Switching Control in DC-DC Converter Circuits: Optimizing Tracking-Energy Tradeoffs

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Abstract: This paper concerns the problem of optimal switching control in voltage converter circuits, where the objective is to minimize a cost-performance function comprised of the sum of a tracking-related measure and the switching energy. Most of the existing approaches to optimal switching are based on continuous-parameter optimization and optimal control techniques, which are mostly suitable to continuous-parameter functions such as tracking-related performance metrics. On the other hand, the switching-energy cost performance is inherently a discontinuous function dependent on the number of switchings, and hence its inclusion in the problem often is done in ad-hoc ways. This paper explores a systematic approach to optimizing performance - energy tradeoffs by extending an algorithm for optimizing tracking, developed by the authors, to include the energy performance via an averaging technique. The problem is posed in the setting of Pulse Width Modulation, and the controlled variables are the cycle time and duty ratios at each cycle. Extensive simulation results suggest the potential generality of the proposed approach.

Keywords: Switched-mode hybrid systems, gradient-descent algorithms, voltage-converter circuits.

1. INTRODUCTION AND PROBLEM STATEMENT

The problem of optimal mode-switching in hybrid dynamical systems has been the focus of extensive research in the past ten years. One of the main application areas is in switching circuits, where one seeks to optimize the schedule of the states of various switches in order to have a particular circuit variable, such as voltage or current, track a given reference. The circuit can be viewed as a switched-mode hybrid dynamical system, where each combination of states of the switches corresponds to a mode, and within the time-span between consecutive switchings the system evolves according to continuous-time dynamics. The optimal mode-switching problem has the structure of an optimal control problem whose control variable consists of the schedule of modes. This endows the problem with a particular structure that has been investigated from the standpoint of theory and computation, and several effective algorithms for its solutions have been developed.

A general algorithmic approach has been proposed by Morari et al. in Almér at al. (2007); Geyer et al. (2008); Almér et al. (2010) and references therein. It is based on model-predictive control over a finite time horizon, and on piecewise-affine interpolations of measurement-data obtained at certain sampling times. These data are used in mixed-integer linear or quadratic programs to compute the optimal switching times. An alternative algorithmic approach has been proposed by DeCarlo et al.; see Bengea and DeCarlo (2005); Neely et al. (2010a,b) and references therein. It is based on relaxing the optimal mode-switching problem, solving the relaxed problem by nonlinear-programming algorithms in conjunction with model predictive control, and projecting the results onto the space of mode-schedules. A third approach has been explored by the authors of this paper, and it is based on applications of gradient-descent techniques in the space of switching schedules (Ding et al. (2008); Kawashima et al. (2011)).

These approaches focus on performance criteria J_p having the form

$$J_p = \int_0^T L_p(x)dt, \tag{1}$$

where $x(t) \in R^n$, $t \in [0,T]$, is the continuous state variable of the circuit, and $L_p : R^n \to R$ is a suitable differentiable cost function. They are largely based on nonlinear programming and optimal control techniques, which generally require continuity of the performance criteria. However, power- and energy-related cost functions, which are becoming increasingly important in circuit-switching control, may not fit within this framework and have to be handled by ad-hoc methods, as in Ding et al. (2008).

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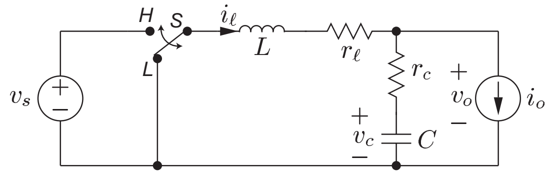

Fig. 1. Step-down DC-DC converter.

This paper provides an initial exploration of a new idea for including switching-energy costs in the above framework. Generally, power leakage at a switching circuit element can be neglected during long periods of its constant state (open or closed), since either voltage or current are close to zero. However, during the process of a state transition both voltage and current vary rapidly, and this results in the dissipation of power and energy that might not be neglected. Thus, the average power or energy associated with the switching schedule is closely related to the number of state transitions at the switches, a discrete variable; and hence does not have the form of J_p (1). This paper proposes a way to adequately approximate the power/energy performance function via a term similar to (1), uses this formulation to define an aggregate cost function comprised of energy-tracking balance, and optimizes it by the aforementioned algorithmic approach.

To explain our idea we use the example of the DC-DC converter circuit shown in Figure 1. This circuit was defined in Almér et al. (2010) to illustrate an algorithm for computing optimal duty ratios in a Pulse Width Modulation (PWM) switching control scheme designed to regulate the output voltage v_0. The same problem was considered in Kawashima et al. (2011) as well, and this paper extends it by adding the switching energy to the cost function that is to be minimized.

The rest of the paper is organized as follows. Section 2 defines the problem and presents relevant background material, Section 3 describes our solution technique and presents simulation experiments, and Section 4 concludes the paper.

2. PROBLEM DEFINITION AND SURVEY OF ESTABLISHED RESULTS

The circuit shown in Figure 1, defined in Almér et al. (2010), depicts a two-port DC-DC power converter, consisting of a switch, an LC filter, a voltage source to supply the power that flows into the input port, and a current source that absorbs the power that flows from the output port. The resistors r_ℓ and r_c represent parasitic resistances in series with the inductor and capacitor, respectively. As in Almér et al. (2010), the current source acts as a load of the circuit.

The switch can be in either state High (H) or Low (L) as shown in the figure, and we note that in the H state the voltage source supplies v_s volts to the input port, while in the L state the voltage source supplies 0 volts. Correspondingly, let $v(t) \in \{0,1\}$ denote the state of the switch at time t, where $v(t) = 1$ when the switch is in the High state, and $v(t) = 0$ when the switch is in the Low state. We consider the function $\{v(t)\}_{t\geq 0}$ as the control input to the system comprised of the circuit.

An application of physical laws yields the dynamic model

$$\frac{d}{dt}\begin{bmatrix} v_c \\ i_\ell \end{bmatrix} = \begin{bmatrix} 0 & C^{-1} \\ -L^{-1} & -L^{-1}(r_c + r_\ell) \end{bmatrix}\begin{bmatrix} v_c \\ i_\ell \end{bmatrix} + \begin{bmatrix} -C^{-1} \\ L^{-1}r_c \end{bmatrix} i_o + \begin{bmatrix} 0 \\ L^{-1} \end{bmatrix} v_s v, \quad v \in \{0,1\} \tag{2}$$

where v_c is the voltage across the capacitor and i_ℓ is the current through the inductor; we assume given initial conditions $v_c(0)$ and $i_\ell(0)$. The quantity of interest is the voltage across the current source, v_o, which is given by the equation

$$v_o \;=\; v_c + r_c(i_\ell - i_o). \tag{3}$$

Consider the problem of controlling the circuit so as to have the voltage v_o track a given constant reference value v_r. Since the control consists of the switching schedule, hence discrete, it is typically impossible to obtain exact tracking. Instead, we consider the tracking-performance function J_p defined by

$$J_p \;=\; \frac{1}{2}\int_0^T (v_o(t) - v_r)^2 dt, \tag{4}$$

where $T > 0$ is a given horizon interval. Observe that J_p is a special case of the cost measure defined in (1).

References (Almér et al. (2010); Kawashima et al. (2011)) considered controlling the circuit by a PWM with a fixed switching rate and variable duty ratios. More specifically, given a constant cycle time T_c and a fixed number of cycles, N, the duty ratios determine the fraction of time the switch is in the respective states H and L (in this order) during each cycle. Thus, denoting by $\rho_k \in [0,1]$ the kth duty ratio, and noting that the kth cycle is the interval $[(k-1)T_c, kT_c)$, $k = 1,\ldots,N$, we have that

$$v(t) \;=\; \begin{cases} 1, \; t \in [(k-1)T_c, (k-1+\rho_k)T_c) \\ 0, \; t \in [(k-1+\rho_k)T_c, kT_c). \end{cases} \tag{5}$$

Fix T_c and N. The performance measure J_p, defined by Equations (2)-(4), can be viewed as a function of the duty ratios ρ_k, $k = 1,\ldots,N$. Minimizing this function was the problem that was solved in Almér et al. (2010); Kawashima et al. (2011).

We change this problem by appending an energy term to the cost function. Denoting by J_e the switching-energy spent during the horizon interval $[0,T]$, and given a weighting term $w \in [0,1]$, the cost function that we minimize is

$$J \;=\; (1-w)J_p + wJ_e. \tag{6}$$

The energy term J_e consists of the sum of the energy spent during each switching of the state (High to Low and vice versa) during the horizon interval $[0,T]$. Let T_c and N_c denote the cycle time and the number of cycles, respectively; note that N_cT_c is approximately T. Finally, suppose that there are N_r switchings in the horizon interval $[0,T]$, and let τ_i, $i = 1,\ldots,N_r$ denote the switching times in increasing order. Observe that N_r is roughly $2N_c$ since there are two switchings at each cycle possibly except for the last cycle, which may be incomplete. The last cycle spans the time-interval $[(N_c - 1)T_c, T]$. Let ρ_{N_c} denote its duty ratio. We treat this cycle in the following way: If $T < (N_c - 1 + \rho_{N_c})T_c$ then the switch is in the High state throughout the cycle, and otherwise, the switch's state is changed from High to Low at the time $(N_c - 1 + \rho_{N_c})T_c$, where it stays to the end of the cycle at time T.

Suppose that the time to open or close the switch is $t_s/2$ seconds for a given $t_s > 0$ (we assume the same time for both opening and closing the switch). Assuming that the switch is implemented as a transistor-diode pair, the energy spent during the *ith* switching can be adequately approximated by $4^{-1}t_s v_s i_\ell(\tau_i)$ joules; see, e.g., Mohan et al. (1995). [1] In this case, the switching-energy term J_e has the following form,

$$J_e \;=\; \frac{t_s v_s}{4}\sum_{i=1}^{N_r} i_\ell(\tau_i) \tag{7}$$

joules. This clearly does not have the form of J_p as defined by Equation (1).

Our starting point is the algorithm described in Kawashima et al. (2011) for minimizing the tracking term J_p defined in Equation (4). It is based on an abstract algorithmic framework for optimizing mode-schedules in switching hybrid dynamical systems, developed in Axelsson et al. (2008). In its general setting the system's dynamic equation has the following form,

$$\dot{x}(t) = f(x(t), v(t)), \tag{8}$$

where $x \in R^n$ is the state variable, the initial state $x(0) := x_0$ is given, the control $v(t)$ is confined to a finite set V, and the vector field $f(\cdot, v) : R^n \to R^n$ is continuously differentiable for every $v \in V$. The cost functional to be minimized is

$$J \;:=\; \int_0^T L(x)dt \tag{9}$$

for a given $T > 0$, where $L : R^n \to R$ is a continuously-differentiable function.

Various algorithmic approaches to this problem have been proposed in Xu and Antsaklis (2002); Bengea and DeCarlo (2005); Shaikh and Caines (2007); Caldwell and Murphy (2010) and references therein, but we follow the one developed in Axelsson et al. (2008). This approach is based on descent directions chosen by needle variations and sensitivity (gradient) of the cost functional J with respect to the mode-switching times, in conjunction with the Armijo step size (Polak (1997)). The gradients have a special form related to the dynamic equation (8) and the cost function (9): defining the costate variable $p(t) \in R^n$ via

$$\dot{p}(t) = -\left(\frac{\partial f}{\partial x}(x, v)\right)^T p(t) - \left(\frac{\partial L}{\partial x}(x)\right)^T \tag{10}$$

with the boundary condition $p(T) = 0$, and denoting by τ_i the *ith* switching time of $v(\cdot)$, the derivative $\frac{\partial J}{\partial \tau_i}$ has the form

$$\frac{\partial J}{\partial \tau_i} = p(\tau_i)^T\big(f(x(\tau_i), v(\tau_i^-)) - f(x(\tau_i), v(\tau_i^+))\big). \tag{11}$$

A convergence analysis of the algorithm was presented in Axelsson et al. (2008), and an implementation of it to the tracking problem of minimizing J_p, defined in (4), exhibited rapid convergence. We point out that the form of the cost functional, given in (9) (of which (4) is a special case) is essential for the algorithm. However, the energy cost function J_e, defined in (7), does not have this form, and therefore, we start the next section by defining an adequate approximation to it that is in the form as (9).

3. TRACKING-ENERGY OPTIMIZATION IN A DC-DC CONVERTER

Consider the problem of minimizing the function J defined by Equation (6), where J_p and J_e are defined via (4) and (7), respectively. We first define an approximation to J_e that has the form of (9), and then describe an extension of the algorithm in Kawashima et al. (2011) for minimizing J.

Suppose that each duty cycle starts at the High state and ends at the Low state of the switch. Thus, each cycle has two state switchings: one from H to L during the cycle, and one from L to H at the end of the cycle except, possibly, at the last one. Denote the number of switchings and the number of cycles by N_r and N_c, respectively. If N_r is even then $N_c = \frac{N_r}{2} + 1$, where the last cycle does not incur any switchings, and Equation (7) can be written as

$$J_e \;=\; \frac{t_s v_s}{2}\sum_{k=1}^{N_c-1}\frac{i_\ell(\tau_{2k-1}) + i_\ell(\tau_{2k})}{2}. \tag{12}$$

On the other hand, if N_r is odd then $N_c = \frac{N_r}{2} + \frac{1}{2}$, and by defining $\tau_{N_r+1} = \tau_{N_r}$, Equation (12) is also satisfied. Dividing and multiplying (12) by the cycle time T_c we obtain the following equation,

$$J_e \;=\; \frac{t_s v_s}{2}\frac{1}{T_c}\sum_{k=1}^{N_c-1}\frac{i_\ell(\tau_{2k-1}) + i_\ell(\tau_{2k})}{2}T_c. \tag{13}$$

Now if N_c is large then the sum-term in the last equation can be approximated by an integral, which yields the following approximation to J_e, denoted by $\tilde{J}_e$:

$$\tilde{J}_e \;=\; \frac{t_s v_s}{2}\frac{1}{T_c}\int_0^T i_\ell(\tau)d\tau. \tag{14}$$

Observe that $\tilde{J}_e$ has the structure of (9), and hence we define our approximation to J, denoted by $\tilde{J}$, as

$$\tilde{J} \;=\; (1-w)J_p + w\tilde{J}_e. \tag{15}$$

The effect of the approximation error on the optimization process will be examined later on a number of test problems and shown to be barely discernable.

Let us denote the variable for our optimization problem by $\xi := (T_c, \rho_1, \ldots, \rho_{N_c})^T$, where T_c is the cycle time and ρ_k are the duty ratios, $k = 1, \ldots, N_c$. The feasible set for the problem consists of the points ξ such that $T_c \geq \epsilon$ for a given (small) $\epsilon > 0$, and $0 \leq \rho_k \leq 1$ for every $k = 1, \ldots, N_c$. The following algorithm is an extension of the one in Kawashima et al. (2011) in that it includes the term $\tilde{J}_e$ in $\tilde{J}$, and it considers T_c as a part of the variable.

Algorithm 1.
Given: Constant parameters $\alpha \in (0,1)$ and $\beta \in (0,1)$.
Step 0: Choose an initial feasible point ξ_0. Set $n = 0$.
Step 1: Compute $\tilde{h}_n$, defined as the projection of $-\nabla\tilde{J}(\xi_n)$ onto the feasible set at ξ_n. If $\tilde{h}_n = 0$, then exit; otherwise, continue.
Step 2: If $\xi_n + \tilde{h}_n$ is feasible, set $h_n := \tilde{h}_n$. Otherwise, compute $\lambda_n := \max\{\lambda \geq 0 : \xi_n + \lambda\tilde{h}_n \text{ is feasible}\}$, and set

[1] This formula implies that v_s has a constant value, otherwise it would be replaced by $v_s(\tau_i)$.

$h_n := \lambda_n \tilde{h}_n$.
Step 3: Compute $j(\xi_n)$ defined as follows:
$j(\xi_n) = \min\Big\{j = 0, 1, \ldots : \tilde{J}(\xi_n + \beta^j h_n) - J(\xi_n)$
$\leq \alpha\beta^j \langle \nabla \tilde{J}(\xi_n), h_n\rangle\Big\}$, and set $\gamma(\xi_n) := \beta^{j(\xi_n)}$.
Step 4: Set $\xi_{n+1} := \xi_n - \gamma(\xi_n)h_n$, set $n = n+1$, and go to Step 1. □

We point out that the various partial derivatives of $\nabla\tilde{J}(\xi)$ are computable by the chain rule and Equation (11) for $\frac{\partial\tilde{J}}{\partial\tau_i}$. To see this, note that for every $k = 1, 2, \ldots$, $\tau_{2k-1} = (k-1+\rho_k)T_c$, while $\tau_{2k} = kT_c$, and this yields ξ explicitly in terms of the switching times.

We ran Algorithm 1 to optimize $\tilde{J}$ for several sets of values of w and initial points. Following Almér et al. (2010) we use normalized, dimensionless circuit elements (except for resistors) and timing variables. The following parameters are common to all the runs.

- *Circuit parameters:* $C = 70/2\pi$, $L = 3.0/2\pi$, $r_c = 0.005$ ohm, $r_\ell = 0.050$ ohm, and the sum of the switch's opening and closing delays at each cycle is $t_s = 0.001$.
- *Problem parameters:* $T = 20$, $v_s = 1.8$ (a constant), $i_o = 4.0$ (a constant), $v_r = 1.0$, and the initial conditions for Equation (2) are $v_c(0) = 1.0$ and $i_\ell(0) = 3.5$.
- *Algorithm parameters:* $\alpha = \beta = 0.5$, $\epsilon = 0.005$, the initial cycle time is $T_c = 0.1$ (meaning that the number of cycles is $N_c = 200$), Equations (8) and (10) are solved numerically via the forward Euler method with a step size of 0.001.

All the runs were made for 100 iterations but convergence is discerned after about 10 iterations, and the figures below show the first 25 iterations. Three values of the weighting factor w were used (see (15)): $w = 0.5$ reflecting on an equal weight between the tracking and energy costs, $w = 0.9$ corresponding to a larger weight on the energy cost, and $w = 0.1$, corresponding to a larger weight on the tracking performance. The results are presented below for various values of w and the initial vector of duty ratios.

(1) $\underline{w = 0.5}$. The duty ratios in the first iteration point, ξ_0, are $\rho_k = 0.5$ for every $k = 1, \ldots, 200$. The graph of the cost $\tilde{J}(\xi_n)$ as a function of the algorithm's iteration count n is shown in Figure 2, where the dashed line indicates the tracking-performance cost $(1-w)J_p(\xi_n)$, the dashed-dotted line shows the energy cost $w\tilde{J}_e(\xi_n)$, and the solid line shows the sum of the two, namely $\tilde{J}(\xi_n)$. The initial cost is $\tilde{J}(\xi_0) = 0.86$ and the final cost is $\tilde{J}(\xi_{100}) = 0.024$. At the end of the run, namely according to ξ_{100}, the number of cycles is $N_c = 14$, and the vector of duty ratios has a mean of 0.66 and a standard deviation of 0.0078, indicating an almost-uniform duty ratio of 0.66. The maximum error, $\varepsilon := \max\{|\tilde{J}_e(\xi_n) - J_e(\xi_n)| : n = 1, \ldots, 100\}$, was computed to be $\varepsilon = 0.0030$, indicating that the approximation yields a good accuracy.

Analogous results were obtained for the case where the initial duty ratios are $\rho_k = 0.8$ for every $k = 1, \ldots, 200$, and the graphs of the cost functions are shown in Figure 3. The initial cost is $\tilde{J}(\xi_0) = 0.68$, and the final cost is $\tilde{J}(\xi_{100}) = 0.024$. At the end of the run the number of cycles is $N_c = 14$, and the vector of duty ratios has a mean of 0.66 and a standard deviation of 0.0074. The energy-approximation error is $\varepsilon = 0.0032$.

A third run was made with the initial duty ratios of $\rho_k = 0.2$ at all cycles, resulting in a high tracking cost at the early stage of the algorithm's run. The results are shown in Figure 4. The initial and final costs are $\tilde{J}(\xi_0) = 4.2$ and $\tilde{J}(\xi_{100}) = 0.025$, respectively, the final number of cycles is 14, and the vector of duty ratios has a mean of 0.66 and a standard deviation of 0.0066. The energy-approximation error is $\varepsilon = 0.0031$.

The results of the three runs with different initial duty ratios are consistent in terms of both the final result ξ_{100} and its cost-value $\tilde{J}(\xi_{100})$.

(2) $\underline{w = 0.9}$. This case corresponds to a large weight on the energy component of the cost. Figures 5-7 show the graphs of the cost measures for three values of the initial duty ratios: $\rho_k = 0.5\ \forall\ k = 1, \ldots, 200$, $\rho_k = 0.8\ \forall\ k = 1, \ldots, 200$, and $\rho_k = 0.2\ \forall\ k = 1, \ldots, 200$. Not surprisingly, in the first two cases the energy cost dominates the total cost as can be seen in Figures 5 and 6. However, when the initial duty ratios are 0.2, the tracking-performance cost is larger in the first 8 iterations of the algorithm (see Figure 7), and this is due to the fact the capacitor is being discharged, and hence v_o is far off v_r, for most of the time.

For the case where the initial duty ratios are 0.5, the initial and final costs are $\tilde{J}(\xi_0) = 0.72$ and $\tilde{J}(\xi_{100}) = 0.027$, respectively; at the end of the run the number of cycles is $N_c = 9$, and the vector of duty ratios has a mean of 0.65 and a standard deviation of 0.035; and the energy-approximation error is $\varepsilon = 0.0028$.

For the case where the initial duty ratios are 0.8, the initial and final costs are $\tilde{J}(\xi_0) = 0.73$ and $\tilde{J}(\xi_{100}) = 0.027$, respectively; at the end of the run the number of cycles is $N_c = 9$, and the vector of duty ratios has a mean of 0.65 and a standard deviation of 0.035; and the energy-approximation error is $\varepsilon = 0.0030$.

For the case where the initial duty ratios are 0.2, the initial and final costs are $\tilde{J}(\xi_0) = 1.3$ and $\tilde{J}(\xi_{100}) = 0.027$, respectively; at the end of the run the number of cycles is $N_c = 9$, and the vector of duty ratios has a mean of 0.65 and a standard deviation of 0.035; and the energy-approximation error is $\varepsilon = 0.0034$. Again, we discern a consistency in the final results of the three runs.

(3) $\underline{w = 0.1}$. This corresponds to the case where the tracking-performance dominates the total cost. Figures 8-10 show the respective graphs of the cost measures for three values of the initial duty ratios: $\rho_k = 0.5\ \forall\ k = 1, \ldots, 200$, $\rho_k = 0.8\ \forall\ k = 1, \ldots, 200$, and $\rho_k = 0.2\ \forall\ k = 1, \ldots, 200$. In all cases the tracking performance dominates, and this is due to the low value of w.

For the case where the initial duty ratios are 0.5, the initial and final costs are $\tilde{J}(\xi_0) = 1.0$ and

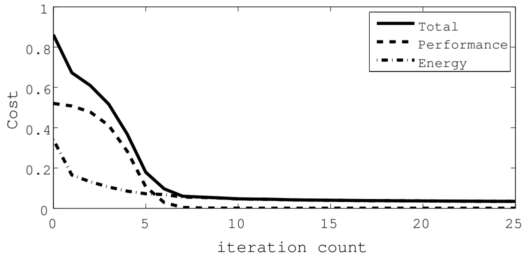

Fig. 2. J ($w = 0.5,\ \rho_{init} = 0.5$).

$\tilde{J}(\xi_{100}) = 0.0081$, respectively; at the end of the run the number of cycles is $N_c = 22$ and the vector of duty ratios has a mean of 0.66 and a standard deviation of 0.014; and the energy-approximation error is $\varepsilon = 0.0030$.

When the initial duty ratios are 0.8, the initial and final costs are $\tilde{J}(\xi_0) = 0.62$ and $\tilde{J}(\xi_{100}) = 0.0088$, respectively; at the end of the run the number of cycles is $N_c = 25$ and the vector of duty ratios has a mean of 0.67 and a standard deviation of 0.012; and the energy-approximation error is $\varepsilon = 0.0033$.

For the case where the initial duty ratios are 0.2, the initial and final costs are $\tilde{J}(\xi_0) = 7.1$ and $\tilde{J}(\xi_{100}) = 0.0080$, respectively; at the end of the run the number of cycles is $N_c = 22$ and the vector of duty ratios has a mean of 0.66 and a standard deviation of 0.017; and the energy-approximation error is $\varepsilon = 0.0029$. Once again, the three runs yield similar results.

For each one of the above values of w, the three runs with the respective initial duty ratios yielded similar graphs of $v_o(t)$, $t \in [0, T]$. These are shown in Figures 11-13 for $w = 0.5$, $w = 0.9$, and $w = 0.1$, respectively, and all with initial duty ratios of 0.5. All three cases show tracking of the reference value of $v_r = 1.0$ albeit at different qualities. The case where $w = 0.9$ yields the worst tracking but it has the lowest switching rate, while the case where $w = 0.1$ yields the best tracking with the highest switching frequency. This is due to the fact that larger values of w are associated with a larger weight of the energy cost, and lower weight of the tracking-performance cost, on the total cost.

4. CONCLUSIONS

This paper considers a problem of optimal switching-control in power-electronics converter circuits, whose objective is to compute a switching regimen that balances a tracking performance with a measure of switching energy. It extends an existing algorithm, suitable for optimal tracking, to include energy metrics as well. Simulation results exhibit fast convergence of the algorithm, and suggest its potential utility in a broader class of switching-control applications in power electronics.

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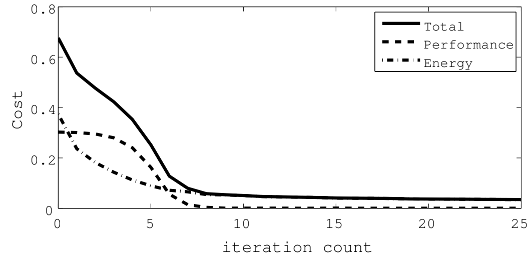

Fig. 3. J ($w = 0.5,\ \rho_{init} = 0.8$).

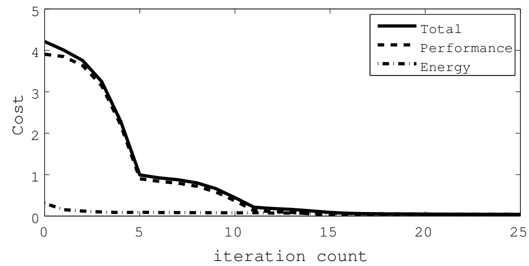

Fig. 4. J ($w = 0.5,\ \rho_{init} = 0.2$).

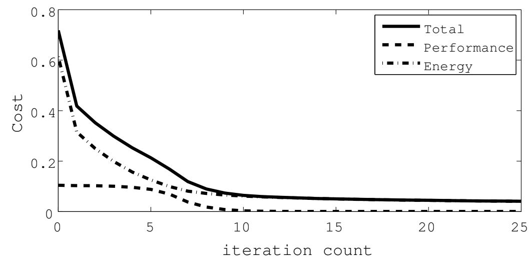

Fig. 5. J ($w = 0.9,\ \rho_{init} = 0.5$).

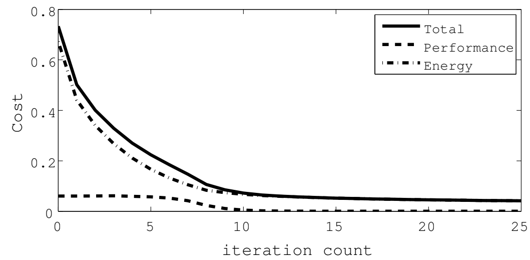

Fig. 6. J ($w = 0.9,\ \rho_{init} = 0.8$).

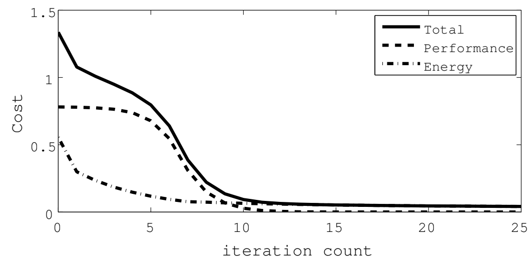

Fig. 7. J ($w = 0.9,\ \rho_{init} = 0.2$).

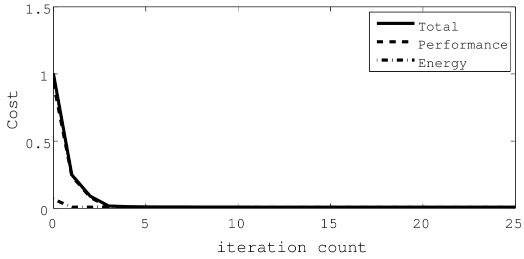

Fig. 8. J $(w = 0.1,\ \rho_{init} = 0.5)$.

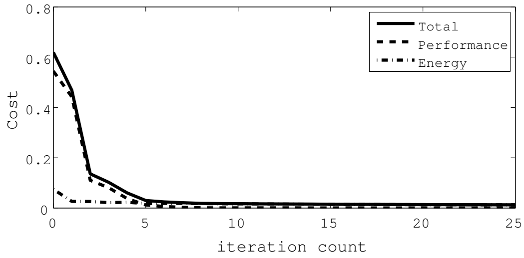

Fig. 9. J $(w = 0.1,\ \rho_{init} = 0.8)$.

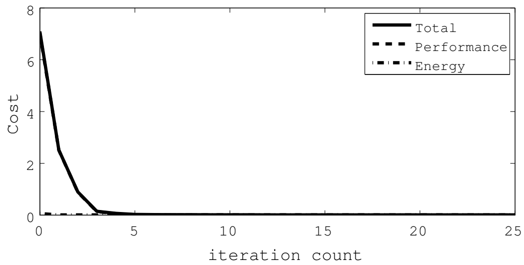

Fig. 10. J $(w = 0.1,\ \rho_{init} = 0.2)$.

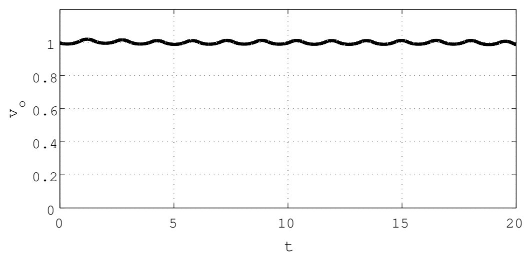

Fig. 11. Final $v_o(t)$ $(w = 0.5, \rho_{init} = 0.5)$.

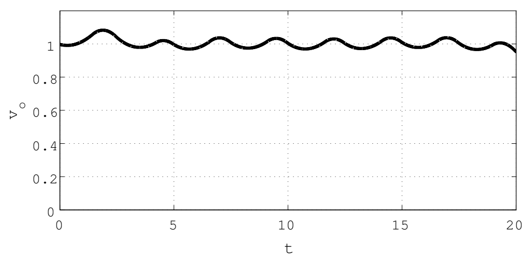

Fig. 12. Final $v_o(t)$ $(w = 0.9, \rho_{init} = 0.5)$.

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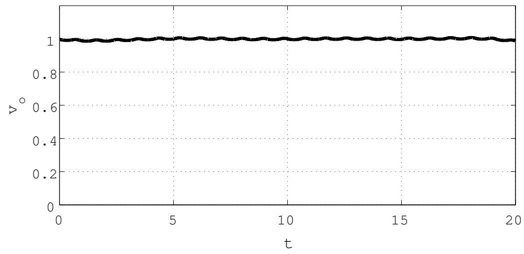

Fig. 13. Final $v_o(t)$ $(w = 0.1, \rho_{init} = 0.5)$.

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