Noname manuscript No.
(will be inserted by the editor)

Timing Control of Switched Systems with Applications to Robotic Marionettes

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Received: date / Accepted: date

Abstract We present an optimal timing control formulation for the problem of controlling autonomous puppets. In particular, by appropriately timing the different movements, entire plays can be performed. Such plays are produced by concatenating sequences of motion primitives and a compiler optimizes these sequences, using recent results in optimal switch-time control. Additionally, we apply saddle-point techniques to approach the problem of timing constraints among interconnected puppets. Experimental results illustrate the operation of the proposed methods.

1 Introduction

This paper addresses the issue of when to switch between different modes of operation when controlling interconnected dynamical systems. This question falls squarely under the *optimal timing control problem* for hybrid systems, which is an area of research that has received a significant interest during the last few years. In these types of problems, the control parameter includes the schedule of the system's modes and the performance metric consists of a cost functional defined on the system's state (see [2,5–7,11,16,17, 23–25,27]).

The work in this paper draws its inspiration from recent results on numerical optimal control of switched-mode systems, in which gradient-descent and second-order algorithms have been developed [2,13,28,29]. However, what is novel in this paper as compared to the previous work is first of all the application of optimal timing control to the problem of controlling a collection of robotic puppets. Secondly, we identify a formal *motion description language* that allows us to specify what the puppets should be doing at a high level of abstraction, and then use optimal control techniques for compiling these high-level specifications into executable control code. Finally, a formulation of the problem is given in such a way that it can be decomposed into subproblems, each involving the optimal timing sequence for the individual puppets. The networking

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aspects, i.e. the way in which these subproblems are combined, are then given a direct interpretation in terms of the Lagrange multipliers in the optimization problem.

When puppeteers execute entire plays, they typically break it down into components, i.e. a play consists of multiple acts and an act consists of multiple scenes. However, within each scene, each piece, e.g. a puppet dance routine, is also broken down into components and the dance is in fact both annotated and executed as a string of movements, each of which has its own characteristics, duration, and intensity. What this research presents is a way of decomposing such complex control tasks in robotics into strings of simpler control tasks, which are then executed on the puppetry system shown in Fig. 1.

(a) Puppet in initial configuration. (b) Puppet in wave motion. (c) Puppet starting a walk. (d) The final step in the walk mode.

Fig. 1 An image sequence of the puppet executing a *wave* followed by a *walk* mode.

We propose formalizing high-level specifications for puppetry using Motion Description Languages (MDLs) [8,12,20,21]. Specifically, a MDL is a string of pairs, each specifying what control law the system should be executing and an interrupt condition corresponding to the termination of this control law. In order for this language to be successful, it is important that it is expressive enough to be able to characterize actual puppet plays, and as such we draw inspiration from the way such plays are staged by professional puppeteers. As an example, consider a part of an actual play, as shown in Fig. 2. The play that this example comes from is the "Rainforest Adventures" - an original puppet play staged at the Center for Puppetry Arts in Atlanta during 2005 [10,19]. It shows how the basic building blocks for a formal language for puppet choreography can be derived from existing practices in puppeteering.

In fact, the standard way in which puppet plays are described is through four parameters, namely *temporal duration*, *agent*, *space*, and *motion* (when?, who?, where?, and what?) [4,15]. Most plays are based on *counts* in that each puppet motion is supposed to happen at a particular count. This property becomes even more important if multiple puppets are acting simultaneously on stage or if the play is set to music. At each specified count, a motion can be initiated and/or terminated.

The outline of this paper is as follows: In Section 2, we recall the basic definitions of a Motion Description Language and show how these definitions can be augmented to be more suitable for specifying puppet plays. We then, in Section 3, use the Calculus of Variations for parsing MDL strings in an optimal way in order to produce effective control programs, as supported by experimental results. We solve the switched-time optimization problem for networked marionettes in Section 4 by utilizing a saddle-

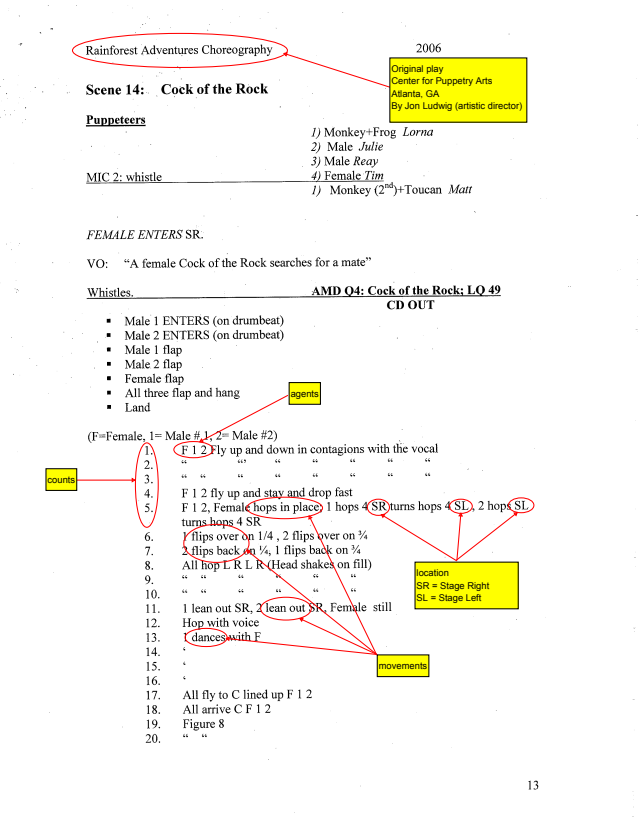

Fig. 2 Rainforest Adventures: This figure is an original puppet choreography sheet from the Center for Puppetry Arts in Atlanta [10]. It shows how the basic building blocks for a formal language for puppet choreography can be derived from existing practices in puppeteering.

point algorithm to maintain timing constraints. This algorithm is then verified using a simulation. Finally, Section 5 concludes this article with some closing thoughts and analysis.

2 Motion Description Languages

As the complexity of control systems increases, due both to the system complexity (e.g. manufacturing systems, [9]) and the complexity of the environment in which the system is embedded (e.g. autonomous robots [3,18]), multi-modal control has emerged as a useful design tool. The main idea is to define different modes of operation, e.g. with respect to a particular task, operating point, or data source. These modes are then combined according to some discrete switching logic and one attempt to formalize this notion is through the concept of a *Motion Description Language* (MDL) [8,12,20,21].

Each string in a MDL corresponds to a control program composed of multiple controllers. Slightly different versions of MDLs have been proposed, but they all share the common feature that the individual modes, concatenated together to form the

control program, can be characterized by control-interrupt pairs. In other words, given a dynamical system

$$\dot{x} = f(x,u),\ x \in X,\ u \in U$$

together with a control program $(\kappa_1,\xi_1),\ldots,(\kappa_p,\xi_p)$, where $\kappa_i : X \to U$ and $\xi_i : X \to \{0,1\}$, the system operates on this program as $\dot{x} = f(x,\kappa_1(x))$ until $\xi_1(x) = 1$. At this point the next pair is read and $\dot{x} = f(x,\kappa_2(x))$ until $\xi_2(x) = 1$, and so on. Note that the interrupts can also be time-triggered, which can be incorporated by a simple augmentation of the state space.

2.1 MDLs for Puppetry

We note that the general MDL outlined in the previous paragraph does not lend itself to the way puppetry plays are specified. In fact, what we will do in this section is augment the standard MDL formulation, as discussed in [14], to include factors such as spatial location. For this, assume that the play starts at time t_0 and that it ends at time t_f. Moreover, let the length of each "count" be Δ, and assume that $(t_f - t_0)/\Delta = M$. Following this, the set of all times over which the play is specified is $\mathcal{T} = \{t_0, t_0+\Delta, t_0+2\Delta,\ldots,t_0+M\Delta\}$.

Moreover, assume that the stage is divided into N different sections (typically this number is 6, namely LowerLeft, LowerCenter, LowerRight, MiddleLeft, MiddleCenter, MiddleRight, UpperLeft, UpperCenter, UpperRight), whose planar center-of-gravity coordinates are given by $r_1,\ldots,r_N$, with the set of regions being given by $\mathcal{R} = \{r_1,\ldots,r_N\}$.

As before, assume that the puppet under consideration has the dynamics

$$\dot{x} = f(x,u).$$

Now, we have construct a number of control laws $\kappa_j,\ j = 1\ldots,C$, corresponding to different moves that the puppet can perform. Each control law is a function of x (state), t (time), and α (a parameter characterizing certain aspects of the motion such as speed, energy, or acceleration, as is the normal interpretation of the parameterization of biological motor programs); therefore, we can let the set of moves that puppet can perform be given by $\mathcal{K} = \{\kappa_1,\ldots,\kappa_C\}$. In fact, we will often use the shorthand $f_j(x,t,\alpha)$ to denote the impact that control law κ_j has through $f(x,\kappa_j(x,t,\alpha))$.

As already pointed out, each instruction in the puppet play language is a four-tuple designating the *who*, *what*, *when*, and *where* that the puppets should be doing. In other words, we let the motion alphabet be given by $\mathcal{L} = \mathcal{I}\times\mathcal{K}\times\mathcal{T}\times\mathcal{R}$. Each element in $\mathcal{L}$ is thus given by (i,κ,τ,r), where the interpretation is that puppet-i should execute the motion κ until time τ, largely in region r.

Following the standard notation in the formal language field, we let $\mathcal{L}^\star$ denote the set of all finite-length concatenations of elements in $\mathcal{L}$ (including the empty string), and let puppet plays be given by words $\pi \in \mathcal{L}^\star$. In particular, if we let

$$\pi = (i,\kappa_1,\tau_1,r_1)(i,\kappa_2,\tau_2,r_2)\cdots(i,\kappa_p,\tau_p,r_p),$$

be a word of length p, then the puppet evolves with this string according to

$$\dot{x} = \begin{cases} f_1(x,t,\alpha_1),\ t\in[t_0,\tau_1) \\ f_2(x,t,\alpha_2),\ t\in[\tau_1,\tau_2) \\ \quad\vdots \\ f_p(x,t,\alpha_p),\ t\in[\tau_{p-1},\tau_p]. \end{cases}$$

This description seems fairly natural, but two essential parameters have been left out. First, the motion parameters $\alpha_1, \ldots, \alpha_p$ have not yet been specified. Moreover, the desired regions $r_1, \ldots, r_p$ have not been utilized in any way. In order to remedy this, we need to construct not just a *parser* for puppet plays, as given above, but also a *compiler* that selects the "best" parameters and durations for the different moves so that the play is executed as efficiently as possible, which is the topic of the next section.

3 Compiling MDL Strings Through Optimal Control

In this section we present a compiler that takes as inputs strings in a MDL and optimizes over these strings by adjusting the interrupt times as well as the parameters defining the specifics of the individual control laws. Rather than solving a large-scale problem explicitly, we start with the canonical two-primitive MDL string. In fact, consider the following optimal control problem:

$$\min_{\tau,\alpha_1,\alpha_2} J(\tau,\alpha_1,\alpha_2) = \int_0^{t_f} L(x,t)dt + C_1(\alpha_1) + C_2(\alpha_2) + \Delta(\tau) + \Psi_1(x(\tau)) + \Psi_2(x(t_f)),$$

where

$$\begin{aligned} \dot{x} &= \begin{cases} f_1(x,t,\alpha_1), & t \in [0,\tau) \\ f_2(x,t,\alpha_2), & t \in [\tau,t_f] \end{cases} \\ x(0) &= x_0. \end{aligned}$$

This optimal control problem is the atomic problem involving how a single puppet should execute the two-instruction play $(i,\kappa_1,\tau,r_1)(i,\kappa_2,t_f,r_2)$ under the interpretation that $\Delta(\tau)$: is a cost that penalizes deviations from the presepcified, nominal switching time τ, $C_i(\alpha_i)$ measures how much energy it takes to use parameter α_i for mode i, $\Psi_i(\cdot)$: ensures that the puppet is close to r_1 at time τ (and similarly for $x(t_f)$), and $L(x,t)$ is a trajectory cost that may be used to ensure that a reference trajectory is followed.

We can apply calculus of variations techniques, under suitable assumptions of continuous differentiability, to the cost functional. The derivations are straightforward and follow those of [2,13], and they result in the optimality conditions

$$\begin{aligned} \frac{\partial J}{\partial \tau} &= \lambda(\tau_-)f_1(x(\tau)) - \lambda(\tau_+)f_2(x(\tau)) + \frac{\partial \Delta}{\partial \tau} \\ \frac{\partial J}{\partial \alpha_2} &= \mu(\tau_+) \\ \frac{\partial J}{\partial \alpha_1} &= \mu(0), \end{aligned}$$

where the co-states λ and μ satisfy the following discontinuous (backwards) differential equations:

$$\dot{\lambda} = -\frac{\partial L}{\partial x} - \lambda \frac{\partial f_2}{\partial x},\ t \in (\tau, t_f),\ \lambda(t_f) = \frac{\partial \Psi_2}{\partial x}(x(t_f))$$
$$\dot{\lambda} = -\frac{\partial L}{\partial x} - \lambda \frac{\partial f_1}{\partial x},\ t \in [0, \tau),\ \lambda(\tau_-) = \lambda(\tau_+) + \frac{\partial \Psi_1}{\partial x}(x(\tau))$$
$$\dot{\mu} = -\lambda \frac{\partial f_2}{\partial x},\ t \in (\tau, t_f),\ \mu(t_f) = \frac{\partial C_2}{\partial \alpha_2}$$
$$\dot{\mu} = \lambda \frac{\partial f_1}{\partial x},\ t \in [0, \tau),\ \mu(\tau_-) = \frac{\partial C_1}{\partial \alpha_1}$$

By a direct generalization to more than two modes, this construction allows us to produce a compiler that takes *plays* and outputs *strings of control modes with an optimized temporal duration and mode parameterization.*

3.1 Numerical Results

We use a kinematic model of the puppet for the experimental simulations of our proposed algorithm. The state of the puppet is given by its joint angles for the arm rotation (θ_l and θ_r), arm lift (ϕ_l and ϕ_r), and leg lift (ψ_l and ψ_r). Therefore, the model of the puppet is given by

$$\dot{q} = Iu$$

where $q = [\theta_l\ \theta_r\ \phi_l\ \phi_r\ \psi_l\ \psi_r]^T$, I is the identity matrix, and u is the mode input. The mode inputs are sinusoidal in this application, but other input functions may be designed to achieve particular motion tasks.

As an illustrative example, consider the following two mode play for puppet-1

$$(1, \kappa_1(\alpha_1), 4, r_1)(1, \kappa_2(\alpha_2), 6, r_1),$$

where κ_1 is "wave left arm" and κ_2 is "walk" and each mode is driven with scaling parameter $\alpha_1 = \alpha_2 = 1$. We formulate the following cost functional,

$$J(\tau, \alpha_1, \alpha_2) = \int_0^{t_f} q^T P q\ dt + \rho(T - \tau)^2 + w_1\alpha_1^2 + w_2\alpha_2^2,$$

where P is a suitable weight matrix, and ρ, w_1, w_2 are cost weights that prescribe relative weights to deviations from the nominal switch time and motion intensity parameters.

Figure 3 shows the joint angle trajectories corresponding to the initial choices for α_1, α_2 and selecting $\tau = 4$. Note that this initial trajectory results in the left arm's odd looking behavior, where the wave motion stops the left arm in "mid-air" as the walk motion begins. A more desirable trajectory would lower the left arm completely before initiating a walk.

To remedy this behavior, we defined the weight matrix, P, such that the joint angles are penalized for deviations from the puppet's "home" position, i.e. the left arm initial joint angles $\theta_l = \phi_l = 0$. After an iterative, descent-based optimization algorithm has terminated, the new values become $\tau = 4.3423$, $\alpha_1 = 1.3657$, $\alpha_2 = 0.9566$, with the corresponding joint angles shown in Figure 4. This plot shows that at termination the joint angles were close to 0 before starting the walk mode, resulting in a more natural looking motion. Additionally, examining the total cost plot in Figure 5 reveals that the cost is indeed reduced as the gradient descent algorithm ran past 20 iterations.

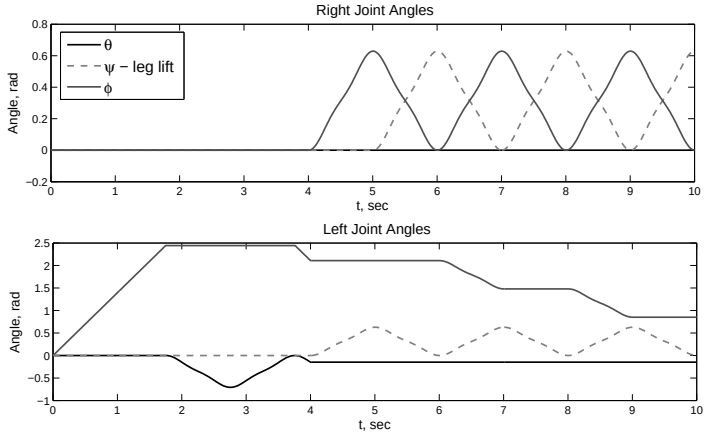

Fig. 3 Original joint angle trajectories.

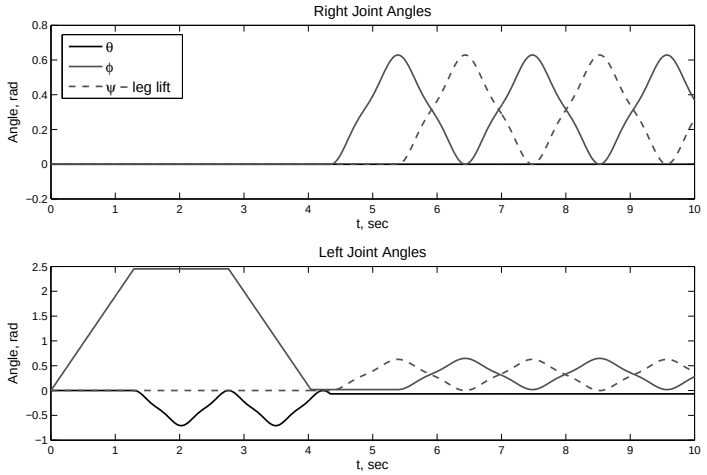

Fig. 4 Optimized joint angle trajectories.

3.2 Experimental Platform

In conjunction with the simulation results above, we have also developed a hardware platform, as shown in Fig. 1. In fact, the movement in that figure is the one obtained in the previous section, in which the puppet switches between a wave and a walk mode.

The puppet system is comprised of three components: hardware system, a Java control application, and Matlab optimization routines. A diagram of the architecture is seen in Fig. 6.

The puppet hardware is controlled by six Robotis Dynamixel motors [26]. These motors are suitable for this application since they can be issued position *and* velocity commands. Additionally, they are linked together by a multipoint serial interface

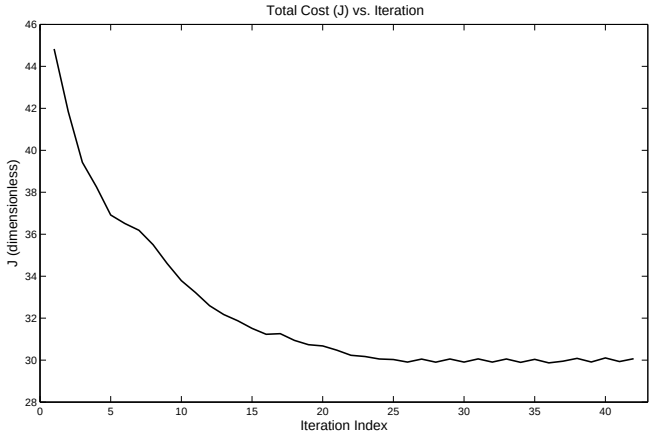

Fig. 5 Total cost as a function of iteration.

(the RS-485 standard), which enables a single command to generate motion on the six motors simultaneously. A microcontroller connects all of the motors together and communicates with a host computer via a serial port.

The Java application has several functional pieces. First, it manages the serial port so that commands may be sent to the underlying puppet hardware. Additionally, it can parse MDL files and generate a string of modes based on what is available in the MDL library. These modes are then fed into an *MDLp Engine*, which applies a particular mode's control action to generate a motion command. Once a user has constructed a play, it may be fed into a Matlab optimization routine, where the algorithm of Section 3 and the kinematics of the puppet are implemented. The Matlab routine then outputs the optimal switch times ($\bar{\tau}^\star$) and scaling factors ($\bar{\alpha}^\star$).

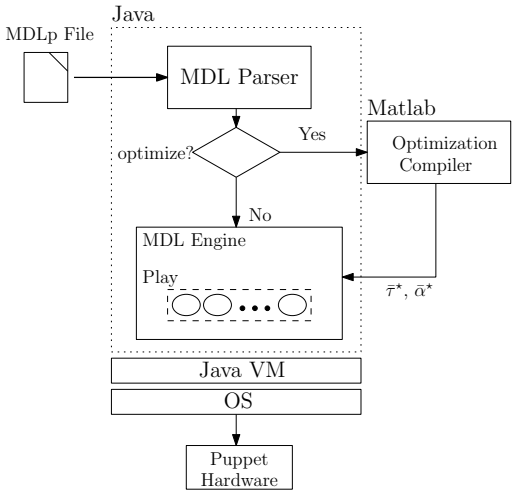

Fig. 6 The puppet system software architecture.

4 Networked Timing Control

Since most interesting puppet plays include more than one marionette, we will be forced to handle situations in which two (or more) puppets need to execute a movement in a coordinated fashion. Due to the risk of tangling strings, multi-puppet coordination is mainly done through spatial and temporal adjacency.

For this, we assume that the play is comprised of n puppets, each operating under their own dynamics. Additionally, each puppet switches between m_i control modes, with (as before) the terminal time denoted by $t_f = \tau_{m_i},\ i = 1, \ldots, n$.

In other words, a direct modification to the previous formulation gives that each puppet be governed by the dynamics,

$$\dot{x}_i(t) = \begin{cases} f_{i,1}(x,t),\ t \in [0, \tau_{i,1}) \\ f_{i,2}(x,t),\ t \in [\tau_{i,1}, \tau_{i,2}) \\ \vdots \\ f_{i,m_i}(x,t),\ t \in [\tau_{i,m_i-1}, \tau_{i,m_i}] \end{cases}$$

for puppets $i = 1, \cdots, n$. Let moreover the cost functional be defined as

$$J(\bar{\tau}_1, \ldots, \bar{\tau}_n) = \int_0^T \sum_{i=1}^{n} D_i(x_i, t)\ dt = \sum_{i=1}^{n} J_i(\bar{\tau}_i)$$

where $D_i(x,t)$ is the cost associated with operating system i for a particular control mode's time duration, without taking the other systems into account.

Now, to illustrate the way in which the temporal constraints show up, we assume, without loss of generality, that the temporal constraint only affects the d^{th} switch for systems j and k, where $j, k \in \{1, \cdots, n\}$, as $\tau_{j,d} - \tau_{k,d} \leqslant 0$.

It is clear that the way this optimization problem can be solved is by simply augmenting the cost with the Lagrangian term $\nu(\tau_{j,d} - \tau_{k,d})$, and then solve the problem jointly across all the switching times for all the puppets. However, we do not want to solve this problem as a joint problem across all subsystems, since the ultimate goal is to have several autonomous puppets optimize their plays in a decentralized fashion. The fact that our optimization problem is *separable* allows us break up the solution process. We specifically choose the approach known as *team theory*, recently explored in [22]. (Note that the details given below are not due to us, but rather that we highlight their application to the problem of distributed timing control as it pertains to the robotic marionette application.)

4.1 Distributed Coordination

Let, as before, puppets j and k ($j \neq k$) be temporally constrained via the d^{th} switch as $\tau_{j,d} - \tau_{k,d} \leq 0$. The constrained problem becomes

$$L(\tau_{j,d}, \tau_{k,d}, \nu) = J_j(\tau_{j,d}) + J_k(\tau_{k,d}) + \nu(\tau_{j,d} - \tau_{k,d}) \tag{1}$$

where we have assumed, without loss of generality, that the only control parameters are $\tau_{j,d}$ and $\tau_{k,d}$, and the other switching times are considered to be fixed. It should be noted that the cost functionals are decoupled (i.e. cost J_j depends *only* on system j's

dynamics). Therefore, taking the derivative of the Lagrangian with respect to ν results in the expression,

$$\frac{\partial L}{\partial \nu} = \tau_{j,d} - \tau_{k,d},$$

in combination with the previously defined gradient expressions for the switching times.

Now, algorithmically, this formulation is interesting in that the dual problem becomes $g^\star = \max_\nu g(\nu),\ \nu \geqslant 0$, where

$$g(\nu) = \inf_{\tau_{j,d},\tau_{k,d}} \{J_j(\tau_{j,d}) + J_k(\tau_{k,d}) + \nu(\tau_{j,d} - \tau_{k,d})\}.$$

As such, this joint problem can be solved using saddle-point search algorithms, such as *Uzawa's algorithm* [1]. Using a gradient descent for the switch times, and a gradient ascent for the Lagrange multiplier ν allows us to largely decouple the numerical solution process and let the networking aspect be reflected only through the update of the multiplier, as was done in [22]. In fact, if we let

$$\begin{aligned}
\dot{\tau}_{j,d} &= -\tfrac{\partial J_j}{\partial \tau_{j,d}} - \nu \\
\dot{\tau}_{k,d} &= -\tfrac{\partial J_k}{\partial \tau_{k,d}} + \nu \\
\dot{\nu} &= \tau_{j,d} - \tau_{k,d}
\end{aligned}$$

all that needs to be propagated between the two systems is the value of the Lagrange multiplier ν. This observation in [22] leads us to a general architecture for solving networked switching time optimization problems, as shown in Figure 7.

This numerical architecture lets us optimize the switch times individually at each algorithm iteration, denoted by the index i. First, we initialize both systems with feasible switch times and scaling parameters based on a play of length p. These values we denote with the arrays, $\bar{\tau}^0 = [\tau_1^0\ \cdots\ \tau_{p-1}^0]$ and $\bar{\alpha} = [\alpha_1^0\ \cdots\ \alpha_p^0]$, respectively. Then we perform the forward-backward integration of the x_j and x_k systems and their associated co-states (λ_j, λ_k). In parallel to those integrations, the ν state is incremented using the current values for the switch times. These values are then passed to the individual systems so that they can take their gradient descent steps with the new ν values.

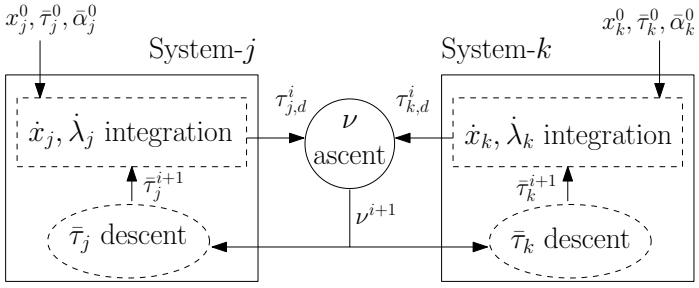

Fig. 7 This figure shows how information propagates between the two subsystems (puppets) in order to solve the networked timing problem. The initial values for system j are denoted $\bar{\tau}_j^0$, $\bar{\alpha}_j^0$ and similarly for system k.

4.2 Numerical Results

The method presented in Section 4.1 is simulated in an expanded version of the Matlab routines in Section 3.1. Additionally, in order to capture more realistic multi-puppet behavior, we modify the cost functionals to include the spatial cost of each puppet. For each puppet, indexed by i,

$$J_i(\bar{\tau}_i, \bar{\alpha}_i) = \int_0^{t_f} q^T P q \; dt + \sum_{l=1}^{p} w_l \alpha_l^2 + \sum_{m=1}^{p-1} \rho(T_m - \tau_m)^2 + (q - r_m)^T Q (q - r_m),$$

where p is the total number of modes in the play to optimize for puppet i. Additionally, we introduce the spatial cost function, $\Psi_m(x(\tau_m)) = (x - r_m)^T Q (x - r_m)$, where r_m is the particular reference trajectory of the current mode and Q is the identity matrix.

Using the collection of control laws $\mathcal{K} = \{\kappa_1 = \texttt{waveLeft}, \kappa_2 = \texttt{walk}, \kappa_3 = \texttt{walkInCircles}\}$ we define a multi-puppet play as follows,

$$(1, \kappa_1(1.2), 2.5, r_1)\ (1, \kappa_2(1.3), 3, r_1)\ (1, \kappa_3(1), 3, r_1)$$
$$(2, \kappa_1(1.2), 2.5, r_3)\ (2, \kappa_3(1.5), 3, r_3)\ (2, \kappa_2(1.3), 2.5, r_3).$$

This play uses two agents, both executing three modes with various timing requirements and scaling parameters.

Following the discussion of switch-time constraints in Section 4.1 we choose to constrain the *first* switch of each puppet, i.e. $d = 1$. If we denote $\bar{\tau}_i = [\tau_{i,1}\ \tau_{i,2}]$ as the switch times and $\bar{\alpha}_i = [\alpha_{i,1}\ \alpha_{i,2}\ \alpha_{i,3}]$ as the scaling parameters for puppet i, then the constrained minimization problem for these two puppets is stated as

$$\min_{\bar{\tau}_1, \bar{\tau}_2, \bar{\alpha}_1, \bar{\alpha}_2} J_1(\bar{\tau}_1, \bar{\alpha}_1) + J_2(\bar{\tau}_2, \bar{\alpha}_2)$$
$$\text{s.t. } \tau_{2,1} \leqslant \tau_{1,1}.$$

We formulate a Lagrangian for this problem in a similar way as equation (1),

$$L(\bar{\tau}_1, \bar{\alpha}_1, \bar{\tau}_2, \bar{\alpha}_2, \nu) = J_1(\bar{\tau}_1, \bar{\alpha}_1) + J_2(\bar{\tau}_2, \bar{\alpha}_2) + \nu(\tau_{2,1} - \tau_{1,1}),$$

and then apply the proposed algorithm approach visualized in Figure 7.

Figure 8 displays the cost graphs for the two puppets after the execution of the distributed algorithm. The cost is indeed reduced for both puppets and, furthermore, Figure 9 shows that the required inequality constraint is satisfied. The optimal switch times and scaling parameters for puppet 1 are $\bar{\tau}_1 = [2.9906\ 3.0463]$ and $\bar{\alpha}_1 = [1.1903\ 1.3249\ 1.0204]$, respectively. Additionally, the results for puppet 2's parameters are $\bar{\tau}_2 = [2.9683\ 2.9157]$ and $\bar{\alpha}_2 = [1.1901\ 1.5228\ 1.3124]$.

5 Conclusions

In this paper we presented the a motion description language for specifying and encoding autonomous puppetry plays in a manner that is faithful to standard puppetry choreography. The resulting strings of control-interrupt pairs are then compiled in the

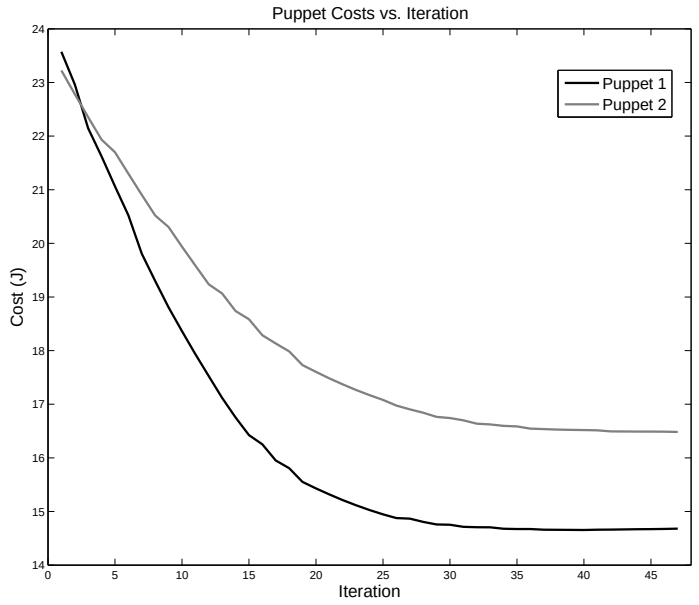

Fig. 8 The cost of both puppets using the distributed switch time constraint architecture.

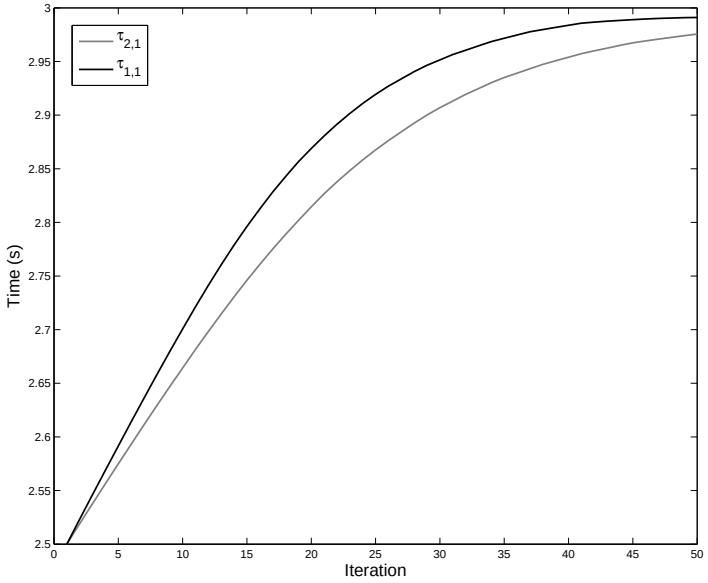

Fig. 9 A graph of the constrained switch time values $\tau_{2,1}$ and $\tau_{1,1}$.

sense that they are parsed by a dynamical system that produces optimized, hybrid control laws corresponding to strings of motions, locations, and temporal durations for each motion primitive. Also, the problem of interconnected puppetry plays was solved using a distributed saddle-point based algorithm. Experimental and simulation results illustrate the viability of these proposed approaches.

Acknowledgements

This work was sponsored by the U.S. National Science Foundation through grant number 0757317.

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