strategy to find points of at least depth r. This in turn can be used to devise a new motion strategy for the searcher S. Let $C_0 = W$ and choose $r > 0$. C_0 denotes the initial set-valued estimate of the target location x_T. The searcher S then samples a large number of points from the C_0. If r is small enough, the probability $\frac{\lambda(X_r)}{\lambda(C)}$ would be close to 1 thus ensuring that sampling produces a point p of at least depth r in C_0. Then the searcher S drives to the sampled point p of at depth r in C_0 and makes a measurement M_1 and constructs the new estimate $C_1 = C_0 \cap M_1$. Since $d_{C_1} \leq d_{C_0}$, the probability $\frac{\lambda(X_r)}{\lambda(C_1)}$ would be lesser than the probability $\frac{\lambda(X_r)}{\lambda(C_0)}$. In general, the probability $\frac{\lambda(X_r)}{\lambda(C_n)}$ would approach 0 as n increases. In fact, it attains 0 after finitely many measurements. This is because C_0 had unit volume and infinitely many measurements from depth r would force the measure $\lambda(C_i)$ to become negative which is not possible. So after finitely many measurements, $d_{C_n} \leq r$ and sampling would fail. At this point the searcher reduces r to $\frac{r}{2}$ and starts making measurements from points of depth $\frac{r}{2}$. Repeating this process, we can see that d_{C_i} would approach 0 as r approaches 0. This allows us to implement a search strategy which does not involve the computation of d_{C_i}.

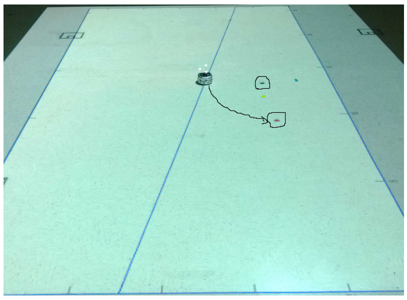
6. HARDWARE IMPLEMENTATION

We implement the motion strategy devised in the previous sections on a Khepera III robot in a lab environent. The position of the robot is tracked by a Opti-track motion capture system. The environment W is a rectangle whose height and width is approximately 5 meters and 3 meters respectively. The target T is located at approximately 1.2 metres 3.5 metres relative to the lower right corner of the environment.

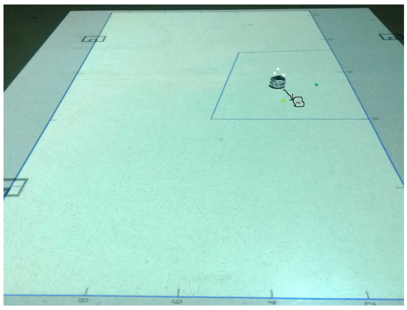
The inital depth r from which the robot will make its measurement is set to 0.5. The minimum depth which is used to determine convergence is $r_{min} = 0.0625$. The convex polygon in which the target T is located is represented by the vertices of the polygon ordered in a clockwise manner. Every point in a convex polygon is a convex combintation of its vertices (i.e of the form $\sum_i \alpha_i v_i$ where v_i represents the ith vertex and $\sum_i \alpha_i = 1$). This allows us the robot to sample from the polygon by sampling a from a n-simplex where n denotes the number of vertices. The robot then makes a measurement by sampling points and checking whether the depth of the point is greater than r and then drives to that point to make a measurement. If sampling fails, the robot reduces r to $\frac{r}{2}$ and tries to sample again. This process terminates when $r \leq r_{min}$. Fig 4 shows this algorithm at different stages of its execution.



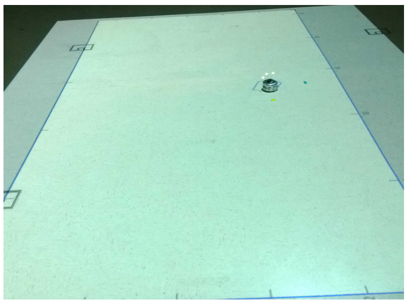
(a) After a single measurement



(b) The blue polygon located in the interior of the rectangle represents the uncertainty associated with the location of the target after 3 measurements



(c) After 5 measurements (target point obscured by the Khepera)



(d) Converges to a set of depth less than 0.0625 after 10 measurements

Fig. 4. The Khepera III robot makes a sequence of measurements from the circled red dots and finally converges to the stationary target represented by the circled black dot. The blue polygons located in the interior of the rectangle represent the region in which the target is located.

7. CONCLUSION

In order to study the impact of limited sensing modalities on coordination algorithms, we have developed a probabilistic set valued sensor model and characterized conditions under which the searcher S can locate the target using this sensor. We have also developed a motion strategy which allows us to combat the uncertainty inherent in the sensor model and achieve two-agent rendezvous. Our results show that the motion strategy which has been devised allows the searcher to locate the target upto a set of arbitrary small measure in finite time.

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