Distributed Reactive Power Sharing Control for Microgrids with Event-Triggered Communication

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***Abstract*—Due to the inherently distributed and heterogeneous nature of the microgrids, distributed control can be a promising approach to improving the stability, reliability and scalability of the microgrids compared with centralized control strategies. This work studies the distributed reactive power sharing problem for a microgrid with connected AC inverters. Under the standard decoupling approximation for bus angle differences, the reactive power flow of each inverter is dependent on the voltage amplitudes of its neighboring inverters connected by electrical power lines. Using the Lyapunov approach, a novel distributed voltage controller with nonlinear state feedback is proposed for reactive power sharing of the microgrid. It is proven that the inverters can achieve accurate reactive power sharing under the proposed controller if the communication network of inverters is connected. Then by introducing the sampling and holding scheme, we extend the proposed controller to the nonlinear state feedback control with event-triggered communication among inverters. The new event-triggered control approach can dramatically reduce the amount of communication of the microgrid, and significantly relax the requirement for precise real-time information transmission among the inverters. Both of the proposed controllers are validated by simulations on a group of inverters with time-varying loads.**

***Index Terms*—Microgrid, Reactive Power Sharing, Event-Triggered Control, Distributed Control**

I. Introduction

Microgrids provide a new paradigm for operation of the distributed generation units. In the integration of a microgrid, distributed generation units including micro-turbines, photovoltaic sets, fuel cells, wind energy systems and so on [1], [2]. Loads and energy storage elements are implemented to form a locally controllable system. The microgrid can be operated in either grid-connected or islanded modes.

In addition to the frequency/voltage stabilization and restoration problems [3]–[7], the power sharing task is a common practice in a microgrid [2], [8]–[10], as different types of generators may have different power generation capacities. Then appropriate power sharing strategies should be developed to adjust the power injections of generators to satisfy the capacity constraint, building a desired distribution of the power outputs to meet the load demand in the microgrid. Effective sharing of active and reactive powers among generators is thus an important performance criterion.

Since the generators are heterogeneous and may be placed at different locations, distributed control and management strategies are required to improve the stability, scalability, and security within the microgrid [11]–[15]. For the power sharing problem, primary droop control is a widely used method and can be implemented in a decentralized manner with centralized information processing [8], [16]. Active and reactive powers are measured for each generator to perform feedback control. The errors between the power setpoints and the measured power flows are taken as feedback signals to set the frequency and voltage amplitude for the inverters. Although this method can achieve power sharing with simple local controllers for inverters, the accuracy is low, especially when the loads change. In [17], a new droop control method based on error reduction and voltage recovery operations has been proposed to improve the accuracy for reactive power sharing.

Recently, the distributed power sharing problem has been investigated using the multi-agent consensus approach [18]. Inverters are considered as agents, which can exchange information with neighboring inverters through a communication network. Since the consensus control protocol can drive the agent group to an agreement of some interested quantity, a global objective can be achieved using only local control and agent-to-agent communication [19]. In particular, a consensus-based P-f droop control has been proposed in [20] to dynamically regulate the frequency when the load is time-varying. In [9], [10], the stability of closed-loop voltage controlled micrigrids is investigated by matrix analysis. It has been proven that the proposed consensus-based Q-V droop control can achieve proportional reactive power sharing asymptotically.

In microgrid coordination control, e.g., the consensus-based power sharing control, different generators need to communicate with each other to exchange their state information. A conventional solution for the communication network is to adopt the data sampling for the generator state. Then the information is exchanged periodically among the inverters. As the sampling period is fixed, the sampling rate should be high enough to guarantee that even in the worst cases, the magnitude of the error between the sampled signal and the realtime signal is small enough such that it will not affect the system stability and the coordination task. However, the communication network of a gird usually has limited bandwidth, and thus an efficient use of the communication infrastructure is desirable [21]. In this circumstance, introducing the aperiodic sampling scheme such as event-triggered sampling, in which the sampling time period is based on the prescribed coordination task, will significantly reduce those unnecessary samplings and make effective use of the communication network [22]–[24]. In the event-triggered diagram, a sampling execution is triggered when a state measurement error exceeds a given threshold, which is a fixed, time-varying, or state-dependent tolerance determined by the performance. For multi-agent

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coordination problems, distributed event-triggered consensus with fixed and time-varying thresholds has been studied in [25], while both the centralized and distributed event-triggered consensus algorithms with state-dependent thresholds have been investigated in [26]. Most recently, the event-triggered consensus approach has been applied in distributed generation control for microgrids in [27]. Using the idea in [26], both centralized and distributed secondary P-Q controllers for microgrid power balance with event-triggered communication have been proposed [27]. Moreover, to avoid continuous monitoring of neighboring inverters' power states, the self-triggered sampling approach, in which the next event time instant is determined by the previous received neighboring inverters' states, has also been studied and validated by experiments [27].

In this work, we consider the reactive power sharing problem for a group of inverters in a microgrid operating in islanded mode. Under the standard decoupling approximation for bus angle differences, the microgrid model for the reactive power injection of the inverter group has been established. Then a distributed droop controller involving nonlinear state feedback has been proposed based on the Lyapunov approach. It is proven that the closed-loop system has unique voltage distribution and moreover, the inverters can achieve proportional reactive power sharing under the proposed controllers. Then we adopt event-triggered communication to reduce the amount of communication among inverters. By proper design, each inverter is only required to transmit the reactive power state to its neighboring inverters at its own event time instants and thus continuous communication is avoided. The novelty of this work lies in the following two aspects. Firstly, we propose a Lyapunov-based design approach for reactive power sharing control. Compared with the consensus-based approach in [9], [10], our approach is simpler in convergence analysis and the condition for achieving power sharing is clearer in expression and easier to verify. Secondly, the proposed event-triggered controller is droop-based without introducing additional secondary control. Thus it can achieve faster and more accurate reactive power sharing than the control approach in [27].

The rest of this paper is organized as follows. Section II presents the electrical and communication network models, inverter models of the microgrid and the definition of proportional power sharing. In Section III, the proposed continuous and event-triggered power sharing controllers are provided with convergence analysis. In Section IV, simulations for microgrid with six inverters are conducted under the two proposed controllers to illustrate their effectiveness in proportional power sharing with varying loads. Finally the paper is concluded in Section V.

II. Preliminaries and Problem Formulation

A. Electrical network model

The generic model of a microgird is adopted. The microgrid is composed of N nodes, denoted by $\mathcal{V} = \{1, 2, \ldots, N\}$. Each node represents a distributed generator (DG) unit interfaced by an AC inverter. The power generator in each DG can be renewable energy rescouses such as PV panels and wind turbines, with a management circuit and an energy storage system. The inverters may have IGBT modules controlled by PWM signals. By using the inner current-loop and voltage-loop controllers, the output of the inverter can be regulated to the prescribed references. The electrical network of the microgrid can be captured by an undirected and complex-weighted graph $\mathcal{G}_e = (\mathcal{V}, \mathcal{E}_e)$, where the edge set $\mathcal{E}_e \subset \mathcal{V} \times \mathcal{V}$ represents the electrical connections. Each node i is associated with a phase angle $\theta_i(t)$ and a voltage magnitude $V_i(t)$. The connection between nodes i and l, i.e., the edge $(i, l) \in \mathcal{E}_e$, is associated with a complex admittance $Y_{il} = Y_{li} = G_{il} + jB_{il}$, where G_{il} and B_{il} represent the conductance and the susceptance, respectively [10]. If nodes i and l are not directly connected, then $Y_{il} = 0$. If node l is connected with node i, then node l is called as an electrical neighbor of node i. All the electrical neighbors of node i constitute its electrical neighbor set $N_i^e = \{l \in \mathcal{V} | (i, l) \in \mathcal{E}_e\}$. In graph $\mathcal{G}_e$, a path of length $m + 1$ from node i to node l consists of a set of edges with the form of $(i, v_1), (v_1, v_2), \ldots, (v_m, l)$. If for any distinct pair of nodes i and l, there exists a path, then graph $\mathcal{G}_e$ is called connected. We make the assumption that $\mathcal{G}_e$ is connected for the microgrid.

In this section the power flow model in [28] is presented and then simplified. Let the angle difference between node i and node l be $\theta_{il} = \theta_i - \theta_l$. The active and reactive power flows from node i to node l are

$$P_{il} = G_{il}V_i^2 - \sum_{l \in N_i^e} V_i V_l (G_{il} \cos(\theta_{il}) + B_{il} \sin(\theta_{il})), \quad (1)$$

$$Q_{il} = -B_{il}V_i^2 - \sum_{l \in N_i^e} V_i V_l (G_{il} \sin(\theta_{il}) - B_{il} \cos(\theta_{il})), \quad (2)$$

where V_i is the voltage amplitude of node i. The total active and reactive power flows at node i are

$$P_i = G_{ii}V_i^2 - \sum_{l \in N_i^e} V_i V_l (G_{il} \cos(\theta_{il}) + B_{il} \sin(\theta_{il})), \quad (3)$$

$$Q_i = -B_{ii}V_i^2 - \sum_{l \in N_i^e} V_i V_l (G_{il} \sin(\theta_{il}) - B_{il} \cos(\theta_{il})), \quad (4)$$

where $G_{ii} = \hat{G}_{ii} + \sum_{l \in N_i^e} G_{il}$ and $B_{ii} = \hat{B}_{ii} + \sum_{l \in N_i^e} B_{il}$ with $\hat{G}_{ii}$ and $\hat{B}_{ii}$ being the shunt conductance and shunt susceptance at node i, respectively. The apparent power flow at node i is $S_i = P_i + jQ_i$.

The objective of this work is to develop distributed controllers for accurate reactive power sharing. To simplify the electrical network model and facilitate both presentation and mathematical development, the standard decoupling approximation which adopts small bus angle differences is employed [9], [28]–[30], i.e., we assume that $\theta_{il} = \theta_i - \theta_l \approx 0$ and then $\sin(\theta_{il}) \approx 0$ and $\cos(\theta_{il}) \approx 1$. Then from (4) the reactive power flow at node i [9] is

$$Q_i = |B_{ii}|V_i^2 - \sum_{l \in N_i^e} |B_{il}| V_i V_l, \quad (5)$$

where $|B_{ii}| \geq \sum_{l \in N_i^e} |B_{il}|$. From this equation, the reactive power flow can be controlled by the voltage amplitudes of itself and its electrical neighbors.

B. Communication network model

We assume that the reactive power flow at each node is measured and then transmitted to its neighboring nodes in the group for the distributed voltage control use in the feedback loop. The communication connections of the inverters can be represented by an undirected communication graph $\mathcal{G}_c = \{\mathcal{V}, \mathcal{E}_c\}$ with $\mathcal{E}_c \subset \mathcal{V} \times \mathcal{V}$ being the communication edge set. If two nodes i and l can communicate with each other, then there is a communication link between these two nodes, denoted by $(i, l) \in \mathcal{E}_c$, and nodes i and l are called as communication neighbors. All of the communication neighbors of node i constitute its communication neighbor set $N_i^c = \{l \in \mathcal{V} | (i, l) \in \mathcal{E}_c\}$. The adjacency matrix of graph $\mathcal{G}_c$ is an $N \times N$ matrix A defined as $A = (a_{il})_{N\times N}$ with

$$a_{il} = \begin{cases} 1, & \text{if } l \in N_i^c, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Here we assume that there is no self-edge in the graph and thus $a_{ii} = 0$ for any $i \in \mathcal{V}$. The (in-)degree of node i is defined by $d_i = \sum_{l\in N_i^c} a_{il}$. The degree matrix is then defined by $D = diag(d_1, \dots, d_N)$. Using these definitions, the Laplacian matrix of graph $\mathcal{G}_c$ is given by $L = D - A$.

According to the results for graph Laplacian matrix in [19], L is a symmetric and positive semi-definite matrix. If $\mathcal{G}_c$ is connected, then its Laplacian L is of rank $N-1$. Moreover, 0 is L's eigenvalue of multiplicity 1 if and only if $\mathcal{G}_c$ is connected. Its associated eigenvector is a column vector of all ones, i.e., $L\mathbf{1}_N = 0$ with $\mathbf{1}_N = (1, \dots, 1)^T$. For an undirected graph, all the eigenvalues are nonnegative real numbers. Thus they can be ordered sequentially in an ascending order as

$$0 = \lambda_1 \le \lambda_2 \le \dots \le \lambda_N. \tag{7}$$

If $\mathcal{G}_c$ is connected, then the second smallest eigenvalue λ_2, called as the algebra connectivity of $\mathcal{G}_c$, is positive. According to Gershgorin theorem, all the eigenvalues of L in the complex plane are located in a closed disk centered at $\max_i(d_i) + j0$ with a radius of $\max_i(d_i)$. Thus we have $\lambda_N \le 2\max_i(d_i)$.

C. Inverter model, power sharing, and droop control

The inverter model of node i can be represented by the following equations [10],

$$\begin{aligned} \dot{\theta}_i &= u_i^\theta, &(8)\\ \tau_i \dot{V}_i &= -V_i + u_i^V, &(9) \end{aligned}$$

where θ_i and V_i are the output angle and voltage amplitude of inverter i, respectively. u_i^θ and u_i^V are the control inputs to be determined. τ_i represents the input delay constant of the voltage. This model implies that the inverter is considered as a controllable AC voltage sources and its amplitude and frequency can be controlled by the inputs u_i^θ and u_i^V. Since we only consider the reactive power flows, the dynamics of θ_i are neglected in the sequel of this paper. Moreover, we assume that the voltage of the inverter has a very fast adjusting rate and thus $\tau_i \approx 0$. Then the inverter voltage and the reactive power of node i has the following model

$$\begin{aligned} V_i &= u_i^V, &(10)\\ Q_i &= |B_{ii}|V_i^2 - \sum_{l\in N_i^e} |B_{il}|V_i V_l, &(11) \end{aligned}$$

where Q_i is the reactive power of inverter i, B_{ii} is the susceptance at inverter i, B_{il} is the susceptance between inverter i and j, and N_i^e represents the electrical neighbor set of inverter i. For reactive power sharing, let $\chi_i > 0$ be the weight factor of the reactive power at node i. Then two inverters at nodes i and l are said to share their reactive powers proportionally according to the weights χ_i and χ_l [9] if

$$\frac{Q_i^s}{\chi_i} = \frac{Q_l^s}{\chi_l}, \tag{12}$$

where Q_i^s and Q_l^s are the reactive power flows at nodes i and l at the steady state, respectively. A practical choice for the weight of node i is $\chi_i = S_i^N$ with $S_i^N > 0$ being the nominal power rating. For a prescribed set of weights $\{\chi_1, \dots, \chi_N\}$, we define

$$\bar{Q}_i = \frac{Q_i}{\chi_i}, i = 1, \dots, N \tag{13}$$

as the weighted reactive power.

Conventional droop control for the voltage regulation has the following form [2]

$$V_i = V_i^d - \kappa_i^Q (Q_i - Q_i^d), \tag{14}$$

where $V_i^d > 0$ is the desired voltage amplitude and Q_i^d is the setpoint for the reactive power output of the inverter at node i. In this controller, the feedback signal is the locally injected reactive power Q_i with a droop gain κ_i^Q. Similarly, the conventional frequency droop control has the form [2]

$$\omega_i = \omega_i^d - \kappa_i^P (P_i - P_i^d), \tag{15}$$

where ω_i^d is the desired frequency, κ_i^P is the frequency droop gain, and P_i^d is the desired setpoint of inverter i. To achieve proportional power sharing, the setpoints are usually designed using a secondary controller.

III. Distributed Reactive Power Sharing

A. Distributed voltage control

Different from the conventional droop control, we will propose a distributed voltage controller with inverters' state communication for reactive power sharing. Since the voltage control model is simplified in (10), $u_i^V(t)$ becomes the voltage reference for inverter i. We design the following reference for inverter i

$$u_i^V(t) = V_i^d - \int_0^t u_i(\tau) d\tau, \tag{16}$$

where V_i^d is the desired voltage magnitude and $u_i(t)$ is the voltage control input for inverter i and will be determined by feedback signals. Such design implies that for reactive power sharing, the convectional Q-V droop control has been replaced by Q-$\dot{V}$ droop control, which can improve the accuracy in power sharing [9]. By differentiating V_i with respect to t, the dynamics for node i are

$$\dot{V}_i = u_i, \tag{17}$$

$$Q_i = |B_{ii}|V_i^2 - \sum_{l\in N_i^e} |B_{il}|V_iV_l. \tag{18}$$

We will show that by using a consensus-like nonlinear state feedback of the signals V_i and Q_i, the reactive power sharing can be achieved asymptotically.

Assume that each node measures the voltage amplitude V_i and the reactive power Q_i by power meter and transmits the signal Q_i to all its communication neighbors via the communication network. We propose the following distributed feedback controller for reactive power sharing,

$$u_i = -\frac{\kappa}{\chi_i}V_i \sum_{l\in N_i^c}(\frac{Q_i}{\chi_i} - \frac{Q_l}{\chi_l}), \tag{19}$$

where $\kappa > 0$ is the feedback gain to be determined, V_i, Q_i are local information of node i and can be measured by node i itself, and $Q_l, l \in N_i^c$ can be received through the communication network. There is no central node to deal with the control of all the nodes. Only local neighboring inverters' information is required and thus the controller is distributed.

The proposed controller (19) employs the product of voltage amplitude and the reactive power and the controller has a nonlinear feedback form. Compared with existing voltage controllers for reactive power sharing, the advantage of this controller is that no linearization around the operating point is involved and thus global convergence analysis of the system can be conducted.

For the closed-loop dynamics of the system with the proposed distributed controller, we will first investigate the equilibrium of the entire microgrid and then provide the convergence analysis. To simplify the analysis, the following notations are employed:

$$V = col(V_i), \quad Q = col(Q_i), \quad u = col(u_i), \tag{20}$$

$$C = diag(\tfrac{1}{\chi_1}, \dots, \tfrac{1}{\chi_N}), \quad \widetilde{V} = diag(V_1, \dots, V_N). \tag{21}$$

Then the closed-loop dynamics of the microgrid are

$$\dot{V} = -\kappa C\widetilde{V}LCQ, \tag{22}$$

$$Q = \widetilde{V}BV, \tag{23}$$

where L is the Laplacian matrix of the communication graph $\mathcal{G}_c$ and the matrix B is the susceptance matrix defined as $B = (b_{il})_{N\times N}$ with elements

$$b_{il} = \begin{cases} |B_{ii}|, & \text{if } l = i, \\ -|B_{il}|, & \text{if } l \in N_i^e, \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

1) Equilibrium analysis: The existence of the equilibrium for the dynamic system (22) and (23) can be analyzed at the steady state. Denote V^s and Q^s as the voltage vector and the reactive power vector of the microgrid at the steady state, respectively. Setting $\dot{V} = 0$ yields the steady state equation of the system

$$-C\widetilde{V}^sLCQ^s = 0, \quad Q^s = \widetilde{V}^sBV^s, \tag{25}$$

which implies

$$-C\widetilde{V}^sLC\widetilde{V}^sBV^s = 0. \tag{26}$$

Since the voltages are positive, we have $LC\widetilde{V}^sBV^s = 0$ at the steady state. For this equation, there are two cases to be investigated. One is

$$BV^s = 0 \tag{27}$$

and the other one is

$$BV^s \neq 0 \text{ and } LCQ^s = 0. \tag{28}$$

In the following we will show that $BV^s = 0$ cannot hold and thus the conclusion is $LCQ^s = 0$.

Since $B_{ii} = \hat{B}_{ii} + \sum_{l\in N_i^e} B_{il}$ [28], we have

$$B = \hat{B} + L_B \tag{29}$$

where $\hat{B} = diag(\hat{B}_{11}, \dots, \hat{B}_{NN})$ with $\hat{B}_{ii} > 0$ being the shunt susceptance at node i and $L_B = (L_{il})_{N\times N}$ is a symmetric matrix with elements

$$L_{il} = \begin{cases} \sum_{l\in N_i^e} |B_{il}|, & \text{if } l = i, \\ -|B_{il}|, & \text{if } l \in N_i^e, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

According to the definition of a Laplacian matrix, L_B can be considered as a weighted Laplacian matrix for the electrical graph $\mathcal{G}_e$. Since the electrical graph is connected, L_B is positive semidefinite. Thus B is positive definite.

Recall the two cases for equilibrium analysis. Since B is positive definite and $V_i^s > 0$ by assumption, $BV^s \neq 0$. Thus it is required that $LCQ^s = 0$ at steady state. Since it is also assumed that the communication graph $\mathcal{G}_c$ is connected, the null space of the Laplacian matrix L is $span\{\mathbf{1}_N\}$ [19]. Thus $LCQ^s = 0$ implies that $CQ^s = \alpha\mathbf{1}_N$, where α is a nonzero real number and $\mathbf{1}_N$ is an order N vector of all ones. From $Q^s = \widetilde{V}^sBV^s$, we have $\alpha > 0$. If there exists a vector V^s such that

$$C\widetilde{V}^sBV^s = \alpha\mathbf{1}_N, \tag{31}$$

then the equilibrium of the closed-loop system exists.

Simple computation shows that $C\widetilde{V}^sBV^s = \alpha\mathbf{1}_N$ is equivalent to

$$-\alpha C^{-1}(\widetilde{V}^s)^{-1}\mathbf{1}_N + BV^s = 0. \tag{32}$$

Denote

$$F(V^s) = col(f_i(V_i^s)) = -\alpha C^{-1}(\widetilde{V}^s)^{-1} \tag{33}$$

as a vector function. Then the i-th entry of this function is $f_i(V_i^s) = -\frac{\alpha\chi_i}{V_i^s}$. It is noted that $f_i(V_i^s)$ is monotonically increasing in V_i^s and B is positive semidefinite. Then using a similar argument as that in the proof of Proposition 1 in [9], we have the following result.

Lemma 1. *For each $\alpha > 0$, there exists a unique V^s such that $C\widetilde{V}^sBV^s = \alpha\mathbf{1}_N$.*

This lemma shows the existence of the equilibrium. In the following we will prove that this equilibrium is stable.

2) Convergence analysis: Let the communication topology be represented by an undirected graph $\mathcal{G}_c$. At steady state, inverters i and l share their reactive powers proportionally according to the weights χ_i and χ_l if equation

$$\lim_{t\to\infty} \frac{Q_i}{\chi_i} - \frac{Q_l}{\chi_l} = 0 \tag{34}$$

holds. From the multi-agent control perspective, this implies that all $\frac{Q_i}{\chi_i}$ achieve consensus as time goes to infinity. Since the communication topology is connected, $LCQ^s = 0$ guarantees such state consensus [19].

Since the electrical graph $\mathcal{G}_e$ is connected, matrix B is positive definite. Then we can choose the Lyapunov functional candidate as

$$W = \frac{1}{2} V^T BV. \tag{35}$$

Since $Q_i = b_{ii}V_i^2 - \sum_{l\in N_i^e} b_{il}V_iV_l$, the Lyapunov function can also have the form of $W = \frac{1}{2}\sum_{i=1}^{N} Q_i$.

Computing the time derivative of the Lyapunov function yields

$$\begin{aligned} \dot{W} &= V^T B\dot{V} = V^T B\widetilde{V}\widetilde{V}^{-1}u \\ &= (\tilde{V}BV)^T \widetilde{V}^{-1}u = Q^T\widetilde{V}^{-1}u. \end{aligned} \tag{36}$$

Considering the proposed distributed controller (19), the augmented form of the controller for the microgrid dynamics (22) and (23) is

$$u = -\kappa C\widetilde{V}LCQ, \tag{37}$$

then

$$\begin{aligned} \dot{W} &= Q^T\widetilde{V}^{-1}(-\kappa C\widetilde{V}LCQ) \\ &= -\kappa (CQ)^T L(CQ) \\ &= -\kappa \bar{Q}^T L\bar{Q} \le 0, \end{aligned} \tag{38}$$

where $\bar{Q}_i$ is the weighted reactive power. By the LaSalle's invariance principle, we have $\lim_{t\to\infty} L\bar{Q} = 0$. Since the communication graph $\mathcal{G}_c$ is connected, the Laplacian matrix is positive semidefinite. It has an eigenvalue 0 with multiplicity 1, with the eigenvector being the vector of all ones. Thus we have

$$\lim_{t\to\infty} (\bar{Q}_i(t) - \bar{Q}_l(t)) = 0 \tag{39}$$

for any i and l. This is equivalent to $\frac{Q_i^s}{\chi_i} = \frac{Q_l^s}{\chi_l}$ for any i and l, which implies that the reactive powers will achieve proportional sharing asymptotically. We have the following result for the proposed control strategy.

Theorem 1. *Consider the voltage control system for reactive power sharing in (17). Assume that the electrical graph $\mathcal{G}_e$ and the communication graph $\mathcal{G}_c$ are connected. Then the proportional reactive power sharing problem is solved asymptotically by the distributed controller (19).*

B. Event-triggered communication

In the distributed control approach developed in the previous section, each inverter is required to monitor the reactive power flows of itself $Q_i(t)$ and its neighbors $Q_l(t)$ to generate the control input $u(t)$ to achieve the reactive power sharing task. Generally, it is assumed that such power flow state monitoring is physically realized by using node-to-node wired/wireless communication. A practical way for such communication is to implement the signal transmission among different nodes by the traditional periodical sampling strategy. Each node adopts a fixed sampling period T, and sends the sampled power flow states $Q_i(kT)$ to its neighboring nodes.

However, since the controller design is conducted in the continuous-time form using the states of $V_i(t)$, $Q_i(t)$ and $Q_l(t)$, an ideal choice is to adopt a high communication frequency $\frac{1}{T}$ to guarantee that realtime signal $Q_i(kT)$ is transmitted to all the neighbors. Thus the neighboring nodes can timely access the state information of each other without introducing any degradation to the control performance. Consequently, the periodic sampling control strategy will generate a large amount of communication during the operating process of the microgrid.

Actually, to achieve proportional power sharing, many samplings in a periodic sampling scheme may be redundant. The frequency of the state sampling can be reduced to a lower level by a proper design of the sampling strategy without affecting the achievement and even the accuracy of the power sharing. As a result, the amount of communication can be reduced significantly. Recently, the event-triggered control method has been employed in control systems to reduce the efforts on communication and computation. As a nonuniform sampling control technique, event-triggered control can achieve satisfactory performance while significantly save the usage of bandwidth and signal processing resources [22], [25], [27]. Motivated by this observation, in this section we will propose an event-triggered scheme to reduce the communication load among the nodes.

From the controller design, it is noted that the reactive power flow $Q_i(t)$ should be measured and then transmitted between neighboring nodes. We assume that each node i only sends this information at some prescribed time instants, denoted by

$$t_0^i = 0 < t_1^i < \ldots < t_k^i < \ldots, k \in \mathbb{N}. \tag{40}$$

These time instants are determined by a special set of inequalities, called "event conditions". When a time-dependent event condition is satisfied, it is considered that a prescribed "event" occurs. The time instants t_k^i when events occur are called "event time instants". These time instants t_k^i will be used to determine when a node conducts state sampling and executes communication transmission to its neighboring nodes. Therefore, once an event occurs, the event triggering signal will be send to the state sampling module, the communication module, and the controller module, to trigger the actions of sampling, communicating, and control input updating [26].

Let $Q_i(t_k^i)$ be the transmitted measurement of $Q_i(t)$ at the event time instant t_k^i. Define the measurement error of node i at time t as

$$e_i(t) = Q_i(t_k^i) - Q_i(t). \tag{41}$$

This error can be considered as the difference between the sampled state and the realtime state. The expectation is that if the magnitude of this error can be properly controlled, then $Q_i(t)$ can be replaced by the sampled measurement $Q_i(t_k^i)$ in the controller design while avoiding any obvious degradation on the control performance. Following the proposed distributed controller (19), we propose an event-triggered voltage controller for reactive power sharing as

$$u_i(t) = -\frac{\kappa}{\chi_i} V_i(t) \sum_{l \in N_i^c} \left(\frac{Q_i(t^i_{k_i(t)})}{\chi_i} - \frac{Q_l(t^l_{k_l(t)})}{\chi_l}\right), t \in [t^i_k, t^i_{k+1}), \tag{42}$$

where

$$k_i(t) = \arg\max_{k \in \mathbb{N}} \{t^i_k | t^i_k \leq t\} \tag{43}$$

represents the subscript of the latest event time instant to the current time t. In this new controller (42), $V_i(t)$ is the voltage amplitude of inverter i. $Q_i(t^i_{k_i(t)})$ and $Q_l(t^l_{k_l(t)})$ are reactive power flow states of inverter i and its neighbor l. These states can be obtained locally by meters and neighbor to neighbor communication. Thus the event-triggered controller (42) is distributed and no central processing unit is needed.

Since the measurement $Q_i(t^i_k)$ is the sampled state of $Q_i(t)$ at the event time instant t^i_k, when it is used in the controller design, the event time instant should be properly chosen such that this sampled measurement of $Q_i(t)$ will not affect the stability of the microgrid and the power sharing task can still be achieved. Thus well-designed event conditions should be established to generate these proper event time instants. To facilitate the event design, denote

$$\hat{Q}_i(t) = Q_i(t^i_k), t \in [t^i_k, t^i_{k+1}) \tag{44}$$

and let $\hat{Q}(t) = col(\hat{Q}_i(t))$. Then from (42), the vector form of the controller for the microgrid is

$$u = -\kappa C \widetilde{V} L C \hat{Q}. \tag{45}$$

Since C and $\widetilde{V}$ are both diagonal matrices, $C\widetilde{V} = \widetilde{V}C$. Consider the same Lyapunov function as in (35). Computing its time derivative yields

$$\begin{aligned} \dot{W} &= V^T B \dot{V} = V^T B(-\kappa C \tilde{V} L C \hat{Q}) \\ &= -\kappa(\tilde{V} B V)^T C L C \hat{Q} \\ &= -\kappa Q^T C L C \hat{Q}. \end{aligned} \tag{46}$$

Using the definition of the measurement error in (41), we have $Q = \hat{Q} - e$. Then

$$\begin{aligned} \dot{W} &= -\kappa(\hat{Q} - e)^T C L C \hat{Q} \\ &= -\kappa \hat{Q}^T C L C \hat{Q} + \kappa e^T C L C \hat{Q}. \end{aligned} \tag{47}$$

For any vectors $x, y \in \mathbb{R}^N$, the inequality $x^T y \leq \frac{1}{2a} x^T x + \frac{a}{2} y^T y$ always holds for a positive constant a. Then

$$\begin{aligned} \dot{W} &= -\kappa \hat{Q}^T C L C \hat{Q} + \kappa (Ce)^T (L C \hat{Q}) \\ &\leq -\kappa \hat{Q}^T C L C \hat{Q} + \kappa\left(\frac{1}{2a}\|Ce\|^2 + \frac{a}{2}\|L C \hat{Q}\|^2\right) \end{aligned} \tag{48}$$

By designing the event time instants, we can require that

$$\|Ce\| < \eta \|L C \hat{Q}\|, \tag{49}$$

where η is a positive constant to be determined. Then we have

$$\dot{W} \leq -\kappa \hat{Q}^T C L C \hat{Q} + \kappa\left(\frac{\eta^2}{2a} + \frac{a}{2}\right)\|L C \hat{Q}\|^2. \tag{50}$$

Using the fact that λ_N is the largest eigenvalue of the Laplacian matrix L, we conclude that

$$\|L C \hat{Q}\|^2 = \hat{Q}^T C L L C \hat{Q} \leq \lambda_N \hat{Q}^T C L C \hat{Q}. \tag{51}$$

Then for $\dot{W}$ we have

$$\dot{W} \leq -\kappa\left(1 - \frac{\lambda_N}{2}\left(a + \frac{\eta^2}{a}\right)\right)\hat{Q}^T C L C \hat{Q}. \tag{52}$$

Thus $\dot{W} \leq 0$ if the constants a and η are chosen such that

$$1 - \frac{\lambda_N}{2}\left(a + \frac{\eta^2}{a}\right) > 0. \tag{53}$$

This condition is equivalent to $a + \frac{\eta^2}{a} < \frac{2}{\lambda_N}$, which can be replaced by letting

$$a < \frac{2}{\lambda_N} \text{ and } \eta^2 < -a^2 + \frac{2}{\lambda_N} a \tag{54}$$

to guarantee $\dot{W} \leq 0$. From the condition (49), it can be concluded that a larger η will allow a larger magnitude of the measurement error e, which results in fewer triggering events. Since

$$-a^2 + \frac{2}{\lambda_N} a = -\left(a - \frac{1}{\lambda_N}\right)^2 + \frac{1}{\lambda_N^2}, \tag{55}$$

the upper bound of η can be chosen as large as possible, e.g., being $\frac{1}{\lambda_N}$. In this case, $a = \frac{1}{\lambda_N}$, which also meets the condition $a < \frac{2}{\lambda_N}$. Based on the above mentioned analysis, by letting $a = \frac{1}{\lambda_N}$, (52) becomes

$$\dot{W} \leq -\frac{\kappa}{2}(1 - \eta^2 \lambda_N^2)\hat{Q}^T C L C \hat{Q}. \tag{56}$$

Thus if (49) holds and in addition $0 < \eta < \frac{1}{\lambda_N}$, then $\dot{W} \leq 0$.

However, checking condition (49) requires global execution of the microgrid since e, L, and $\hat{Q}$ are all defined based on global information. Proper design of the event condition is needed such that the event time instants can be determined in a distributed manner. For any $x \in \mathbb{R}^N$, denote $[x]_i$ as its ith entry. Then condition (49) can be satisfied if

$$|[Ce]_i| \leq \eta |[L C \hat{Q}]_i| \tag{57}$$

holds for any i. This set of conditions is equivalent to

$$|e_i(t)| \leq \eta \chi_i \left| \sum_{l \in N_i^c} \left(\frac{Q_i(t^i_{k_i(t)})}{\chi_i} - \frac{Q_l(t^l_{k_l(t)})}{\chi_l}\right) \right|, i = 1, \ldots, N. \tag{58}$$

From the above analysis, (58) with $0 < \eta < \frac{1}{\lambda_N}$ is sufficient to guarantee $\dot{W} \leq 0$. For each node, condition (58) uses the states of $e_i(t)$, $\hat{Q}_i(t)$, and $\hat{Q}_l(t)$. They can be locally obtained by node i through measurement and communication channels. Thus the process of checking condition (58) can be performed in a distributed manner. Since (58) is sufficient for (49), to guarantee $\dot{W} \leq 0$, the event time instants can be determined as the time when (58) is violated. Once such violation happens, an event is triggered and node i takes a new sampling of $Q_i(t)$ to update $\hat{Q}_i(t)$. According to (41), this sampling action will naturally reset the measurement error $e_i(t)$ to 0. Then (58) will always be satisfied and thus $\dot{W} \leq 0$. Following this idea,

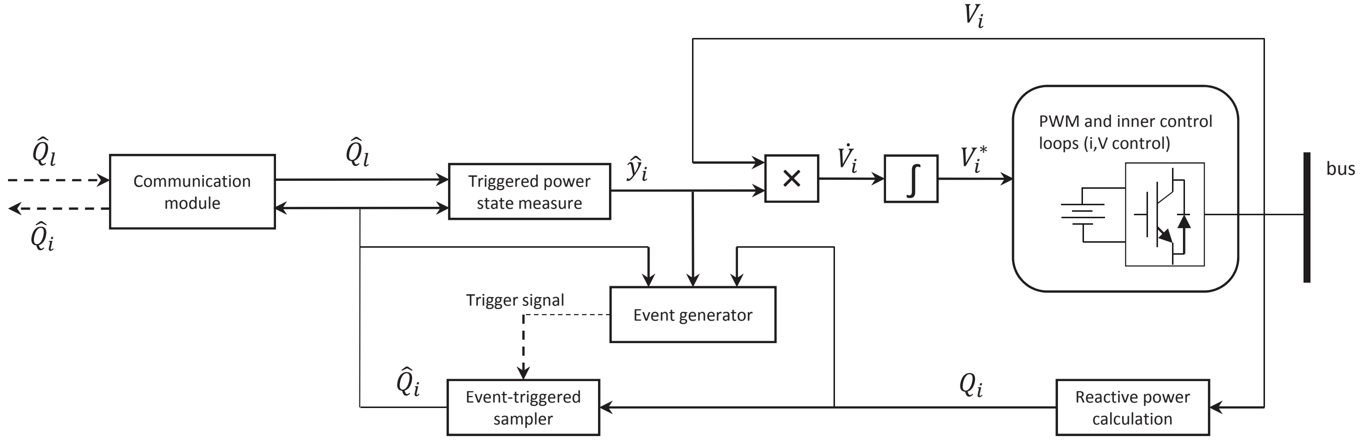

Fig. 1. Structure of the event-triggered controller for a single inverter.

we propose the distributed event condition

$$|\bar{e}_i(t)| \geq \eta \left| \sum_{l \in N_i^c} \left(\bar{Q}_i(t^i_{k_i(t)}) - \bar{Q}_l(t^l_{k_l(t)}) \right) \right|, i = 1, \ldots, N, \tag{59}$$

where $0 < \eta < \frac{1}{\lambda_N}$ and

$$\bar{e}_i(t) = \frac{e_i(t)}{\chi_i} = \bar{Q}_i(t^i_{k_i(t)}) - \bar{Q}_i(t). \tag{60}$$

An event of node i occurs if (59) is satisfied. Then agent i will take the state sampling of $Q_i(t)$ and then update its control input (42) by replacing $Q_i(t^i_{k_i(t)})$ using the new sampled state. Node i will also send the new sampled state to its neighbors through communication. On the other hand, if any neighbor node triggers an event, node i will receive the sampled reactive power state from this neighbor. Following the controller (42), node i will also update its control input to a new level.

From (56), the event-triggered controller (42) with the event time instants determined by (59) are sufficient to guarantee $\dot{W} \leq 0$. By the LaSalle's invariance principle, we have

$$\lim_{t \to \infty} LC\hat{Q}(t) = 0. \tag{61}$$

Since the graph $\mathcal{G}_c$ is connected, $C\hat{Q}(t)$ converges to a vector in $span\{\mathbf{1}_N\}$ as time goes to infinity, which is equivalent to $\frac{\hat{Q}_i^s}{\chi_i} = \frac{\hat{Q}_l^s}{\chi_l}$ for all $i, l = 1, \ldots, N$. On the other hand, $\lim_{t \to \infty} LC\hat{Q}(t) = 0$ implies that

$$\lim_{t \to \infty} [LC\hat{Q}(t)]_i = 0 \tag{62}$$

for any i. Then from (58) one has $\lim_{t \to \infty} e_i(t) = 0$ for any i. Based on the definition of $e_i(t)$, we have $Q_i^s = \hat{Q}_i^s$ and thus $\frac{Q_i^s}{\chi_i} = \frac{Q_l^s}{\chi_l}$ for any $i, l = 1, \ldots, N$, which implies that the reactive powers have achieved proportional sharing asymptotically. This power sharing result under the event-triggered controller can be described in the theorem below.

Theorem 2. *Consider the voltage control system for reactive power sharing in (17). Assume that the electrical graph $\mathcal{G}_e$ and the communication graph $\mathcal{G}_c$ are connected. Then the proportional reactive power sharing problem is solved asymptotically by the distributed event-triggered controller (42), where the event time instants are determined by (59).*

The block structure of the proposed event-triggered reactive power sharing controller for a single inverter is shown in Fig. 1. Events are determined by the event generator using the states of Q_i, $\hat{Q}_i$, and $\hat{y}_i = \sum_{l \in N_i^c} (\frac{\hat{Q}_i}{\chi_i} - \frac{\hat{Q}_l}{\chi_l})$. The voltage control input u_i consists of a nonlinear feedback form of V_i and $\hat{y}_i$. Then the voltage reference V_i^* for the inner-loop control is generated by an integrity of the control input u_i to achieve the reactive power sharing task. The communication module is responsible for sending the event-triggered reactive power states $\hat{Q}_i$ to the neighbor inverters and also receiving those states $\hat{Q}_l$ from the neighbors.

IV. Case Studies

The effectiveness of the proposed distributed controllers is validated by simulations based on an islanded microgrid. The networked system consists of six inverters operating in parallel and supplying a variable load, as shown in Fig. 2. Assume that the frequency of each inverter is dominated by a conventional droop controller described in (15). Since the inverter only requires P_i, i.e., the active power measurement of itself, to conduct conventional frequency droop control, there is no need to transmit the active power states by the communication channels. Thus the frequency droop control is actually distributed. For the proposed voltage controllers, reactive power states should be sent to neighboring inverters through the communication network. In this section for case studies, the communication network of the inverter group is given in Fig. 3. There is no central node/inverter to perform as a reference in this network and thus the entire control structure of the microgrid is distributed.

In this microgrid, the nominal voltage amplitude and frequency of the microgrid are 400v and 50Hz, respectively. Each inverter is associated with a reactive power rating $S^N =$(0.54, 0.72, 0.34, 0.66, 0.45, 0.26)pu and the base reactive power is assumed to be $S_{base} = 3$kvar. The weight coefficients for the reactive power sharing are selected as multiples of the nominal power rating $\chi_i = 3S_i^N$ for $i = 1, 2, 3, 4, 5, 6$. The

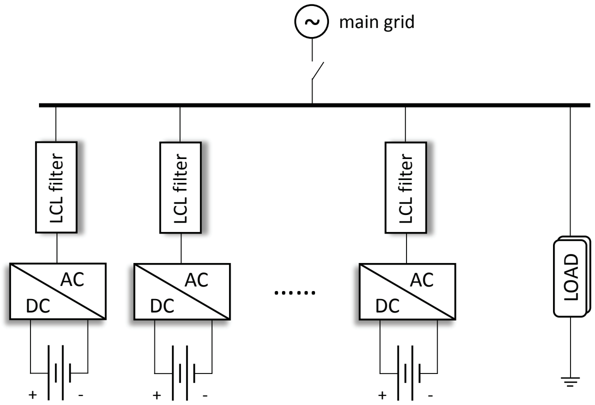

Fig. 2. Microgrid with parallel operating DGs interfaced by AC inverters.

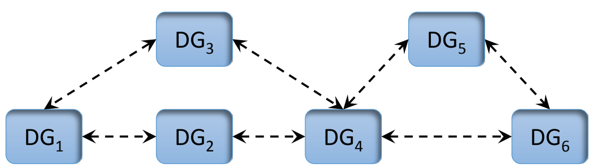

Fig. 3. Communication structure of the DGs.

feedback gains of the controllers in (19) and (42) are chosen as $\kappa = 2 \times 10^{-5}$ var^{-1}. For each inverter, we set the desired voltage amplitude as $V_i^d = 1$pu.

A. Periodic Communication

For the nonlinear feedback controller in (19), consider a variable-load case. To satisfy the realtime information transmission requirement, each DG measures its own reactive power and sends the state Q_i periodically at a frequency of 200Hz to the controller. Initially the microgrid is operated under the nominal load condition. At time $t = 2$s, the load increases to a new level to validate the effectiveness of the controller.

Fig. 4 shows the trajectories of the reactive power and the voltage amplitudes of the inverters under the continuous controller (19) with periodic communication. From the first subfigure, one can notice that the proposed controller can drive the inverters to achieve proportional reactive power sharing before the load changes. This is consistent with the results in Theorem 1. The second subfigure shows that the power sharing is accurate, since consensus of the weighted reactive powers $\bar{Q}_i$ has been reached. In the third subfigure, the voltage amplitudes achieve their corresponding stable distribution as described in Lemma 1. When the load increases, new proportional reactive power sharing according to the weight coefficients χ_i has been established asymptotically by the proposed control strategy. Fig. 5 shows the voltage control inputs of the inverters under the continuous controller. Although the control input performs abrupt changing when the load increases, the voltage amplitude at each inverter changes smoothly, due to the employment of additional dynamic process, i.e., the Q-$\dot{V}$ droop, in the control law.

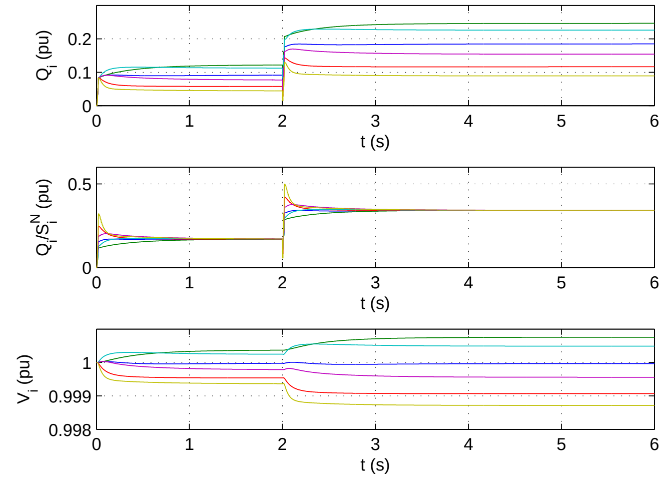

Fig. 4. Trajectories of reactive power, weighted reactive power, and voltage amplitudes of inverters under the controller (19).

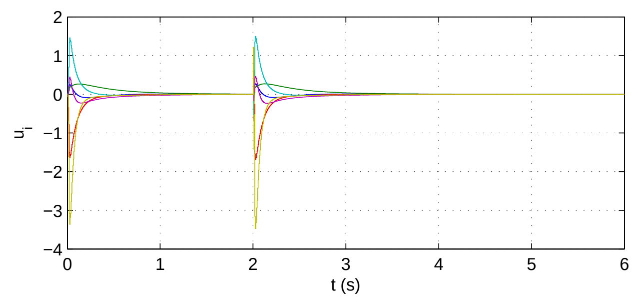

Fig. 5. Voltage control inputs with periodic communication.

B. Event-triggered Communication

For the feedback control with event-triggered communication in (42), consider the same load changing scenario as that in the periodic communication case to facilitate reasonable comparison. It is noted that when the microgrid is operating at the steady-state, the right hand side of (59) can be very small. Then in a real microgrid, due to the existence of measurement and computation errors, the event generation can be more often than required. To prevent this, one may devise a filter for the measurement of reactive power flows, or set a small tolerance for $e_i(t)$ in (59). Here we add an ϵ on the right hand side of (59). Without affecting the accuracy of the power sharing, choose $\epsilon = 0.1$ in this case study.

Fig. 6 provides the trajectories of the reactive power and the voltage amplitudes of the inverters under the event-triggered controller with transmission of signal $Q_i(t_k^i)$ via the communication network. Notice that the new controller can drive the inverters to achieve accurate proportional reactive power sharing under time-varying loads. Moreover, compared with the trajectories in Fig. 4, no significant performance loss has been detected for the event-triggered control strategy, although realtime communication cannot be satisfied.

Fig. 7 shows the voltage control inputs of the inverters under the event-triggered controllers. One can observe that each input

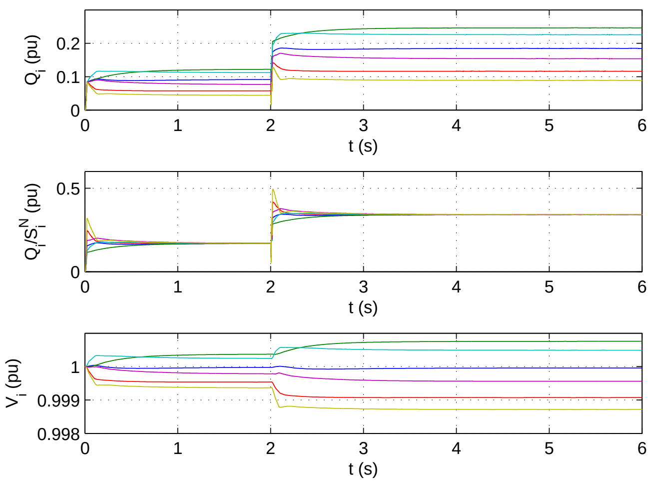

Fig. 6. Trajectories of reactive power, weighted reactive power, and voltage amplitudes of inverters under the controller (42).

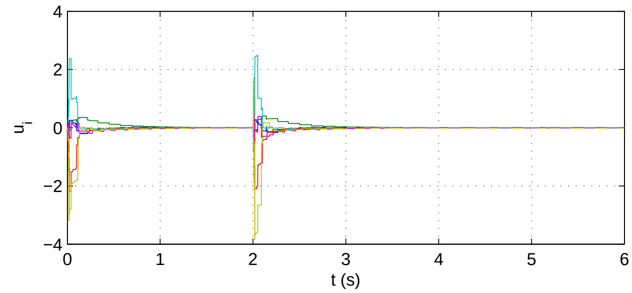

Fig. 7. Voltage control inputs with event-triggered communication.

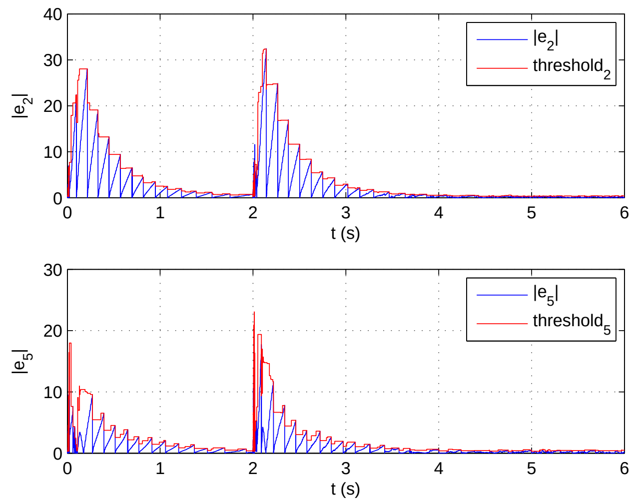

Fig. 8. Evolution of measurement errors under the event-triggered controller.

changes to a new level once an event occurs, which implies that the controller updating of each inverter is also event-driven. The evolutions of the measurement errors of $Q_i(t)$ for inverters under the event-triggered control are presented in Fig. 8. When the error norm $|e_i(t)|$ reaches its threshold given in (58), a new sampling action of the reactive power will be executed and the inverter will transmit this information $\bar{Q}_i(t_k^i)$ to all the neighbor inverters. The measurement error $e_i(t)$ will be reset to 0 correspondingly. The event time instants of each inverter is recorded in Fig. 9. It is noted that at time $t = 2$s, the event generation of the microgrid under the event-triggered controller increases. This implies that when the load changes, more frequent event generation and thus communication is required by the inverters. This result shows that the proposed event-triggered controller is capable of quick adaptation to achieve better dynamic performance.

To show the advantage of the event-triggered strategy compared with the periodic control strategy, the numbers of communication transmission for the inverters under both the periodic communication scheme (19) and the event-triggered communication scheme (42) have been recorded in Fig. 10. With the event-triggered approach, the communication amount can be significantly reduced during the operation. Fig. 11 shows the angle differences between neighboring inverters. By using the conventional droop control for frequency regulation, all the angle differences are very small during the operation of the microgrid. This implies that the standard decoupling approximation is valid for this case study.

C. Event-triggered control with noises

To further study the effectiveness of the proposed controller in power sharing, we consider the case of operation with input noises and show the robustness of the event-triggered controllers. In this case study, the tolerance for the event generation is chosen as $\epsilon = 0.2$. The input noise for each inverter is assumed to be a Gaussian distributed random signal with 0 mean and 0.3 variance.

The trajectories of the reactive power, weighted reactive power, and the voltage amplitudes of the inverters under the event-triggered controller with input noises are shown in Fig. 12. Although there are input noises, the proposed event-triggered control can still drive the inverters to achieve accurate reactive power sharing under time-varying loads. Fig. 13 shows the communication transmission of the microgrid under periodic communication and event-triggered communication with input noises. Compared with the results in Fig. 10, no significant increase of communication occurs in the presence of noises. This shows that the proposed event-triggered controller is robust to input noises to some extent.

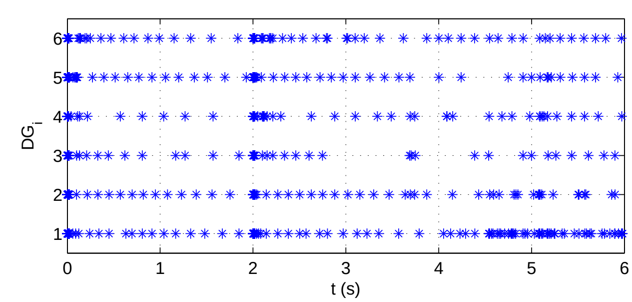

Fig. 9. Event time instants under the event-triggered controller.

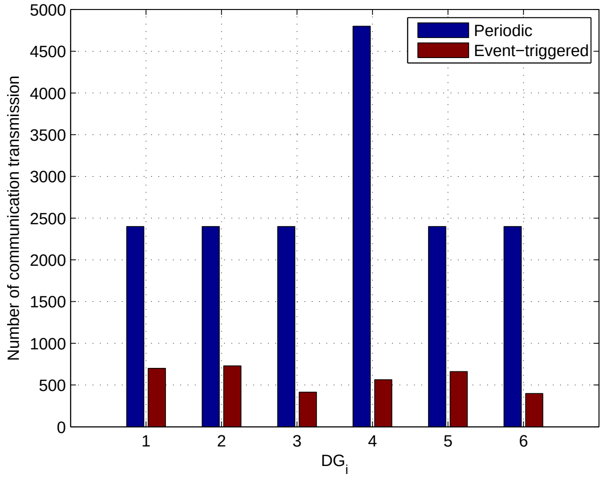

Fig. 10. Communication transmission of the microgrid under the periodic and event-triggered communication schemes.

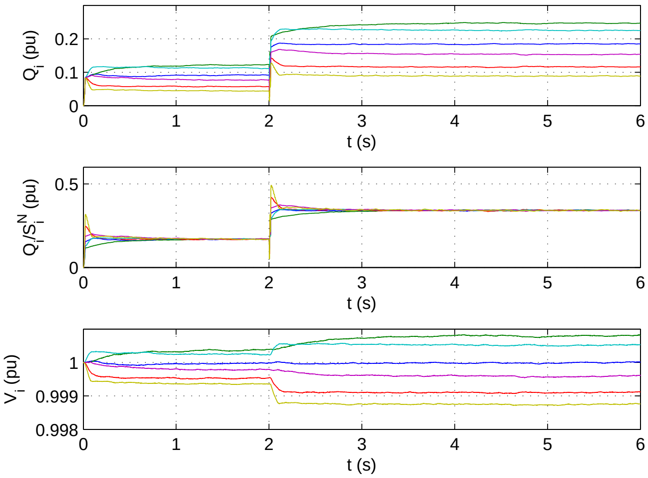

Fig. 12. Trajectories of reactive power, weighted reactive power, and voltage amplitudes of inverters under the event-triggered controller with input noises.

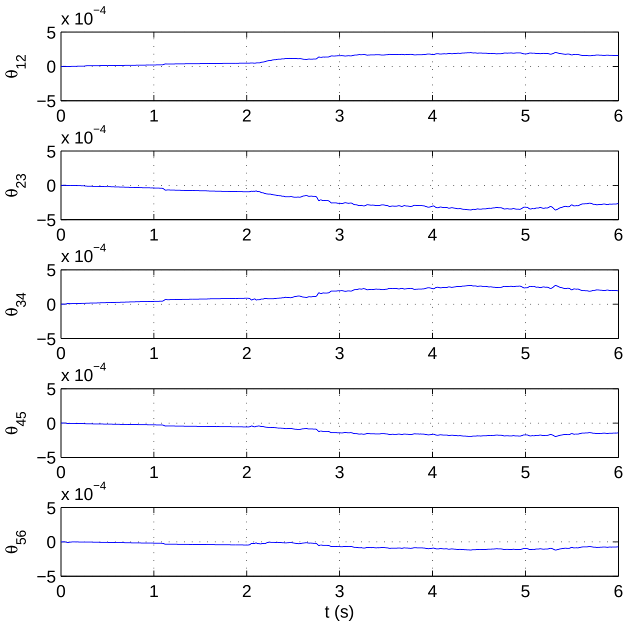

Fig. 11. Angle difference between inverters under the event-triggered control.

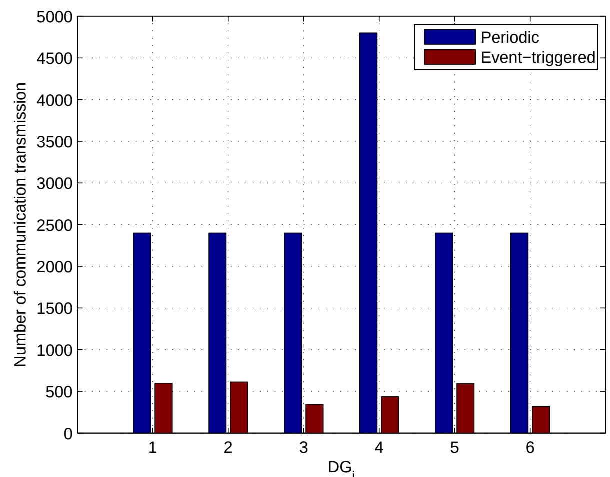

Fig. 13. Communication transmission of the microgrid under periodic communication and event-triggered communication with input noises.

V. Conclusions

We proposed a distributed voltage controller for the reactive power sharing of a microgrid using nonlinear state feedback based on the Lyapunov approach. It is proven that the closed-loop dynamic system has unique stable voltage distribution and the inverters can achieve proportional reactive power sharing asymptotically under the proposed controller. To reduce the consumption of communication bandwidth, a distributed event-triggered control strategy was developed based on the proposed continuous controller. It is proven that with proper event design, each inverter only communicates with its neighboring inverters aperiodically at the event time instants. The event-triggered approach can significantly reduce the amount of communication for the microgrid, while achieving nearly the same accuracy in reactive power sharing as those using continuous controllers with traditional periodic sampling schemes.

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