Merging and Spacing of Heterogeneous Aircraft in Support of NextGen

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FAA's NextGen program aims to increase the capacity of the national airspace, while ensuring the safety of aircraft. This paper provides a distributed merging and spacing algorithm that maximizes the throughput at the terminal phase of flight, using information communicated between neighboring aircraft through the ADS-B framework. Aircraft belonging to a mixed fleet negotiate with each other and use dual decomposition to reach an agreement on optimal merging times, with respect to a pairwise cost, while ensuring proper inter-aircraft spacing for the respective aircraft types. A set of sufficient conditions on the geometry and operating conditions of merging forks are provided to identify when proper inter-aircraft spacing can always be achieved using the proposed algorithm for any combination of merging aircraft. Also, optimal decentralized controllers are derived for merging air traffic when operating under such conditions. The performance of the presented algorithm is verified through computer simulations.

I. Introduction

Next Generation Air Transportation System (NextGen) is the FAA's vision to address the impact of air traffic growth by increasing the National Airspace System's capacity and efficiency, while improving the safety and reducing environmental impacts [1]. It is expected that under NextGen,

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the so-called performance-based navigation (PBN) will allow aircraft to fly negotiated trajectories, thereby changing the air traffic controller's tasks from clearance-based control to trajectory management. One of NextGen's goals is to explore improvements in terminal area operations, namely the automatic merging and spacing of incoming flight paths, in order to increase the air traffic capacity of the terminal phase and save fuel by reducing extraneous flight maneuvers, e.g., holding patterns. Current systems completely rely on air traffic controllers to safely route aircraft. As a result, conflicts in merging routes are often identified too late and merging aircraft are asked to hold or redirect to wait for an opening, thus creating an excessive separation between the aircraft.

Safe and efficient merging of air traffic in support of the FAA's NextGen program is an active area of research and is the subject of a few large-scale tests of systems developed based on Automatic Dependent Surveillance-Broadcast (ADS-B) information. ADS-B, a crucial component of NextGen, relays highly accurate traffic information between equipped aircraft and a network of satellites and ground stations [2]. SafeRoute, which is implemented on UPS aircraft, is an example of a centralized and large-scale ADS-B based technology. Air traffic controllers instruct the pilot to follow a particular aircraft, while an on-board system actively computes and displays a recommended aircraft speed such that a safe distance is maintained with the leading aircraft and safe merging is guaranteed at the merge points [3]. In Point Merge, another centralized merging and spacing solution, aircraft approaching the terminal area achieve the desired separation by flying on one of the vertically spaced sequencing legs to extend the flight path as necessary [4]. The National Aeronautics and Space Administration (NASA) is also actively involved in air traffic management research [5]. NASA's Aviation Systems Division is focusing on hi-flow airports [6], high density en route operations, and automated separation assurance by using trajectory based tactical air traffic management [7].

A central theme in air traffic management research is the problem of conflict resolution amongst aircraft. Tomlin *et al.* use a game theoretic approach for conflict resolution of noncooperative aircraft [8]. Mao *et al.* provide sufficient conditions for stable conflict avoidance of two intersecting aircraft flows [9]. Rahmani *et al.* propose a decentralized deconfliction algorithm based on artificial potential functions [10]. Wollkind *et al.* use the bargaining technique of Monotonic Concession

Protocol to detect and pseudo-optimally resolve conflicts [11]. Roy and Tomlin suggest a slot-based model where en-route traffic select an available slot and then maintain its positioning in the traffic flow, hence guaranteeing safety-of-flight [12].

This paper extends previous work on distributed merging and spacing for the terminal phase of flight (i.e., [13] and [14]) to allow for mixed fleets of aircraft, where having heterogeneous aircraft implies that the spacing distances depend on the types of aircraft present. For example, the Boeing 747 is a large aircraft that requires a greater inter-aircraft separation distance than a small plane like the Canadair CRJ700. Therefore, in this paper, the mixed fleet will consist of different types of aircraft with different spacing requirements. Spacing aircraft as closely as possible, while maintaining the proper inter-aircraft separation, increases the landing capacity of the airport and is of great importance to the NextGen initiative. The presented algorithm stays true to safe-operation practices (e.g., flying between predefined waypoints) and utilizes the inter-aircraft ADS-B protocol with the addition of a few extra negotiation parameters. Specifically, binary merging trees are considered where at each branch, two paths merge into one at the merge point. Figure 1 illustrates this concept.
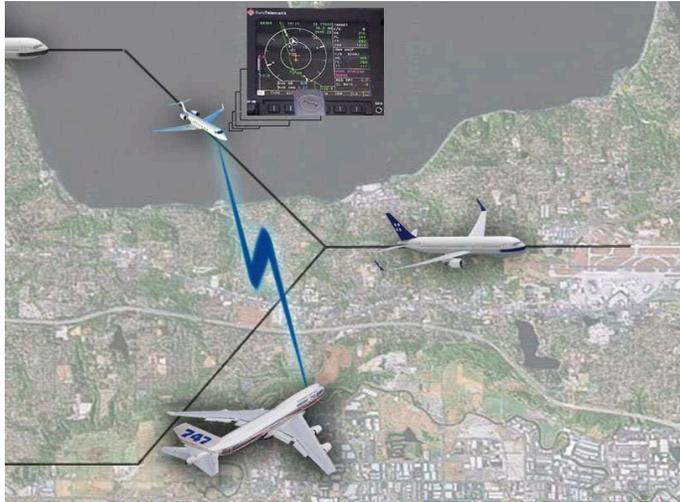

Fig. 1 Autonomous merging and spacing can increase the capacity of the national airspace, while ensuring the safety of aircraft.

In the presented algorithm, aircraft use dual decomposition (e.g., [15]) to negotiate and agree on merging times that minimize a pairwise cost, while maintaining proper inter-aircraft separation based on aircraft type. A set of sufficient conditions are provided to guarantee when proper inter-aircraft spacing can be maintained at all times while merging, and optimal decentralized controllers

are derived for merging air traffic while operating under such conditions. The performance of this algorithm is verified through computer simulations.

II.   Trajectory Based Operations

Within air traffic systems, it is often necessary for multiple aircraft approaching a terminal area to merge onto the same leg of flight, while maintaining a safe inter-aircraft separation at all times. Current systems completely rely on air traffic controllers to safely route aircraft, which often results in an excessive separation between aircraft. It is foreseen that the ADS-B communication protocol will furnish future air traffic control systems with the ability to have tighter spacings amongst aircraft approaching terminals, resulting in an increased throughput of air traffic. This paper presents a distributed algorithm for coordinating multiple heterogeneous aircraft such that adequate spacing is maintained between aircraft during traffic merging maneuvers and the approach to the terminal. Specifically, the NextGen framework will be used to accomplish this coordination and ADS-B communication will be utilized by each aircraft to communicate with other aircraft and the ground station, as shown in Figure 2. In this algorithm, ADS-B type messages will be passed between aircraft to communicate aircraft states, identification, negotiation parameters, and intended flight plans. Most importantly, they will also be used to negotiate changes in these flight plans, in the case of a merging conflict, and to coordinate merging and spacing actions.
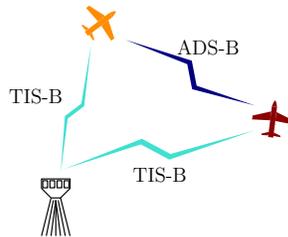

Fig. 2 NextGen's system level communication framework.

The software module for the merging and spacing algorithm within the NextGen/ADS-B framework is shown in Figure 3, and was designed while referring to [2]. Within this module, each aircraft determines if immediate collisions or potential conflicts are projected to occur by using state and flight plan information received from other aircraft in the terminal area. For immediate collisions, the aircraft plans evasive actions and sets a priority flag in its outgoing ADS-B message. In the
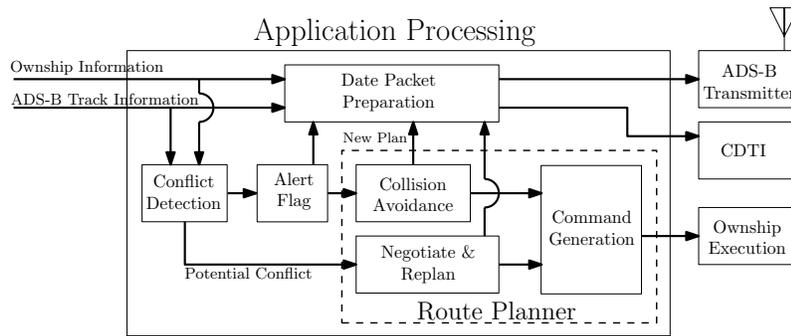
Fig. 3 Application processing, avoiding conflicts by negotiating and replanning.

event of a potential conflict, the Negotiate and Re-plan block within the Route Planning sub-module takes into account state, flight plan, suggested flight plan, and negotiation parameters passed by other aircraft through ADS-B in order to negotiate flight plan changes that result in proper merging and spacing. The resulting flight plan, suggested flight plans for other aircraft, and updated cost parameters are then sent to the Data Packet Preparation sub-module where it is packaged and sent out via ADS-B, as well as being sent to the pilot via the Cockpit Display of Traffic Information (CDTI). Furthermore, the updated flight plan is also sent to the Command Generation block where it will be translated to commands that are appropriate for execution.

The remainder of this paper will focus on a distributed implementation of the Negotiate and Replan Block shown in Figure 3. Specifically, the following sections will describe the problem in detail, present a distributed solution, and conclude with simulation results.

III. Problem Statement

The problem addressed in this paper is to devise a merging and spacing procedure that increases the air traffic throughput for a mixed fleet during the terminal phase of flight, while guaranteeing the safety of each aircraft. Specifically, the scenario where air traffic on two legs of flight must merge onto a single leg with a predetermined ground track speed, and separation distances determined by the type of aircraft, is addressed. Furthermore, this two-track merging fork can be generalized to a binary tree structure such that any arbitrary number of tracks can be merged pairwise onto a single terminal leg. This generalization will be further discussed in Section V.

It should be noted that similar to some other existing merging and spacing solutions (e.g.,

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SafeRoute [3] and Point Merge [4]), our approach uses a more-or-less static structure for merging aircraft. One of the main appeals in having a static structure is that it provides predictability in flight paths. This allows for an aircraft controller to easily monitor merging aircraft to see if they are following procedure correctly, and to easily reroute aircraft as needed in the event of emergencies. Moreover, a predictable flight path drastically lessens the burden on the pilot.

To phrase the merging and spacing problem concisely, first assume that each aircraft has a unique ID, given by some positive integer. Let Ξ denote the set of all aircraft types and let $y : \mathbb{N} \to \Xi$ map each aircraft's ID to its associated aircraft type, i.e., the function $y(i)$ returns the type of Aircraft i. Furthermore, denote the spacing required for an aircraft of type k following an aircraft of any type as $\Delta_{\mathrm{III}}^{k}$, for each $k \in \Xi$. Finally, let the set of all inter-aircraft separations be given by $\mathcal{D} = \{\Delta_{\mathrm{III}}^{k} \mid \forall k \in \Xi\}$.

Referring to Figure 4, the goal is to merge two legs of air traffic onto a single terminal leg, where each Aircraft i must maintain a spacing of at least $\Delta_{\mathrm{III}}^{y(i)} \in \mathcal{D}$ with the aircraft merging in front of it at all times. The merging and spacing procedure is divided into three phases. In Phase I, the *Negotiation Phase*, aircraft approach waypoints WP1 and WP2 with a constant ground track speed V_{I}, spaced at least Δ_{I} apart from the aircraft in front of it on the same leg. During this approach, aircraft on opposing legs will conduct pairwise negotiations to determine arrival times at WP3 and flight plans over Phase II so as to maintain a safe separation with other aircraft. In Phase II, the *Action Phase*, each aircraft executes the negotiated flight plan to travel from WP1/WP2 to WP3. As seen in Figure 4, both WP1 and WP2 are assumed to be a distance d from WP3 at an angle θ apart, and the two dimensional problem is considered where tracks refer to the ground track of the aircraft.
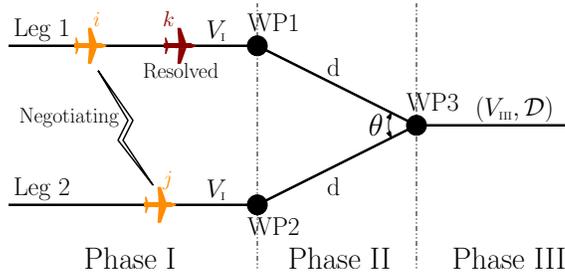

Fig. 4 Top view of a two-track merging fork at the terminal phase of flight.

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The flight plan constitutes a ground track speed $V_{\mathrm{II}} \in [V_{\min}, V_{\max}]$ and a path deviation $h \in [0, h_{\max}]$ from the straight line path between WP1/WP2 and WP3. As in [4] and [3], changing V_{II} and h modifies the arrival time at WP3, which will be used to space merging aircraft. This is illustrated in Figure 5. Note that path deviations occur in the direction opposite to the other leg of flight.
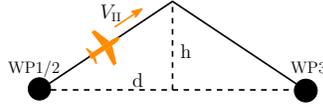

Fig. 5 Overhead view of ground track speed and path deviations during Phase II.

In Phase III, the *Terminal Approach Phase*, each Aircraft i approaches the terminal with constant ground track speed V_{III} and must be at least $\Delta_{\mathrm{III}}^{y(i)}$ away from the aircraft in front of it, as shown in Figure 6.
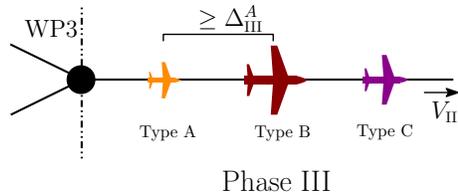

Phase III

Fig. 6 Ground track speed and aircraft separation during Phase III.

Throughout the three phases, all aircraft are assumed to have access to the following global information as labeled in Figures 4 - 6: V_{I} and Δ_{I} are the ground track speed and minimum spacing that aircraft fly at in Phase I, d is the minimum distance required to fly in Phase II, $V_{\min}$ and $V_{\max}$ are respectively the minimum and maximum ground track speeds that aircraft can fly during Phase II, $h_{\max}$ is the maximum allowable path deviation in Phase II, and V_{III} and $\mathcal{D}$ are respectively the constant ground track speed of all aircraft and the set of minimum required separation by aircraft type during Phase III. The mapping of aircraft to aircraft type, y, is also known to all aircraft.

A.   Kinematic Considerations

The proposed merging structure is shown as line segments, as is common practice in merging and spacing literature (e.g. SafeRoute [3] and PointMerge [4]). In this common practice, it is understood that at lower spatial scales, the transitions between line segments have curvatures that allow for smooth aircraft flight paths.

As an illustration of how one would go about producing smooth flight paths from line segments that take into account kinematic considerations, we present a method for explicitly defining arcs with constant curvature κ that yield the same path length as the triangular path deviation in Phase II, as parameterized by h.

Here, we show how given a triangular deviation in Phase II parameterized by h, a corresponding flight path with constant curvature κ can be constructed which yields the same arc length. Therefore, the corresponding constant curvature arc can be used in implementation to respect aircraft kinematics.

Proposition 1. *Consider the triangular path deviation parameterized by h in Phase II, as shown in Figure 5. The deviated path has the same arclength as a constant curvature arc between WP1/WP2 and WP3 with curvature κ, such that*

$$h^2 = \frac{1}{\kappa^2}\left(\sin^{-1}\left(\frac{d\kappa}{2}\right)\right)^2 - \frac{d^2}{4}, \tag{1}$$

as long as

$$\kappa \leq \frac{2}{d} \quad, \text{ or equivalently,} \quad h \leq \frac{d}{4}\sqrt{\pi^2 - 4}. \tag{2}$$

Proof. Looking at Figure 7, we see that half the arclength of the triangular path deviation is given by the hypotenuse of the right triangle (shown in solid lines).
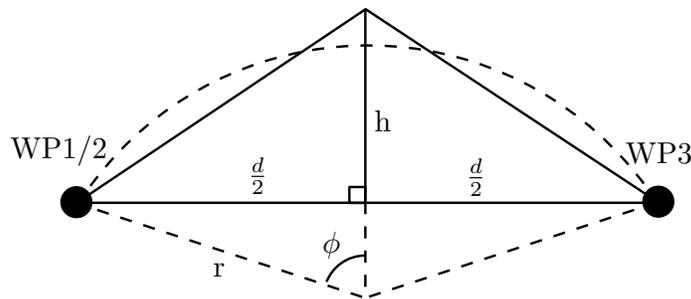

Fig. 7 Diagram for computing a constant curvature arc with the same arclength as the triangular path deviation for Phase II.

Letting L be that quantity, we want it to equal half the arclength of the constant curvature arc

(shown in dotted lines). Therefore,

$$\sqrt{h^2 + \frac{d^2}{4}} = L = r\phi = \frac{\phi}{\kappa}. \tag{3}$$

Moreover, computing the sine of the angle ϕ yields

$$\sin(\phi) = \frac{d}{2r} = \frac{d\kappa}{2}, \text{ meaning that } \phi = \sin^{-1}\left(\frac{d\kappa}{2}\right). \tag{4}$$

Substituting this into the first equation, squaring both sides, and moving $\frac{d^2}{4}$ to the right side of the equal sign then gives us the relationship shown in 1. $\qquad\square$

B. Negotiation and Local Information

Recall during the negotiation phase (Phase I) that aircraft on opposing legs negotiate for arrival times at WP3 that ensures separation. Assume that all aircraft initially start as being 'unresolved'. The two unresolved aircraft that are closest to the merge point (one from each leg) will be the first pair to negotiate for an arrival time. After the pair has negotiated, the aircraft with the earliest arrival time will be assigned that arrival time and hence, be labeled the most recently resolved aircraft. The other aircraft, still unresolved, will then conduct pairwise negotiations with the next unresolved aircraft on its opposing leg for an arrival time at WP3. This arrival time should not only allow for the two aircraft to maintain a separation with each other, but also with the most recently resolved aircraft, when merging. Pairwise negotiation is continued in this manner until all merging aircraft on both legs of flight are assigned arrival times.

Let Aircraft i be the next unresolved aircraft on Leg 1 and Aircraft j be the next unresolved aircraft on the Leg 2, while Aircraft k is the most recently resolved aircraft. The following information is known to Aircraft i: $t_i^{\text{WP1/2}}$ is Aircraft i's expected arrival time at WP1/WP2, $t_{i,0}^{\text{WP3}}$ is Aircraft i's Estimated Time of Arrival (ETA) at WP3 if choosing $V_{\text{II}} = V_{\text{I}}$ and $h = 0$ in Phase II, τ_i is the set of Aircraft i's *feasible* arrival times at WP3 while maintaining $\Delta_{\text{III}}^{y(i)}$ separation from Aircraft k in Phase III, and t_k^{WP3} is Aircraft k's resolved time of arrival at WP3. Similarly, Aircraft j will know $t_j^{\text{WP1/2}}$, $t_{j,0}^{\text{WP3}}$, τ_j, and t_k^{WP3}. Aircraft i and j will also need to communicate additional information to

each other throughout the negotiation process. These additional parameters are explained in detail in Section V as part of the proposed distributed solution.

C.   Relating Ground Track Speed, Path Deviation, and Arrival Times

A fixed arrival time at WP3 for any Aircraft i on either leg leads to a corresponding set of possible (V_{II}, h) pairs that can be chosen for Phase II to meet the arrival time. Vice versa, bounds on V_{II} and h limit which arrival times at WP3 can be achieved. The fastest that Aircraft i can plan to arrive at WP3 is when it flies in a straight line using the maximum ground track speed, corresponding to $V_{\text{II}} = V_{\max}$ and $h = 0$. Thus, $T_{\min}$, the soonest that Aircraft i can reach WP3, is given by

$$T_{\min} = t_i^{\text{WP1/2}} + \frac{d}{V_{\max}}. \tag{5}$$

The slowest that Aircraft i can reach WP3 is by flying at the minimum ground track speed with the greatest path deviation, corresponding to $V_{\text{II}} = V_{\min}$ and $h = h_{\max}$. As a consequence, $T_{\max}$, the latest that Aircraft i can reach WP3, is

$$T_{\max} = t_i^{\text{WP1/2}} + \frac{2}{V_{\min}} \sqrt{h_{\max}^2 + \frac{d^2}{4}}. \tag{6}$$

The set of *reachable arrival times* at WP3 for Aircraft i arriving at WP1/WP2 at time $t_i^{\text{WP1/2}}$ is therefore given by $R_i = [T_{\min}, T_{\max}]$. Let $\delta_{\text{III}}^i = \frac{\Delta_{\text{III}}^{y(i)}}{V_{\text{III}}}$ denote the amount of time that Aircraft i must arrive later than the most recently resolved aircraft at WP3 so as to ensure the appropriate Phase III spacing of $\Delta_{\text{III}}^{y(i)}$. Suppose Aircraft k is the most recently resolved aircraft with arrival time t_k^{WP3} at WP3, then let the set of *feasible arrival times* at WP3 for Aircraft i be $\tau_i = R_i \cap [t_k^{\text{WP3}} + \delta_{\text{III}}^i, \infty)$. Aircraft negotiating for arrival times must therefore choose from their respective sets of feasible arrival times.

Assuming that WP1/WP2 is a distance d from WP3 and it is desired to reach WP3 at time

$t_i^{\text{WP3}} = t_i^{\text{WP1/2}} + T \in R_i$, it is possible to do so with any choice of $(V_{\text{II}}, h) \in \mathbb{S}(d, T)$ where

$$\mathbb{S}(d, T) = \left\{ (V, h) \in [V_{\min}, V_{\max}] \times [0, h_{\max}] \,|\, V = \frac{2}{T}\sqrt{h^2 + \frac{d^2}{4}} \right\}. \tag{7}$$

Having defined the merging and spacing problem, the next step is to develop a distributed negotiation procedure, along with a set of feasibility conditions, to determine terminal phase arrival times that maintain inter-aircraft separation. Furthermore, the negotiated arrival times must minimize pairwise aircraft costs. Optimal flight plan parameters also need to be derived for aircraft on Phase II so as to realize the negotiated arrival times.

IV.   Feasibility

Before discussing the negotiation aspect of this framework, we will identify a set of sufficient conditions on the geometry and operating conditions of merging forks for our algorithm to merge any combination of incoming aircraft (out of those which are specified initially), while ensuring that the minimum separation distance (respective to aircraft type) is maintained at all times. It is first shown that conditions exist on the interval length and intersections of the reachable time sets R_i, for all Aircraft i, such that aircraft on opposite legs performing pairwise negotiation can agree on reachable arrival times at WP3 that guarantees a minimum separation between each other and also the previously resolved aircraft when in Phase III. This leads to conditions on the allowable choices of $V_{\min}$, $V_{\max}$, and $h_{\max}$ on Phase II, which in turn gives conditions for choosing the ground track speed V_{I} and minimum aircraft spacing Δ_{I} for each leg during Phase I.

Define the length of a reachable time set $R_i = [a_i, b_i]$ as $|R_i| = |b_i - a_i|$. Denote the largest required inter-aircraft time separation by $\delta_{\text{III}}^{\max} = \frac{\max \mathcal{D}}{V_{\text{III}}}$. The first proposition will give conditions as to when it is always possible for a pair of airplanes on two different legs to find arrival times that ensures separation, irregardless of what arrival times previous aircraft had chosen.

Proposition 2. *If R_i, R_j, and R_{i+1} are such that $|R_x| \geq 2\delta_{\text{III}}^{max}$, for $x \in \{i, j, i+1\}$, and $b_i \leq a_{i+1}$, then for all $c_i \in R_i$, there exists $c_j \in R_j$ and $c_{i+1} \in R_{i+1}$ such that $|c_i - c_j| \geq \delta_{\text{III}}^{max}$, $|c_i - c_{i+1}| \geq \delta_{\text{III}}^{max}$, and $|c_{i+1} - c_j| \geq \delta_{\text{III}}^{max}$.*

Proof. Choose

$$c_j = a_j \qquad \text{and} \quad c_{i+1} = b_{i+1}, \ \text{ if } a_j \leq c_i - \delta_{\mathrm{III}}^{\mathrm{max}} \text{ or } a_j \leq a_{i+1}$$

$$c_j = c_i + \delta_{\mathrm{III}}^{\mathrm{max}} \quad \text{and} \quad c_{i+1} = b_{i+1}, \ \text{ if } c_i + \delta_{\mathrm{III}}^{\mathrm{max}} \in R_j$$

$$c_j = b_j \qquad \text{and} \quad c_{i+1} = a_{i+1}, \ \text{ otherwise.}$$

$\square$

Suppose Aircraft i and $i+1$ are on one leg and Aircraft j is on the opposite leg. The above proposition says that as long as certain conditions on the feasible time sets are met, any choice of arrival time at WP3 by Aircraft i has corresponding choices of arrival times at WP3 for Aircraft $i+1$ and j such that the three maintain a separation of at least $\max \mathcal{D}$ from each other in Phase III. The maximum aircraft type separation is used here to ensure that all smaller spacings are accommodated. This result can be used to show that the proposed pairwise negotiation algorithm is guaranteed to result in arrival times for each aircraft that ensure separation in Phase III.

Theorem IV.1. *If the following conditions are satisfied for every Aircraft i and $i+1$ following behind it on the same leg:*

R1 : $|R_i| \geq 2\delta_{III}^{max}$, where $|R_i| = \frac{2}{V_{min}}\sqrt{h_{max}^2 + \frac{d^2}{4}} - \frac{d}{V_{max}}$,

R2 : $b_i \leq a_{i+1}$, for $R_i = [a_i, b_i]$ and $R_{i+1} = [a_{i+1}, b_{i+1}]$,

then pairwise negotiation will allow all aircraft to agree on arrival times at WP3 that guarantee an inter-aircraft separation of least $\max \mathcal{D}$ in Phase III for all types of aircraft.

Proof. Suppose some Aircraft $i+1$ and j are engaging in pairwise negotiation, with a previously resolved Aircraft i (if one exists). Proposition 1 guarantees that independent of what arrival time t_i^{WP3} Aircraft i chose, there is a set of $(t_{i+1}^{\mathrm{WP3}}, t_j^{\mathrm{WP3}})$ pairs that allow all three aircraft to maintain a separation of at least $\max \mathcal{D}$ in Phase III. Pairwise negotiation chooses a pair of arrival times for Aircraft $i+1$ and j within that set that occur after t_i^{WP3}. Without loss of generality, assume that $t_j^{\mathrm{WP3}} < t_{i+1}^{\mathrm{WP3}}$. Aircraft j now becomes the next resolved aircraft, where t_j^{WP3} is chosen such that Aircraft j is guaranteed a separation of at least $\Delta_{\mathrm{III}}^{y(j)}$ from all other previously resolved aircraft in

Phase III. This process then continues inductively, where Aircraft $i + 1$ and $j + 1$ must perform pairwise negotiation to determine a $(t_{i+1}^{\mathrm{WP3}}, t_{j+1}^{\mathrm{WP3}})$ pair, and repeats until all aircraft have negotiated arrival times that guarantee the minimum separation requirement is met in Phase III. □

Condition R2 requires aircraft on the same leg in Phase I to have reachable arrival time sets that overlap at most only at the boundary of the intervals. This condition can be transformed to equivalent conditions on spacing for incoming aircraft on Legs 1 and 2.

Theorem IV.2. *Condition R2 mentioned in Theorem 3.1 is equivalent to the distance between any two consecutive aircraft on the same leg during Phase I, Δ_I, being greater than or equal to $V_I|R_i|$, where $|R_i|$ is as given in Theorem IV.1.*

Proof. Assume at time t_0, Aircraft i is a distance $x_{\mathrm{WP1/2}} - x_i$ from WP1/WP2 and Aircraft $i + 1$ is following behind at a distance $x_{\mathrm{WP1/2}} - x_{i+1}$ from WP1/WP2. Therefore, the arrival times at WP1/WP2 are

$$
\begin{aligned}
t_i^{\mathrm{WP1/2}} &= t_0 + \frac{x_{\mathrm{WP1/2}} - x_i}{V_{\mathrm{I}}} \\
t_{i+1}^{\mathrm{WP1/2}} &= t_0 + \frac{x_{\mathrm{WP1/2}} - x_{i+1}}{V_{\mathrm{I}}}.
\end{aligned}
$$

From Equations (5) and (6) and letting $R_i = [a_i, b_i]$ and $R_{i+1} = [a_{i+1}, b_{i+1}]$, it follows that

$$
b_i = t_i^{\mathrm{WP1/2}} + \frac{2}{V_{\mathrm{min}}} \sqrt{h_{\mathrm{max}}^2 + \frac{d^2}{4}} \quad \text{and} \quad a_{i+1} = t_{i+1}^{\mathrm{WP1/2}} + \frac{d}{V_{\mathrm{max}}}.
$$

Substituting into Condition R2 results in

$$
\Delta_{\mathrm{I}} = x_i - x_{i+1} \geq V_{\mathrm{I}} \left(\frac{2}{V_{\mathrm{min}}} \sqrt{h_{\mathrm{max}}^2 + \frac{d^2}{4}} - \frac{d}{V_{\mathrm{max}}} \right) = V_{\mathrm{I}}|R_i|.
$$

□

Sufficient conditions also exist that ensure aircraft on Phases I and II do not violate the minimum separation requirement, which are presented in the following theorem.

Theorem IV.3. *Assuming conditions R1 and R2 are met, a sufficient condition on θ, the angle between Legs 1 and 2, which guarantees that aircraft on Phase II maintain their required minimum separation is given by*

$$V_{min} \cos\left(\frac{\theta}{2}\right) \geq V_{III} \quad \text{and} \quad \pi \geq \theta \geq \max\{\theta', \theta^*\},$$

where θ' and θ^* are given by

$$\alpha_1 \cos^2(\theta^*) + \alpha_2 \cos(\theta^*) + \alpha_3 \geq 0, \text{ and}$$

$$d \sin\left(\frac{\theta'}{2}\right) \geq \frac{\max \mathcal{D}}{2},$$

with α_1, α_2, α_3 defined in (9), (10), (11) respectively.

Proof. For the purposes of analysis, let $t = 0$ be the time at which Aircraft 1 is at WP3 while Aircraft 2 is trailing behind and spaced as closely as possible at a distance of $\Delta_{\text{III}}^{y(2)}$. Tracing aircraft trajectories backward in time by defining $s = -t$, the minimum distance between two aircraft occurs when they do not deviate from the straight path, and where Aircraft 1 travels at $V_{\max}$ while Aircraft 2 travels at $V_{\min}$ in Phase II. As shown in Figure 8, the distance from Aircraft 1 to WP3
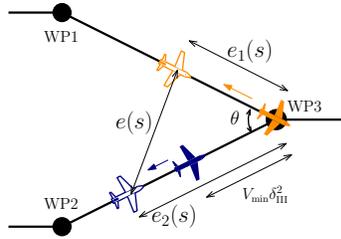

Fig. 8 Diagram of Phase II used for proof of Theorem 3.3

is $e_1(s) = V_{\max}s$ and from Aircraft 2 to WP3 is $e_2(s) = V_{\min}(s + \delta_{\text{III}}^2)$, while the distance between the two Aircraft, $e(s)$, can be computed from the law of cosines. Solving for the time $s^* \geq 0$ when the minimum distance is achieved and making sure that $e(s^*) \geq \Delta_{\text{III}}^{y(2)}$, gives a condition on the minimum allowed inter-leg angle θ^*, such that

$$\alpha_1 \cos^2(\theta^*) + \alpha_2 \cos(\theta^*) + \alpha_3 \geq 0 \tag{8}$$

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with

$$\alpha_1 = \frac{-V_{max}^2 V_{min}^2}{V_{III}^2}, \tag{9}$$

$$\alpha_2 = 2V_{max}V_{min}, \tag{10}$$

$$\alpha_3 = \frac{V_{max}^2 V_{min}^2}{V_{III}^2} - V_{max}^2 - V_{min}^2. \tag{11}$$

In addition, to ensure that aircraft on opposing legs maintain a separation from each other while in Phase I, Legs 1 and 2 must be at least $\max \mathcal{D}$ apart, meaning that $d \sin \left(\frac{\theta'}{2} \right) \geq \frac{\max \mathcal{D}}{2}$. Hence, it is required that angle $\theta \geq \max\{\theta', \theta^*\}$.

Finally, it must be checked that aircraft flying on Phase II and approaching the merge point maintain spacing with aircraft already flying in Phase III. In order to do this, let Aircraft 1 and 2 both be in Phase III where Aircraft 2 is at the merge point and is $\Delta_{III}^{y(2)}$ behind Aircraft 1. For purposes of analysis, let this time be $t = 0$. An expression for the inter-aircraft distance traced back in time as Aircraft 2 moves back into Phase II, while Aircraft 1 remains in Phase III, is given by the following expression:

$$r^2(t) = (V_{II}t)^2 + (\Delta_{III}^{y(2)} - V_{III}t)^2 - 2V_{III}t(\Delta_{III}^{y(2)} - V_{III}t)\cos\left(\pi - \frac{\theta}{2}\right) \tag{12}$$

for all $t \in [-\delta_{III}^2, 0]$. In order to ensure spacing, it is required that $\min_{t \in [-\delta_{III}^2, 0]} r^2(t) \geq \left(\Delta_{III}^{y(2)}\right)^2$. This function is clearly a quadratic of the form $at^2 + bt + c$, so to ensure concavity and that the minimum exists at $t = 0$ (where it is known that the two aircraft have adequate separation), it is required that $a \geq 0$ and $b \geq 0$. Therefore, the 'b' term gives the following condition:

$$2V_{II}\Delta_{III}^{y(2)}\cos\left(\frac{\theta}{2}\right) - 2\Delta_{III}^{y(2)}V_{III} \geq 0 \tag{13}$$

which implies that $V_{II}\cos\left(\frac{\theta}{2}\right) \geq V_{III}$. To ensure that any V_{II} chosen will satisfy the previous condition, it is required that

$$V_{min}\cos\left(\frac{\theta}{2}\right) \geq V_{III}. \tag{14}$$

In summary, the following conditions are sufficient to guarantee complete feasibility:

C1 $\Delta_{\mathrm{I}} \geq V_{\mathrm{I}}|R_i| \geq 2V_{\mathrm{I}}\delta_{\mathrm{III}}^{\max}$.

C2 $V_{\min} \cos\left(\frac{\theta}{2}\right) \geq V_{\mathrm{III}}$.

C3 $\pi \geq \theta \geq \max\{\theta', \theta^*\}$.

Having shown the conditions for which pairwise negotiation will ensure inter-aircraft separation throughout all three phases of flight, the original two-track merging fork will now be generalized to a binary tree that can merge an arbitrary number of legs of flight onto a single terminal leg.

V.   Merging Multiple Legs

The proposed two-track merging fork, as shown in Figure 4, allows for air traffic from two separate legs to safely merge into one with guarantees that all aircraft will maintain a safe spacing from one another at all times. The feasibility results derived thus far in this section can be used to generalize the two-track merging fork to allow for the merging of multiple legs of air traffic using a binary tree configuration as shown in Figure 9. In the figure, air traffic from legs 1 through 5 on the
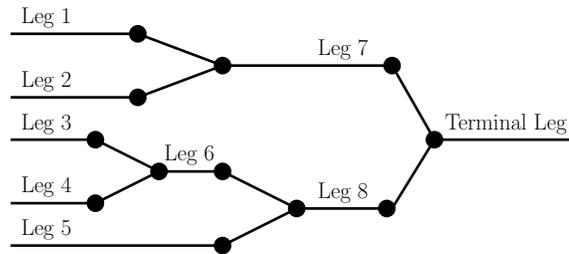

Fig. 9 Binary tree structure for merging multiple tracks.

left all merge onto the terminal leg on the right, making use of intermediate legs 6, 7, and 8. The binary tree can be treated as a collection of two-track merging forks, where each leg in Phase I of a fork can be viewed as Phase III of another fork consisting of that leg and the two merging onto it. Thus, the speed and separation requirements on the terminal leg can be propagated backwards throughout the branches of the tree until feasible parameters for all legs have been determined.

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As an example, let legs 7, 8, and the terminal leg of Figure 9 be Fork A, while legs 1, 2, and 7 form Fork B. The desired conditions $\mathcal{D}_A$ and $V_{\text{III,A}}$ on the terminal leg will determine a range of options for choosing $\Delta_{\text{I,A}}$ and $V_{\text{I,A}}$ on legs 7 and 8 during the design of Fork A. However, leg 7 is both Phase I of Fork A and Phase III of Fork B, so for whatever $\Delta_{\text{I,A}}$ and $V_{\text{I,A}}$ values that are chosen, it is necessary for $\mathcal{D}_B = \{\Delta_{\text{I,A}}\}$ and $V_{\text{III,B}} = V_{\text{I,A}}$. With the conditions for Phase III of Fork A established, the feasibility conditions can then be used to determine a range of valid choices of $\Delta_{\text{I,B}}$ and $V_{\text{I,B}}$ on legs 1 and 2.

It should be noted that the discussion above only addresses how to maintain a safe spacing amongst aircraft on the same fork. Additional care must be made in choosing the geometry of the fork (d and θ) so as to ensure that aircraft traveling on parallel forks, such as those on legs 2 and 3, are also able to maintain the necessary separations. Moreover, while a range of options for the ground track speed and inter-aircraft separation are available when designing a merging leg, it is expected that once chosen, all aircraft adhere to that one option when flying on the leg.

In order to avoid spacing conflicts between adjacent legs not on the same merging fork, a sufficient condition is presented based on the maximum path deviation regardless of the inter-leg angles for each fork. This condition is determined using the worst-case scenario where the two legs in question have parallel merging phases as in Figure 10. Referring to Figure 10, the adjacent fork spacing between Fork A (Legs 1, 2, and 7) and Fork B (Legs 3, 4, and 6), given by h_{AB} must allow for the maximum path deviation for each fork $h_{max,A}$ and $h_{max,B}$ as well as the safe spacing distance, $\max(\Delta_{\text{I,A}}, \Delta_{\text{I,B}})$. Therefore, $h_{AB} = h_{max,A} + h_{max,B} + \max(\Delta_{\text{I,A}}, \Delta_{\text{I,B}})$. Hence, any two adjacent forks, a and b, must be separated by

$$h_{ab} = h_{max,a} + h_{max,b} + \max(\Delta_{\text{I,a}}, \Delta_{\text{I,b}}). \tag{15}$$

VI. Pairwise Optimization Problem

The pairwise negotiations for arrival times at WP3 will minimize a pairwise cost for both aircraft, consisting of the sum of Maneuvering and Delay costs for each aircraft and a joint Separation Cost. For an Aircraft i moving into Phase II, its Estimated Time of Arrival (ETA) at WP3, called $t_{i,0}^{\text{WP3}}$, is
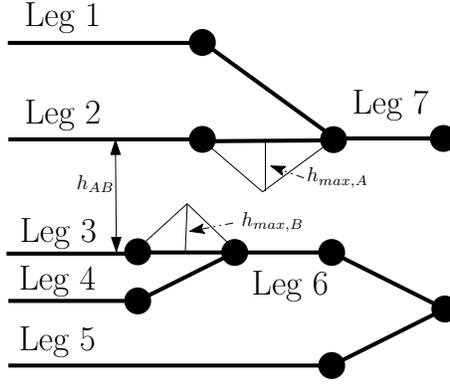
Fig. 10 Binary tree structure for merging multiple tracks.

the time it takes to fly a straight line from WP1/WP2 to WP3 using the same ground track speed as in Phase I. Any deviation in path, as well as changes in speed when switching between flight phases, correspond to an increase in fuel consumption and so is penalized.

Given an arrival time at WP3, the Maneuvering and Arrival Delay cost for an Aircraft i is

$$J_i(t_i^{\mathrm{WP3}}) = \min_{(V_{\mathrm{II}}, h)} \left(k_{1,i} h^2 + k_{2,i} \left((V_{\mathrm{II}} - V_{\mathrm{I}})^2 + (V_{\mathrm{III}} - V_{\mathrm{II}})^2 \right) \right) + k_{3,i} \left(t_i^{\mathrm{WP3}} - t_{i,0}^{\mathrm{WP3}} \right)^2, \qquad (16)$$

such that $(V_{\mathrm{II}}, h) \in \mathbb{S}(d, t_i^{\mathrm{WP3}} - t_i^{\mathrm{WP1/2}})$. The weights $k_{1,i}$, $k_{2,i}$, $k_{3,i} \in \mathbb{R}_+$ may be chosen differently for each aircraft. The minimum term chooses the optimal maneuver (V_{II} and h pair) to arrive at WP3 at time t_i^{WP3}, which minimizes the penalty on deviations in path and ground track speed. In particular, $k_{1,i}$ penalizes path deviations, $k_{2,i}$ penalizes the changes in ground track speed when switching between flight phases, and $k_{3,i}$ penalizes the change in the aircraft's estimated time of arrival.

The Separation Cost penalizes a proposed pair of arrival times if they lead to aircraft having a separation greater than the minimum aircraft-type dependent separation in Phase III. The idea is to encourage aircraft to space themselves as closely as possible, without losing separation, so that later aircraft can have a wider range of feasible arrival times to choose from. This cost is referred to as a joint cost since it relies on both t_i^{WP3} and t_j^{WP3}. Therefore, J_{ij} denotes the Separation cost if Aircraft i arrives first

$$J_{ij}\left(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}} \right) = \gamma_{ij} (|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| - \delta_{\mathrm{III}}^j)^2, \ \gamma_{ij} > 0,$$

and $J(ji)$ denotes the Separation cost if Aircraft j arrives first:

$$J_{ji}\left(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}\right) = \gamma_{ij}(|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| - \delta_{\mathrm{III}}^i)^2, \ \gamma_{ji} > 0.$$

Note that the desired separations, δ_{III}^i and δ_{III}^j, depend on which aircraft is arriving at WP3 second.

There are two constraints on allowable choices of WP3 arrival times. The first is that they must be feasible for the aircraft and so it is required that $t_i^{\mathrm{WP3}} \in \tau_i$, $t_j^{\mathrm{WP3}} \in \tau_j$. The negotiated arrival times must also ensure that inter-aircraft separation, as determined by the type of the second aircraft to arrive, is achieved in Phase III, which is accomplished by the constraint $|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| \geq \delta_{\mathrm{III}}^i$ when Aircraft j arrives first, and $|t_j^{\mathrm{WP3}} - t_i^{\mathrm{WP3}}| \geq \delta_{\mathrm{III}}^j$ when Aircraft i arrives first.

Letting each Aircraft i and j be responsible for its own Maneuvering and Arrival Delay cost as well as half of the Separation Cost, the individual costs for each aircraft are

$$U_i^i(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_i(t_i^{\mathrm{WP3}}) + \frac{1}{2}J_{ij}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}),$$

$$U_j^i(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_j(t_j^{\mathrm{WP3}}) + \frac{1}{2}J_{ij}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}),$$

where the superscript i denotes that Aircraft i arrives first. If Aircraft j arrives first, the costs are

$$U_i^j(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_i(t_i^{\mathrm{WP3}}) + \frac{1}{2}J_{ji}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}),$$

$$U_j^j(t_i^{\mathrm{WP3}}, t_j^{\mathrm{WP3}}) = J_j(t_j^{\mathrm{WP3}}) + \frac{1}{2}J_{ji}(t_j^{\mathrm{WP3}}, t_i^{\mathrm{WP3}}).$$

Letting $a \in \{i, j\}$ denote which aircraft arrives first and $b \in \{i, j\}$ denote which aircraft arrives second, these costs can be combined to create the pairwise cost, and hence the following pairwise optimization problem:

Problem VI.1.

$$\min_{t_i^{WP3} \in \tau_i, t_j^{WP3} \in \tau_j} (U_i^a(t_i^{WP3}, t_j^{WP3}) + U_j^a(t_i^{WP3}, t_j^{WP3})),$$

such that $|t_j^{WP3} - t_i^{WP3}| \geq \delta_{III}^b.$

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Note that a pair of negotiating aircraft must solve this problem twice, once for when Aircraft i arrives first and once for when Aircraft j arrives first. If both scenarios have valid solutions, then the two aircraft must decide who goes first by seeing which scenario results in the lowest pairwise cost. In the next section, a distributed pairwise negotiation is used to solve this problem.

VII.   Distributed Solution

Dual decomposition will be used for a pair of aircraft to reach agreement (as seen in [15]) on arrival times at WP3 that minimizes the pairwise cost between them, while satisfying the separation constraint. First, let t_{ij}^{WP3} be Aircraft i's estimate of what Aircraft j's arrival time at WP3 should be. The dual optimization problem to the primal Problem VI.1 is now written as

$$\max_{\lambda_1,\lambda_2} \min_{t_*^{\text{WP3}}} U_i^a(t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}) + U_j^a(t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}})$$

$$+\lambda_1(t_{ii}^{\text{WP3}} - t_{ji}^{\text{WP3}}) + \lambda_2(t_{jj}^{\text{WP3}} - t_{ij}^{\text{WP3}}),$$

such that $t_{ii}^{\text{WP3}}, t_{ji}^{\text{WP3}} \in \tau_i$ and $t_{jj}^{\text{WP3}}, t_{ij}^{\text{WP3}} \in \tau_j$, with the constraint:

$$|t_{ij}^{\text{WP3}} - t_{ii}^{\text{WP3}}| \geq \delta_{\text{III}}^b \text{ and } |t_{jj}^{\text{WP3}} - t_{ji}^{\text{WP3}}| \geq \delta_{\text{III}}^b.$$

The primal problem has a bounded non-convex cost, meaning the dual problem has weak duality and so its solution cannot be guaranteed to result in a global minimum. Therefore, arrival times are sought which achieve local minima for the pairwise constrained optimization problem.

A.   Dual Decomposition Solution

In [16] and [15], methods are presented for decomposing this dual optimization problem into subproblems that each aircraft can solve. As a result, the negotiation is broken down into steps. First, each Aircraft solves a minimization problem based on its own arrival time estimates and given λ values. Then, arrival time estimates are communicated between the aircraft and each aircraft takes a gradient step to update its value of λ. Finally, the updated λ values are communicated to the other aircraft and the cycle begins again. These steps repeat until the other aircraft's suggested arrival

time agrees with the aircraft's own calculated arrival time. The following describes the subproblems of the dual problem that are solved at each of these steps. Aircraft i solves

$$\min_{t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}} U_i^a(t_{ii}^{\text{WP3}}, t_{ij}^{\text{WP3}}) + \lambda_1 t_{ii}^{\text{WP3}} - \lambda_2 t_{ij}^{\text{WP3}} \tag{17}$$

such that $t_{ii}^{\text{WP3}} \in \tau_i$, $t_{ij}^{\text{WP3}} \in \tau_j$, and $|t_{ij}^{\text{WP3}} - t_{ii}^{\text{WP3}}| \geq \delta_{\text{III}}^b$.

Aircraft j solves

$$\min_{t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}}} U_j^a(t_{ji}^{\text{WP3}}, t_{jj}^{\text{WP3}}) - \lambda_1 t_{ji}^{\text{WP3}} + \lambda_2 t_{jj}^{\text{WP3}} \tag{18}$$

such that $t_{ji}^{\text{WP3}} \in \tau_i$, $t_{jj}^{\text{WP3}} \in \tau_j$, and $|t_{jj}^{\text{WP3}} - t_{ji}^{\text{WP3}}| \geq \delta_{\text{III}}^b$.

Next, Aircraft i and j take the gradient steps

$$\lambda_1^+ = \lambda_1 + t_{ii}^{\text{WP3}} - t_{ji}^{\text{WP3}}, \text{ and } \lambda_2^+ = \lambda_2 + t_{jj}^{\text{WP3}} - t_{ij}^{\text{WP3}}. \tag{19}$$

After the gradient step, the process repeats until an agreement on the arrival times is reached.

In order to solve these problems, Aircraft i must communicate t_{ij}^{WP3} and λ_1 to Aircraft j, while Aircraft j must communicate t_{ji}^{WP3} and λ_2 to Aircraft i. Each aircraft must solve the minimization problem once for the scenario when Aircraft i arrives first ($a = i, b = j$), and again for the scenario when Aircraft j arrives first ($a = j, b = i$), and choose the best of the two scenarios (the one with the lowest pairwise cost) to execute.

VIII.   Simulations

This section showcases the performance of the proposed merging and pairwise negotiation protocol in a series of numerical simulations. The simulations show merging and spacing for a mixed fleet, where aircraft sizes are either "large" or "small", each requiring a different spacing distance to ensure separation. The first simulation shows how each aircraft's proposed arrival times converge throughout a pairwise negotiation, in the case when a large aircraft is negotiating with a small aircraft. The second simulation shows the proposed algorithm merging aircraft in a binary tree

setting, where aircraft on three different legs of flight wish to merge onto a single terminal leg.

A. Pairwise Negotiation Simulation

The simulations start by demonstrating the convergence of each aircraft's proposed arrival times during a pairwise negotiation. To show this, only the subtree of the binary tree in Figure 9, consisting of legs 7, 8, and the terminal leg is considered, which collectively is referred to as Fork A. The parameters for Fork A are:

$$V_{\mathrm{I,A}} = 1 \qquad\qquad\qquad \Delta_{\mathrm{I,A}} = 12.6$$

$$V_{\mathrm{min,A}} = 0.64 \qquad\qquad\qquad V_{\mathrm{max,A}} = 1.8$$

$$h_{\mathrm{max,A}} = 2.9 \qquad\qquad\qquad V_{\mathrm{III,A}} = 0.45$$

$$\Delta_{\mathrm{III,A}}^{\mathrm{Large}} = 2.8 \qquad\qquad\qquad \Delta_{\mathrm{III,A}}^{\mathrm{Small}} = 1.5$$

$$d_A = 10 \qquad\qquad\qquad \theta_A = \tfrac{\pi}{2},$$

where $\mathcal{D}_A = \left\{ \Delta_{\mathrm{III,A}}^{\mathrm{Large}}, \Delta_{\mathrm{III,A}}^{\mathrm{Small}} \right\}$. All the above parameters satisfy the derived feasibility conditions. Furthermore, since the air traffic consists of a mixed fleet, "large" aircraft require a larger separation, while "small" aircraft require a smaller separation, when following other aircraft.

The conversion from simulation units to physical units is given by:

$$2.8 \text{ Distance Units } = 2.5 \text{ NM}$$

$$2.8 \text{ Time Units } = 1 \text{ minute}$$

$$1 \text{ Velocity Unit } = 150 \text{ knots} \tag{20}$$

It is important to note that the above parameters were chosen merely to illustrate how one could design a merging fork for usage with the presented algorithm. However, we make no claim as to how practical the merging fork and operating conditions used in this example would be in implementation.

Suppose that Aircraft 1, a "large" aircraft, and Aircraft 2, a "small" aircraft, are on opposing legs in Phase I, with no other aircraft preceding them. Aircraft 1 (large) is scheduled to arrive at WP1 at $t_1^{\text{WP1}} = 13$, while Aircraft 2 (small) is scheduled to arrive at WP2 at $t_2^{\text{WP2}} = 12$. Notice that if the two aircraft did not negotiate and just proceeded with their default flight plans of $V_{\text{II,A}} = V_{\text{I,A}}$ and $h = 0$, Aircraft 2 (small) would be the first to arrive at the merge point but the two aircraft would have a separation of only $V_{\text{III,A}}(t_1^{\text{WP1}} - t_2^{\text{WP2}}) = 0.45 < \Delta_{\text{III,A}}^{\text{Large}}$ in Phase III, and hence lose separation. Therefore, a pairwise negotiation is needed to resolve this merging conflict.

Based on the parameters of Fork A, the feasible time sets for Aircraft 1 and 2 to arrive at WP3 are $\tau_1 = [18.5556\ 31.0629]$ and $\tau_2 = [17.5556\ 30.0629]$, respectively. The goal of pairwise negotiation is to find a pair of feasible arrival times within the aircraft's feasible time sets that ensure separation. If Aircraft 1 (large) is chosen to go first, the distance between the two aircraft must be at least $\Delta_{\text{III,A}}^{\text{Small}}$ upon arriving at WP3. Since the ground track speed in Phase III is constant, maintaining separation is equivalent to the arrival times being at least $\delta_{\text{III,A}}^2 = \frac{\Delta_{\text{III,A}}^{\text{Small}}}{V_{\text{III,A}}} = 3.3333$ apart. Alternatively, if Aircraft 2 (small) is chosen to go first, the separation between the aircraft must be at least $\Delta_{\text{III,A}}^{\text{Large}}$ upon arriving at WP3, which is the same as the arrival times differing by at least $\delta_{\text{III,A}}^1 = \frac{\Delta_{\text{III,A}}^{\text{Large}}}{V_{\text{III,A}}} = 6.2222$.

The pairwise negotiation cost weights for Aircraft 1 (large) are $k_{1,1} = 3$, $k_{2,1} = 8$, and $k_{3,1} = 3$, while the weights for Aircraft 2 (small) are $k_{1,2} = 10$, $k_{2,2} = 2$, and $k_{3,2} = 1$. Recall that k_1 penalizes any deviations in the flight path, k_2 penalizes changes in the ground track speed, and k_3 penalizes deviations from an aircraft's ETA (had it chosen $V_{\text{II,A}} = V_{\text{I,A}}$ and $h = 0$ for Phase II) at the merge point. The values of the weights thus define the preferences of each aircraft. If it is necessary to delay the arrival time at the merge point, Aircraft 1 would rather deviate its path than change its ground track speed, as seen by it weighing changes in ground track speed more in its cost. Aircraft 2, on the other hand, has opposite preferences and would rather change its ground track speed than deviate its path, in order to stall for time. Furthermore, since the k_3 term is larger for Aircraft 1, it wishes to arrive at WP3 at its ETA more than Aircraft 2. The weight in the joint cost was chosen to be $\gamma = 10$ to give some incentive for the two aircraft to space themselves as closely as possible without losing separation.
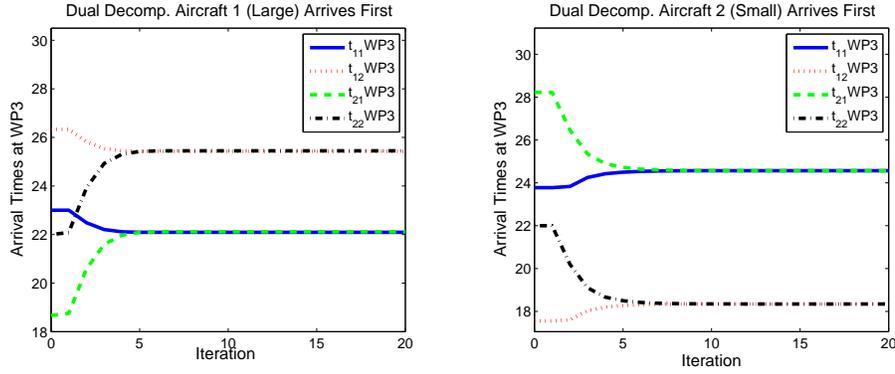
(a) Convergence of proposed arrival times when Aircraft 1 (large) goes first.

(b) Convergence of proposed arrival times when Aircraft 2 (small) goes first.
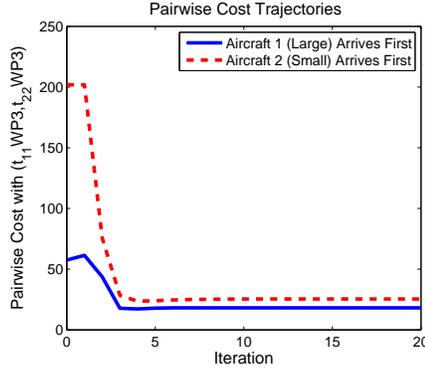
(c) Pairwise cost trajectory using negotiated arrival times per iteration.

Fig. 11 Arrival time agreement and pairwise cost minimization.

The results of performing a pairwise negotiation by running a dual decomposition for 20 iterations between the two aircraft are shown in Figures 11(a) and 11(b). Each iteration in the plots correspond to an exchange of information between the two negotiating aircraft. In both cases, the pairwise negotiations converge in that $|t_{11}^{\mathrm{WP3}} - t_{21}^{\mathrm{WP3}}| \to 0$ and $|t_{22}^{\mathrm{WP3}} - t_{12}^{\mathrm{WP3}}| \to 0$ as the number of iterations increase, i.e., both Aircraft 1 and 2 eventually agree on what Aircraft 1 should do during Phase II, and vice versa.

First, consider the case when Aircraft 1 (large) is chosen to go first. Figure 11(a) shows that the final negotiated arrival times are $t_{11}^{\mathrm{WP3}} = 22.0918$ and $t_{22}^{\mathrm{WP3}} = 25.4471$. Notice that the two negotiated arrival times ensure separation because they differ by 3.3553, which is greater than the required $\delta_{\mathrm{III,A}}^2 = 3.333$. Next, in Figure 11(b), the case when Aircraft 2 (small) is chosen to go first results in final negotiated arrival times $t_{11}^{\mathrm{WP3}} = 24.5681$ and $t_{22}^{\mathrm{WP3}} = 18.3459$. Once again, the two negotiated arrival times ensure separation since they differ by 6.2222, which exactly equals

the required $\delta^1_{\text{III,A}} = 6.2222$. Since both negotiations resulted in arrival times that would ensure a successful merging with separation maintained throughout Phase III, it is necessary to look at the final pairwise costs to determine which aircraft ultimately should go first.

The pairwise cost trajectories for both scenarios are shown in Figure 11(c). Note that the pairwise costs do not necessarily need to be monotonically decreasing throughout the negotiation since forcing t^{WP3}_{11} and t^{WP3}_{22} to satisfy the necessary spacing constraints, when they originally do not, may increase the cost. At the end of the negotiation, the case when Aircraft 1 (large) arrives first results in a final pairwise cost of $J = 18.1243$, while the case when Aircraft 2 (small) arrives first results in $J = 25.4488$. Upon evaluating the final pairwise costs for both of the valid scenarios, the aircraft decide amongst themselves that it is best for Aircraft 1 (large) to arrive at the waypoint first. Thus, Aircraft 1 is marked as being resolved and is scheduled to take the merge point first. It does so by choosing the optimal (V_{II}, h) pair for Phase II that will allow it to reach WP3 at the negotiated time of $t^{\text{WP3}}_{11} = 22.0918$, which is $V_{\text{II}} = 1.1201$ and $h = 0$. The negotiated flight plan for Aircraft 1 corresponds to an increase in its ground track speed during Phase II with no path deviation, in order to get to the merge point earlier than its original ETA. Since Aircraft 2 is still unresolved, it must now negotiate with the next unresolved aircraft behind Aircraft 1 for an arrival time at the merge point.

B. Binary Tree Simulation

Having demonstrated a pairwise negotiation in the previous simulation, a simulation is presented that uses the pairwise negotiation protocol to merge three different legs of air traffic onto a single terminal leg. The three legs of air traffic will be composed of 2 two-track merging forks: Fork A from the previous simulation, and Fork B, composed of legs 1, 2, and 7 from Figure 9, whose parameters are given by

$$V_{I,B} = 1 \qquad\qquad \Delta_{I,B} = 26$$

$$V_{min,B} = 1.42 \qquad\qquad V_{max,B} = 10$$

$$h_{max,B} = 18 \qquad\qquad V_{III,B} = 1$$

$$\Delta_{III,B}^{Large} = 12.6 \qquad\qquad \Delta_{III,B}^{Small} = 12.6$$

$$d_B = 15 \qquad\qquad \theta_B = \tfrac{\pi}{2},$$

where $\mathcal{D}_B = \left\{ \Delta_{III,B}^{Large}, \Delta_{III,B}^{Small} \right\}$. The set of parameters for Fork B also satisfy the feasibility conditions. Recall that when using multiple forks to create a binary tree, the parameters for Phase I of a fork are the same as the parameters that define Phase III of the fork preceding it. Therefore, special care was taken to ensure that $\Delta_{III,B}^{Large} = \Delta_{III,B}^{Small} = \Delta_{I,A}$ and $V_{III,B} = V_{I,A}$.

Since the parameters for each fork were chosen to satisfy the derived feasibility conditions, using the proposed pairwise negotiation protocol amongst merging aircraft will guarantee that aircraft will maintain a separation from each other at all times. The simulation of the binary tree was performed with incoming aircraft randomly inserted into leg 8 of Fork A with at least a separation of $\Delta_{I,B}$, and legs 1 and 2 of Fork B with at least a separation of $\Delta_{I,A}$, from all other aircraft on the same leg. Screenshots from the simulation showing how pairwise negotiation successfully merges the three legs of air traffic onto a single terminal leg are shown in Figure 12.

Although there are many aircraft seen in the simulation, only the actions taken by Aircraft 1 through 4, as marked accordingly in the figures are considered. Aircraft 1 and 3 are "large", while Aircraft 2 and 4 are "small" aircraft. In Figure 12(a), both Aircraft 1 and 2 are approaching the merge point in Fork B. Similarly, Aircraft 3 and 4 are approaching Fork A's merge point. Since both pairs of aircraft are reaching WP1/2 of their respective forks at almost the same time, they will most likely lose separation if they continue using their default flight plans of $V_{II} = V_I$ and $h = 0$ during Phase II. Thus, to resolve these merging conflicts, both pairs of aircraft must perform pairwise negotiations and act accordingly if they wish to maintain a safe separation when merging.
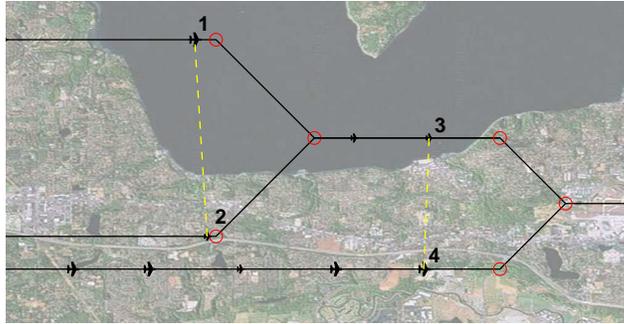
(a) Aircraft 1 and 2 approach the merge point in Fork B, Aircraft 3 and 4 similarly approach the merge point in Fork A.
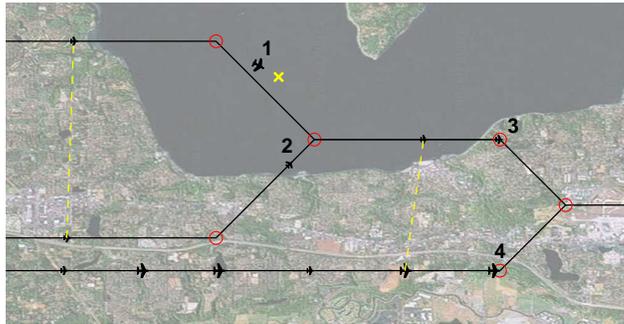
(b) Aircraft 1 yields to Aircraft 2 in Fork B by deviating its path. Aircraft 3 and 4 continue to negotiate in Fork A.
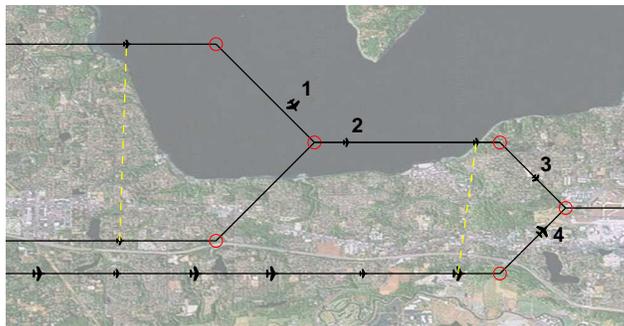
(c) Aircraft 2 has merged in Fork B, Aircraft 1 follows behind at a safe distance. Aircraft 4 speeds up to take the merge point before Aircraft 3 in Fork A.
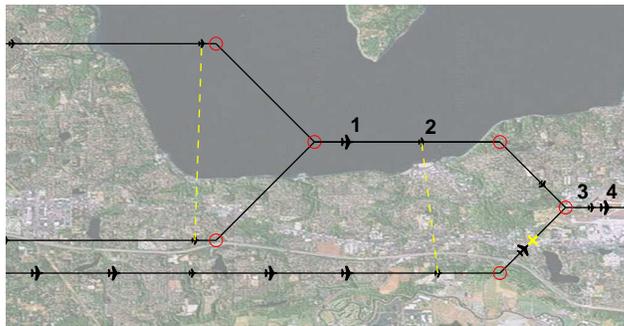
(d) Both Aircraft 1 and 2 have merged in Fork B and are now on same leg in Fork A. Aircraft 3 and 4 successfully merged in Fork A onto the terminal leg.

Fig. 12 A simulation of a binary tree structure merging three legs of air traffic onto a single terminal leg.

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Figure 12(b) shows that Aircraft 1 and 2's negotiation resulted in Aircraft 1 taking a path deviation to delay its arrival time at the merge point. Figure 12(c) shows that Aircraft 3 and 4's negotiation, on the other hand, has determined that the best course of action was for Aircraft 4 to increase its ground track speed, while Aircraft 3 decreases its ground track speed. Figure 12(d) shows that both pairs of aircraft have merged successfully and have maintained a safe separation with other aircraft. Aircraft 1 and 2 have merged onto the same leg in Phase I of Fork A and must now negotiate with aircraft on the opposing leg of Fork A to determine how to engage the next merge point. Aircraft 3 and 4 have both merged onto the terminal leg and can proceed to land in the terminal.

To verify concretely that pairwise negotiation in the preceding simulation had succeeded in maintaining separation amongst aircraft, a plot of inter-aircraft spacing at the merge point of Fork A is shown in Figure 13(a), while a similar plot for the merge point in Fork B is shown in Figure 13(b). Looking at the plots, each arrival of a "large" aircraft on a fork has a separation of at least $\Delta_{\mathrm{III}}^{\mathrm{Large}}$ for that fork, and similarly all arrivals of "small" aircraft have a separation of at least $\Delta_{\mathrm{III}}^{\mathrm{Small}}$ for that fork. Therefore, the simulation confirms that pairwise negotiation was successful in safely merging aircraft from the three incoming legs onto a single terminal leg.

(a) Spacing between consecutive aircraft arriving at the merge point for Fork A.

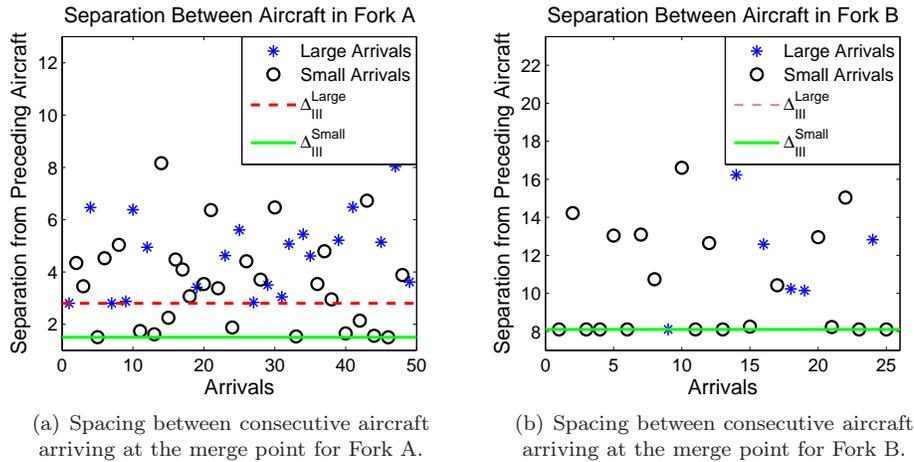(b) Spacing between consecutive aircraft arriving at the merge point for Fork B.

Fig. 13 Plots of separation distances amongst consecutive aircraft arrivals for each two-track merging fork's merge point in the binary tree merging simulation.

IX.   Conclusions

This paper addressed the problem of merging and spacing heterogeneous air traffic during a terminal approach using a distributed scheduling algorithm, where aircraft pass and negotiate information amongst each other through the ADS-B framework. Sufficient conditions were derived for when pairwise negotiations could resolve all merging conflicts amongst the heterogeneous aircraft by maintaining inter-aircraft separations specific to the aircraft type. A distributed algorithm, using dual decomposition, was presented to allow for pairwise negotiations between merging aircraft to determine arrival times at the merge point that not only satisfy spacing requirements, but also minimize a pairwise cost which penalizes fuel consumption and changes in the ETA. Using this information, optimal flight plans could be calculated for each merging aircraft so as to meet the negotiated arrival times. Finally, to verify the performance of the algorithm, a computer simulation was performed where air traffic consisting of two types of aircraft, distributed over three legs of flight, must merge onto a single terminal leg while maintaining inter-aircraft separation specific to the type of aircraft.

X.   References

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