Power-Aware Sensor Coverage: An Optimal Control Approach

Patrick Martin, Rosalba Galvan-Guerra, Magnus Egerstedt, and Vadim Azhmyakov

***Abstract*— Sensor networks primarily have two competing objectives: they must sense as much as possible, yet last as long as possible when deployed. In this paper, we approach this problem using optimal control. We describe a model that relates each sensor's "footprint" to their power consumption and use this model to derive optimal control laws that maintain the area coverage for a specified operational lifetime. This optimal control approach is then deployed onto different sensor networks and evaluated for its ability to maintain coverage during their desired lifetime.**

I. Introduction

Sensor networks are becoming an important component in civilian, commercial, and military cyber-physical system applications. From smart building networks to unmanned reconnaissance, these low-power sensor networks provide information for deployed control systems to make decisions about future operation. One challenge with deploying these devices into real environments is their limited power capacity and, consequently, their ability to sense and communicate. To better utilize these distributed sensing platforms, new algorithms will need to be developed that balance the ability to sense and communicate with the available power supply.

Recent work in wireless sensor networks (WSN) has focused on developing such algorithms to maintain coverage while also extending the lifetime of the network. Alfieri *et al.* developed heuristic algorithms that turn subsets of sensors on/off to conserve energy [1]. Potkonjak and Slijepcevic's heuristic algorithm [8] creates a set cover made up of mutually exclusive sets of randomly placed sensors. This organization maximally covers the area and hence allows for the extension of the network's coverage lifetime. The more recent work by Cardei *et al.* [3], [4] proposes alternative heuristics that produce more set covers, and hence, longer lifetime; however, the algorithm takes longer to compute.

An alternative approach to set covering is using energy efficient protocols, e.g. [10], [11]. Tian and Georganas created a protocol for sensors that uses neighbor information to set sleep intervals [10]. Sensors using the protocol proposed by Ye *et al.* [11] look for active neighbors within a distance threshold. If none are present, the node activates and remains active until it runs out of power.

P. Martin and M. Egerstedt are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA, patrick.martin@gatech.edu, magnus@ece.gatech.edu. Their work was supported by the Office of Naval Research through the MURI, Heterogeneous Unmanned Networked Teams (HUNT).

R. Galvan-Guerra and V. Azhmyakov are with the Departamento de Control Automatico, CINVESTAV, A.P. 14-740, Av. Instituto Politecnico Nacional No. 2508, C.P. 07360, Mexico D.F., Mexico, {rgalvan,vazhmyakov}@ctrl.cinvestav.mx. R. Galvan-Guerra is sponsored by CONACyT.

These sensor network algorithms use fixed range communication and sensing models, e.g. [7], [9]. However, these fixed size disks do not capture the ability of these systems to control their transmission or sensing gains, which in turn, affect their range. In this paper, we consider a collection of sensors distributed in an environment where each sensor's footprint varies with time according to its power levels. This approach to the coverage problem opens up the study of sensor networks operating under cyber-physical constraints, such as power levels or communication ranges. These types of constraints are key to developing sensor networks that can be deployed for extended periods of time.

This paper is structured as follows: first, we describe our sensor power dynamics and formulate a finite-time horizon optimal control problem for finding the sensing and communication gain signals. We solve this problem and use the optimality conditions to numerically solve for the gain signals and deploy the results onto a team of simulated sensor nodes. Our results demonstrate that given any final time, we can calculate an optimal signal for tuning each sensor's gain for maximum coverage.

II. Problem Formulation

Assume we have a collection of N sensors, located at the fixed positions $p_i \in \mathcal{R} \subseteq \mathbb{R}^2, \ i = 1, \cdots, N$. Furthermore, we assume that each sensor's state, $x_i(t)$, represents the current power level of the device at time t. The evolution of this power level is modeled by the dynamics

$$\dot{x}_i(t) = -u_i(t)x_i(t) \tag{1}$$

where $u_i(t) \in \mathbb{R}_+$ represents a signal that alters the gain of the device and, consequently, its sensing and communication range. This assumption is reasonable since new hardware technologies enable the dynamic adjustment of communication and sensing gains on sensor nodes. Note that this model results in an exponentially decaying state over time, which is an approximation to more elaborate models of energy discharge. For further details on battery discharge models of cyber-physical systems, see, for example, the work of Zhang and Shi [12].

We start by basing our sensor range model on the RF power density function for an isotropic antenna [6]:

$$S_{recv} = \frac{P_{trans}}{4\pi r_i^2},$$

where S_{recv} is the power density a distance r_i generated by the transmitted power, P_{trans}. We associate the power transmitted by the node with its current power level, i.e. $P_{trans} = u_i x_i$. By setting a desired threshold power density level for sensing, $\tilde{S}_{recv}$, each sensor's coverage area, or footprint, is given by:

$$\mathcal{R}_i(p_i, x_i(t), u_i(t)) := \pi r_i(t)^2 = \frac{u_i x_i}{4\tilde{S}_{recv}}.$$

Without loss of generality, we let $\tilde{S}_{recv} = \frac{1}{4} Watt/m^2$, resulting in the footprint for sensor i:

$$\mathcal{R}_i(p_i, x_i(t), u_i(t)) = u_i x_i. \tag{2}$$

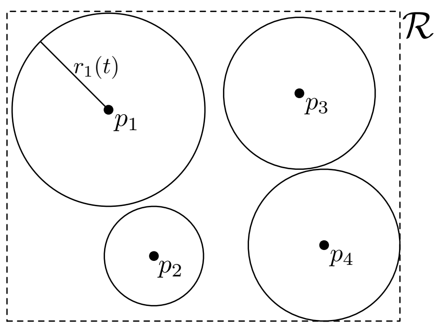


Fig. 1. An illustration of four sensor nodes using the time varying footprint model in equation (2). These sensors are deployed in some region $\mathcal{R} \subseteq \mathbb{R}^2$.

Figure 1 illustrates the sensor footprints of a small set of sensor nodes. Each sensor in our model is placed at their position, p_i, where $i = 1, 2, 3, 4$. Furthermore, each sensor may start at different power levels, $x_i(t)$, which then impacts their effective sensing range, $r_i(t)$.

Using this sensor footprint model, our goal is to select $u_i(t)$ signals such that we maintain coverage of an area of size M m^2, within region $\mathcal{R}$. In other words, we want

$$\Big|\bigcup_{i=1}^{N} \mathcal{R}_i(p_i, x_i(t), u_i(t))\Big| \geqslant M, \ \forall t. \tag{3}$$

First, we make the assumption that at $t = 0$ the footprints of all sensors do not intersect, i.e. $\mathcal{R}_i \cap \mathcal{R}_j = \emptyset, \ \forall i, j \in \{1, \cdots, N\}$. Since the dynamics of each agent (1) always decay from $t = 0$, these regions will never intersect. Therefore,

$$\Big|\bigcup_{i=1}^{N} \mathcal{R}_i\Big| = \sum_{i=1}^{N} |\mathcal{R}_i| = \sum_{i=1}^{N} u_i(t) x_i(t). \tag{4}$$

for all time $t > 0$. Given (4), we can restate our coverage constraint (3) as:

$$\sum_{i=1}^{N} u_i(t) x_i(t) \geqslant M, \ \forall t. \tag{5}$$

For notational convenience, we aggregate all of the sensor states as the vector $x(t) = [x_1(t) \ \cdots \ x_2(t)]^T \in \mathbb{R}^n$. Consequently, we let the total dynamics of all N sensors be given by:

$$\dot{x} = -U(t)x(t), \tag{6}$$

where, $U(t) = diag(u_i), \ i = 1, \cdots, N$.

III. Optimal Control Solution

In this section, we use the system dynamics (6) and coverage constraint (5) to determine the control signals, $u_i(t)$, for the sensor nodes. To do so, we formulate an optimal control problem with the objective to minimize power consumption, while, at the same time, maintain the desired coverage area, M, during the specified lifetime, T.

Under the assumption that $\mathcal{R}_i \cap \mathcal{R}_j = \emptyset, \forall i, j$ we are interested in finding a solution to:

$$\min_u J(u, x, t) = \int_{t_0}^{T} \frac{1}{2}\left(\left(u^T(t)x(t) - M\right)^2 + u^T(t)Ru(t)\right) dt$$

where R is a positive definite matrix, such that the dynamics (6) are satisfied. In this cost functional, we encode the coverage constraint as the cost $\left(u(t)^T x(t) - M\right)^2$, which penalizes deviations from the are M. Furthermore, we include an energy cost $u(t)^T Ru(t)$ in order to generate a $u(t)$ that uses minimal energy.

Following the standard optimal control technique for fixed terminal time (e.g. [2]), we obtain the following Hamiltonian:

$$\begin{aligned} H(u, x, t) \ &= \ -u^T(t)\Lambda(t)x(t) + \frac{1}{2}\left(u(t)^T x(t) - M\right)^2 \\ &+ \ \frac{1}{2}u(t)^T Ru(t). \end{aligned}$$

Here, $\Lambda(t) = diag(\lambda_i(t))$, for $i = 1, \cdots, N$, is a matrix of the co-states that satisfy the backwards differential equation:

$$\dot{\lambda}(t) = \frac{\partial H}{\partial x}^T = \Lambda(t)u(t) - \left(u(t)^T x(t) - M\right) u(t),$$

with $\lambda(T) = 0$. Also, we derive the optimal control signal for the sensor gains by solving $\frac{\partial H}{\partial u} = 0$ for $u(t)$:

$$\begin{aligned} \frac{\partial H}{\partial u} \ &= \ (-x(t)^T \Lambda(t) + (u(t)^T x - M)^2 + u(t)^T R) = 0 \\ &= \ (x(t)x(t)^T + R)u(t) = (MI + \Lambda(t))x(t) \end{aligned}$$

where, I is an $N \times N$ identity matrix. Since the power of each sensor does not decay to exactly 0, we know that each $x_i(t) > 0$ for all $t > 0$. Therefore, $(x(t)x(t)^T + R)$ is a positive definite matrix, which we can invert to solve for $u(t)$:

$$u(t) = \left(x(t)x^T(t) + R\right)^{-1} (\Lambda(t) + MI)\, x(t). \tag{7}$$

This control signal is computed by numerically solving a two-point boundary value problem for a specified final time, T. In the next section we will demonstrate the deployment of this control on simulated sensor nodes.

IV. Simulation Results

In this section we present two specific examples to illustrate the effectiveness of our proposed approach.

Example 1: Let us consider a square region with an area of 16 m^2, where four sensors are distributed and the sensor's initial power levels are set randomly. Our aim is to minimize a cost function of the form

$$J(u,x,t) = \int_0^T \frac{1}{2}\left(\left(u^T(t)x(t) - 5.33\right)^2 + u^T(t)u(t)\right)dt.$$

over the trajectory of the system under consideration; i.e., to make the sensors cover an area of $M = 5.33$ m^2 within $T = 0.65$s.

Applying the approach of section III to our system we obtain the simulations shown in the Figure 2.

Note that the value of $u_i(t)$ is increasing the footprint over time to compensate for the loss of energy (see Figure 2(a)). The area is successfully covered over the specified time interval, as shown in Figure2(b). The final area coverage for the individual sensors is shown in Figure 2(c).

We now examine an example with a larger number of sensor nodes.

Example 2: We deployed the control signal (7) on 16 sensors with randomly set initial power levels, distributed in a region of 64 m^2. The sensors were specified to cover $M = 30$ m^2 over 2 seconds, i.e., minimize

$$J(u,x,t) = \int_0^2 \frac{1}{2}\left(\left(u^T(t)x(t) - 30\right)^2 + u^T(t)u(t)\right)dt.$$

The results of this simulation are shown in Figure 3. Note, again, that the gain signal in Figure 3(a) increases over time to ensure that the area is covered (Figure 3(b)) for the specified time. Figure 3(c) shows the final coverage of all 16 sensors in the region.

Observe that if we increase the simulation time, the area covered will decrease, since our sensor model does not allow for the injection of new energy, i.e. recharging the sensor batteries.

V. Conclusion

In this paper, we developed a sensor footprint model that depends on the current energy capacity of the sensor node. We use this model to develop an optimal controller that adjusts the gains of the sensors to efficiently cover a specified area. The simulation of this controller demonstrates the effectiveness of this optimal control strategy for maintaining sensor coverage, while, at the same time, managing energy consumption.

Acknowledgment

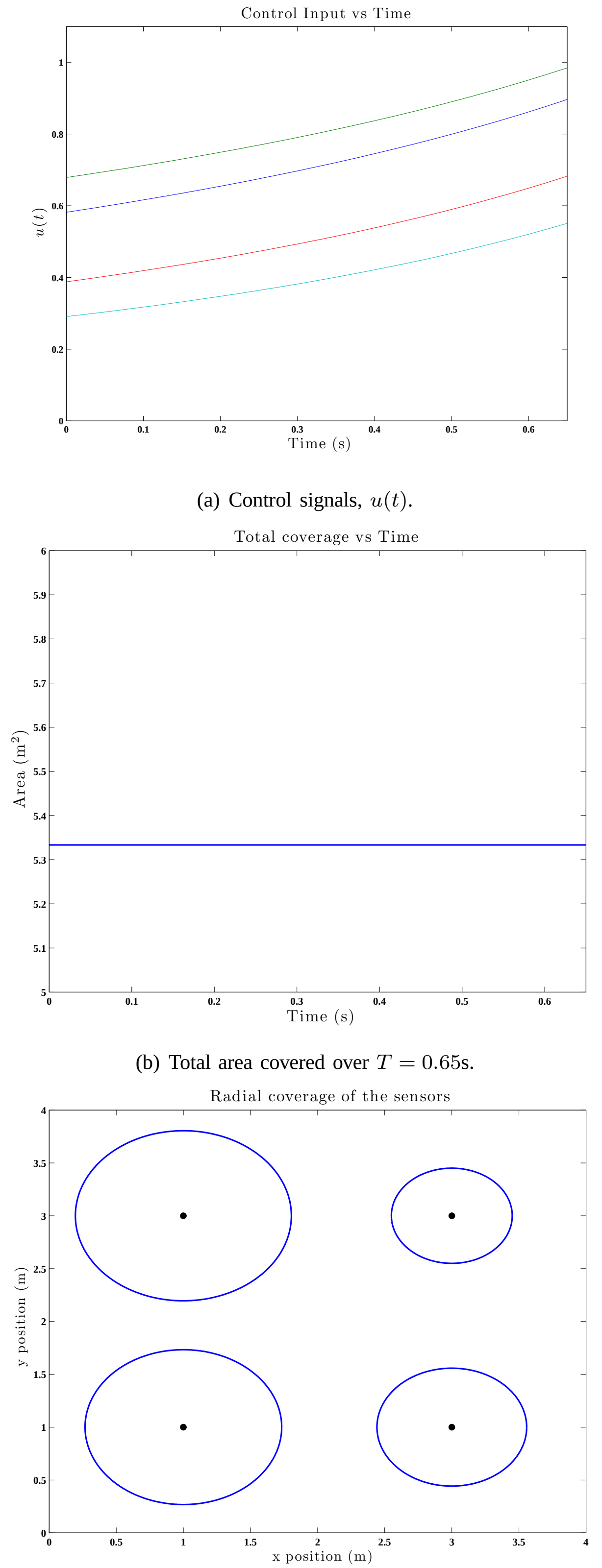
The authors would like to thank Amir Rahmani for helpful discussions.

References

[1] A. Alfieri, A. Bianco, P. Brandimarte, and C.F. Chiasserini. Maximizing system lifetime in wireless sensor networks. *European Journal of Operational Research*, Vol. 181, issue 1, pp. 390-402, August 2007.

[2] A.E. Bryson and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publising Corp., New York, 1975.

[3] M. Cardei and D. Du. Improving Wireless Sensor Network Lifetime through Power Aware Organization. *ACM Wireless Networks*, Vol. 11, No. 3, pp. 333-340, May 2005.

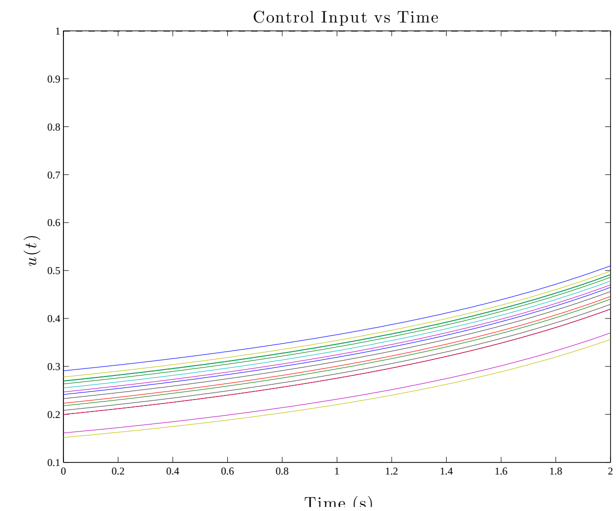


(a) Control signals, $u(t)$.

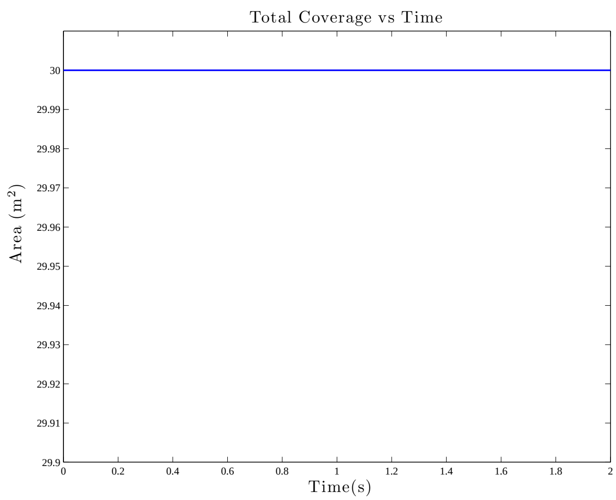
(b) Total area covered over $T = 0.65$s.

(c) Final coverage configuration.

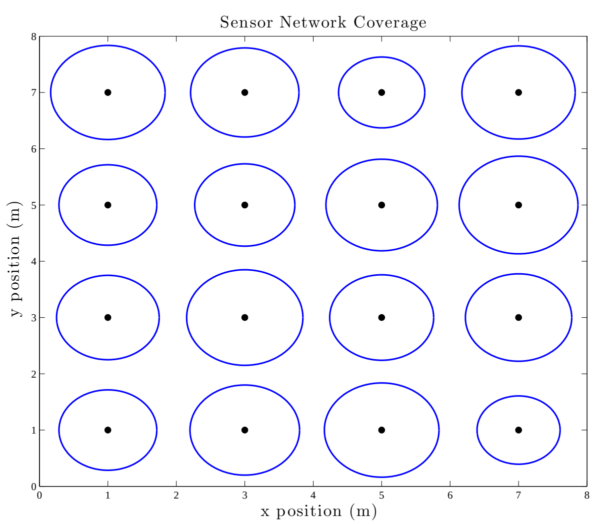
Fig. 2. The simulation results of the sensor coverage example using four sensors.



(a) Control signals, $u(t)$.



(b) Total area covered over $T = 2$s.



(c) Final coverage configuration.

Fig. 3. The simulation results of the sensor coverage example using 16 sensors.

[4] M. Cardei and J. Wu. Energy-efficient coverage problems in wireless ad-hoc sensor networks. *Computer Communications*, Vol. 29, pp. 413-420, 2006.

[5] J. Carle and D. Simplot-Ryl. Energy-Efficient Area Monitoring for Sensor Networks. *Computer*, Vol. 37, No. 2, pp. 40-46, 2004.

[6] K. Chang. *RF and Microwave Wireless Systems*. John Wiley and Sons, 2000.

[7] J. M. McNew, E. Klavins, and M. Egerstedt, Solving Coverage Problems with Embedded Graph Grammars. *Hybrid Systems: Computation and Control*, Springer-Verlag, Pisa, Italy (2007), pp. 413-427.

[8] M. Potkonjak and S. Slijepcevic. “Power Efficient Organization of Wireless Sensor Networks,” in *IEEE International Conference on Communications*, vol. 2, pp. 472-476, June 2001.

[9] V. de Silva and R. Ghrist. Coordinate-free coverage in sensor networks with controlled boundaries via homology. *International Journal of Robotics Research*, Vol. 25, No. 12, pp. 1205-1222, 2006.

[10] D. Tian and N.D. Georganas, A Coverage Preserving Node Scheduling Scheme for Large Wireless Sensor Networks, *Proc. 1st ACM Workshop Wireless Sensor Networks and Applications*, ACM Press, 2002; pp. 32-41.

[11] F. Ye, G. Zhong, J. Cheng, S. Lu, and L Zhan. PEAS: A Robust Energy Conserving Protocol for Long-Lived Sensor Networks, *Proc. IEEE Intl Conf. Network Protocols* (ICNP 2002), IEEE CS Press, 2002, pp. 200-201.

[12] F. Zhang and Z. Shi ”Optimal and Adaptive Battery Discharge Strategies for Cyber-Physical Systems,” in *Proc. 48th IEEE Conference on Decision and Control*, pp. 6232-6237.