OPTIMAL TRAJECTORY GENERATION FOR NEXT GENERATION FLIGHT MANAGEMENT SYSTEMS

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Abstract

Next generation flight management systems require the compliance of temporal and spatial constraints on navigation performance. The problem of generating fuel-efficient trajectories for aircrafts that comply with the required navigation performance is approached from an optimal control framework. By deriving the necessary conditions for optimality, nominal optimal control signals can be generated in a computationally efficient manner. These control signals can be used to generate nominal trajectories for an aircraft model. Using the nominal control and nominal trajectory, a feedforward-feedback control scheme can be implemented to robustify the system's response in the presence of uncertainty and disturbances to still achieve the required navigation performance. The feasibility of the approach is demonstrated through simulation and Monte Carlo runs.

Introduction

Next generation air transportation systems, also known as NextGen, concepts require compliance with temporal and spatial constraints on navigation performance [1]. It is necessary that flight management systems (FMS) are able to comply with 4D trajectories, which needs to be done in computationally efficient ways due to the limited computational resources available. The unpredictable nature of flight conditions and changes in aircraft dynamics over the course of flight also dictates that trajectory compliance should be robust to disturbances. Furthermore, the execution of fuel efficient trajectories by the control system is desirable for multiple reasons, such as economical or environmental.

The problem of obtaining fuel-optimal 4D trajectories with required times of arrival (RTA) compliance has been addressed before. For example, Park and Clarke [2] approach the problem of fuel optimal descent with RTA compliance in a unifying framework called Trajectory Performance Analyzer. Here the performance bounds of descent trajectories were analyzed using optimal control formulations, while trajectories that comply with RTAs lie somewhere between these performance bounds. The optimal control problem is solved with a particular non-linear programing (NLP) solver. However, due to the computational power the NLP solver requires, sub-optimal trajectories (constructed from trajectories that can be generated via the vertical navigation functions of the FMS) that approximate the fuel-optimal solutions and still achieve RTA compliance were proposed.

An alternate approach at solving the 4D trajectory generation problem which results in optimal solutions is proposed. The 4D trajectories are viewed as defined from a set of finite 3D waypoints, a subset of which has an RTA associated with it, thus making these waypoints 4D waypoints, while other 3D waypoints have no RTA associated with them. By connecting consecutive waypoints with linear segments the reference trajectory is completed. The aircraft is required to stay within a certain distance from the center of the trajectory as defined by the required navigation performance (RNP) with a certain statistical confidence level. It is only required that the aircraft passes through 4D waypoints within an allowed RTA window and the spatial RNP. The time at which the aircraft passes through all other 3D trajectory matters little. In essence, the trajectory is given, but the time profile is not.

By linearizing over nominal trajectories the problem can be restated as a linear-quadratic (LQ) problem. The LQ approach to the tracking problem to date offers optimal solutions when the trajectory to be tracked as a function of time is given [3]. This implies that along with the trajectory, the time profile of the desired state trajectory is already specified. The application under consideration, however, does not require the aircraft to track the trajectory point-by-point at each time.

To this end, the introduction of a controlled time variable [4] is proposed, that would serve as a proxy for the time variable that the LQ formulation requires in the tracking problem. This time variable serves to make the connection between the existing LQ approaches that require a specific point in time that must be tracked and the problem that is being addressed, where there is no specific point in time to be tracked (except for a finite number of points with RTA). This 'virtual time' variable is controlled in such a way that by tracking the point on the reference trajectory at this 'virtual time' with the aircraft at real time, fuel optimality and the 4D required trajectory compliance are both achieved. The aircraft control signal is given analytically which allows the proposed solution to be computed rapidly. Furthermore, the control signal is given in feedback form, so that the proposed solution is robust to disturbances. Preliminary results show that RNP and RTA compliance is achieved, even in the presence of uncertainty in the wind and aircraft dynamical model, with the required statistical confidence level.

Optimal Trajectory Generation

In [1], primary technical methods and other considerations for approaching spatial and temporal RNP compliance were presented. Two of the methods therein discussed were RTA predictive guidance, which relies on trajectory prediction to generate a flight trajectory that achieves the desired arrival time, and continuous time control guidance, which manages the flight time of the aircraft along a pre-computed 4D reference trajectory. It seems that a combination of these methods provided the means to compensate shortcomings from both methods. More importantly, it also provided the means of very naturally restating the problem as an optimal control problem. This will be the pursued approach and the focus of the rest of the document.

The basic idea would be to find the fuel-optimal control input for the aircraft that allows the aircraft to perform tracking of a reference trajectory within the allowed RNP. An approach to obtain these reference trajectories is to have air traffic control and the aircraft negotiate a flight path, given as a finite number of 3D waypoints, some of which will have an

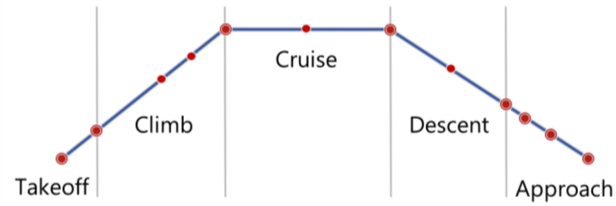

Figure 1. Reference Path Formed from Waypoints

associated RTA effectively converting them into 4D waypoints. Figure 1 showcases an example of a vertical flight path where multiple waypoints with and without time constraints have been selected. All the dots represent waypoints with an associated RNP. The bigger dots also have an RTA associated to them. By connecting these waypoints together the reference trajectory can be effectively produced.

Even though only a finite number of waypoints with and without RTA have been selected, the aircraft must still comply with the RNP throughout the length of the flight. This can be interpreted as the existence of a tube of a prescribed radius surrounding the reference path. The aircraft must remain inside of this tube throughout the duration of the flight. The RTA constraints can be interpreted as disks extending from the waypoint. The aircraft must be within these disks at the RTA. This is exemplified in figure 2.

Before stating the optimal control problem, a system model is needed to constrain any trajectories found by what is dynamically feasible for an aircraft. In general, the aircraft model may be represented as

$$\dot{x} = f(x,u)$$
$$y = h(x)$$

with $x \in \mathbb{R}^n$ being the set of states, $u \in \mathbb{R}^m$ the set of control inputs and $y \in \mathbb{R}^p$ the set of desired outputs. In

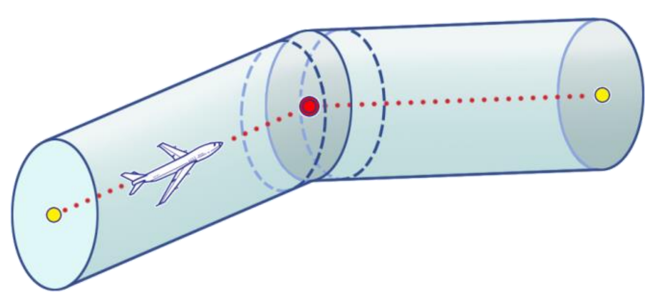

Figure 2. Tube Representation of RNP Constraints

order to obtain realistic results the model needs to be accurate, however this also requires higher order dynamical modeling which can impair computational time. As this work is meant to serve as merely a proof of concept, model accuracy will be traded off for simplicity.

A three degrees-of-freedom, rigid-body dynamical model is used where only force equations are considered. Moment dynamics are neglected as it is assumed that the attitude of the aircraft is controlled by an autopilot. Under some simplifying assumptions, described in more detail in [5], the following is the dynamical model for an aircraft that will be used

$$\dot{x} = \begin{bmatrix} \dot{V}_T \\ \dot{\gamma} \\ \dot{\chi} \\ \dot{E} \\ \dot{N} \\ \dot{H} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(T-D) - g\sin(\gamma) - \cos(\gamma)\left(\dot{U}_W \cos(\chi) + \dot{V}_W \sin(\chi)\right) \\ \frac{1}{mV_T}\ell\cos(\mu) - \frac{1}{V_T} g\cos(\gamma) + \frac{1}{V_T}\sin(\gamma)\left(\dot{U}_W \cos(\chi) + \dot{V}_W \sin(\chi)\right) \\ \frac{1}{mV_T\cos(\gamma)}\ell\sin(\mu) + \frac{1}{V_T\cos(\gamma)}\left(\dot{U}_W \sin(\chi) - \dot{V}_W \cos(\chi)\right) \\ V_T\cos(\gamma)\cos(\chi) + U_W \\ V_T\cos(\gamma)\sin(\chi) + V_W \\ V_T\sin(\gamma) \end{bmatrix}$$

where $\boldsymbol{x}^T = [V_T, \gamma, \chi, E, N, H]$ is the set of states representing true airspeed, aerodynamic flight path angle, aerodynamic yaw angle, East direction, North direction and up, respectively. The set of control signals $\boldsymbol{u}^T = [T, \ell, \mu]$ represent the thrust force, lift force and aerodynamic roll angle, respectively. The terms U_w and V_w correspond to the wind velocity components in the East and North direction, respectively. Other values of interest are the drag force D, the gravitational acceleration constant g and the mass of the aircraft m.

With this choice of model the optimal control problem can be stated in the form

$$\min_u J(u) = \int_{t_0}^{t_f} L(x,u)dt + \Psi\left(x(t_f)\right)$$

subject to

$$\begin{cases} \dot{x} = f(x,u) \\ x(t_0) = x_0 \end{cases}$$

for L: $\mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}_{\geq 0}$ and Ψ: $\mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$, where $\mathbb{R}_{\geq 0}$ is the set of non-negative real numbers, and $[t_0, t_f]$ is the time window from the current waypoint to the next waypoint with RTA. For notational convenience, the initial and final times are defined as $t_0 := 0$ and $t_f := T$. It is of interest to find the control input $u(t)$ that minimizes the cost functional $J(u)$ subject to the dynamical model for the aircraft. In particular, it is of interest to minimize the following cost functional

$$\min_u J(u) = \frac{1}{2}\int_0^T \left(u^T R u + (y-r)^T Q (y-r)\right)\mathrm{d}t + \frac{1}{2}\left(x(T) - x_d\right)^T F\left(x(T) - x_d\right)$$

subject to

$$\begin{cases} \dot{x} = f(x,u) \\ x(t_0) = x_0 \\ \left\| y(t) - r(\tau) \right\| < d, \quad t \in [0,T] \text{ and } \tau \in [0,T] \end{cases}$$

for positive definite matrix $R = R^T$, for positive semi-definite matrices $Q = Q^T$ and $F = F^T$, for the reference trajectory to be tracked $r \in \mathbb{R}^p$ and the desired terminal states $x_d \in \mathbb{R}^n$. This choice of quadratic cost functional is beneficial for multiple reasons in generating the optimal trajectory. The first term ensures that the resulting trajectory is fuel-optimal by penalizing for having large control signals. By tuning R, fuel-consumption can be reduced through this term. The second term penalizes from having the outputs deviating too much from the reference trajectory. By tuning Q, it can be ensured that the aircraft remains inside the RNP tube. The last term ensures that the aircraft will be at the next waypoint within the time window. By tuning F, it can be ensured that the aircraft complies with the RTA.

It is desired that the distance from the reference path is at all times less than some prescribed distance d which is provided by the RNP. That is, for all time $t \in [0, T]$, there exists a time $\tau \in [0, T]$ such that $\|y(t) - r(\tau)\| < d$, i.e. the aircraft is inside the RNP "tube" at all times (figure 2). Since there is only time constraints on the initial and final conditions, that is at time $t = 0$ and $t = T$, it might be beneficial to change time profile of the trajectory. To this end a new variable $s(t)\in\mathbb{R}$ is introduced and the problem changed to:

$$\min_{u(t)} J(u(t)) = \int_0^T L(x(t),u(t),y(t),r(s(t)),v(t))dt + \Psi(x(T))$$

where it is of interest to minimize the functional with respect to $u(t)$ and the new control signal $v(t)\in\mathbb{R}$ subject to the constraints

$$\begin{cases} \dot{x} = f(x,u) \\ x(t_0) = x_0 \\ y = h(x) \\ \dot{s} = v \\ s(0) = 0 \end{cases}$$

The new variable $s(t)$ whose rate of change is being controlled directly serves as a proxy for τ. Its control signal is also introduced into the cost functional to further minimize the cost. In the following section the first order optimality conditions are derived for this type of problem.

First Order Optimality Conditions

It is now of interest to find the first order optimality conditions for the kind of problem presented in the previous section. The problem is first constrained by the system dynamics. This is achieved through the inclusion of a Lagrange multiplier function for each of the system dynamics $p_x(t)\in\mathbb{R}^n$ and $p_s(t)\in\mathbb{R}$, coupled with the state differential equations. The following equation shows how this is done.

$$\begin{aligned} &J_0(u,v) = \\ &\qquad \int_0^T \left[L(x,u,y,r(s),v) + p_x^T(f(x,u)-\dot{x}) + p_s^T(v-\dot{s})\right]\mathrm{d}t \\ &\qquad + \Psi(x(T)) \end{aligned}$$

Calculus of variations dictates that a necessary condition for the optimal control signals to have been found is that for any *admissible* variations, that is variations in the control signals that still comply with the boundary constraints, the resulting cost functional is less than that of the cost functional with small variations in the control signals. This is,

$$J_0(u(t),v(t)) \le J_0(u(t)+\varepsilon\beta(t), v(t)+\varepsilon\alpha(t))$$

for all $\varepsilon > 0$ and for all admissible $\alpha(t)$ and $\beta(t)$, $t\in[0, T]$. A necessary condition for this is to find where the functional derivative vanishes, that is inserting admissible variations in J_0 and finding δJ where

$$\delta J(u,v) = \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\left(J_0(u+\varepsilon\beta, v+\varepsilon\alpha) - J_0(u,v)\right).$$

After performing Taylor expansions and collecting same type variations the following optimality conditions arise.

$$\begin{cases} \dot{p}_x(t) = -\dfrac{\partial f}{\partial x}^T p_x(t) - \dfrac{\partial L}{\partial x}^T \\ p_x(T) = \dfrac{\partial \Psi}{\partial x}^T (x(T)) \end{cases},$$

$$\begin{cases} \dot{p}_s(t) = -\dfrac{\partial r}{\partial t}^T (s)\dfrac{\partial L}{\partial r}^T \\ p_s(T) = 0 \end{cases},$$

$$\begin{cases} \dfrac{\partial f}{\partial u}^T p_x(t) + \dfrac{\partial L}{\partial u}^T = 0 \\ p_s(t) + \dfrac{\partial L}{\partial v}^T = 0 \end{cases}$$

where $p_x(t)$ and $p_s(t)$ are co-states to $x(t)$ and $s(t)$, respectively. These optimality conditions together with the constraints

$$\begin{cases} \dot{x}(t) = f(x(t),u(t)) \\ x(0) = x_0 \\ y(t) = h(x(t)) \end{cases}, \qquad \begin{cases} \dot{s}(t) = v(t) \\ s(0) = 0 \end{cases}$$

provide enough information to find the optimal control signals $u(t)$ and $v(t)$.

It should be noted that as these conditions are first order necessary optimality conditions, they are not conditions for global optimality. And, moreover, both minimizers as well as maximizers satisfy them. As will be seen in the “Simulation Results” section, a descent algorithm is employed to solve for the control signals which ensures that the signals are indeed minimizers as opposed to maximizers. However, the potential for a locally optimality solution remains. As global optimality is computationally costly, and certainly not achievable on the computational resources available to the FMS, only locally optimal solutions were pursued since they at least improve upon whatever nominal path is generated from the vertical navigation function of the FMS. And, as will be seen in the "Simulation Results" section, this suffices in all example scenarios considered. However, a case when the optimality conditions are indeed global is first considered, namely the linear-quadratic case.

Improving Robustness

The previous section provided optimality conditions that can be used to generate nominal optimal trajectories for a given initial state and desired terminal state, which represent the current and next waypoints with RTA. However, this results in an open-loop control scheme, where the nominal control signal may be computed offline and stored for later use. This kind of strategy is fragile to disturbances, which could potentially result in the system not complying with all the RNP. Therefore, in order to compensate for uncertainties and other kind of disturbances, it is favorable to change from an open-loop strategy to a closed-loop strategy where the control signal becomes a function of the states. This is very difficult to do and is seldom possible for general non-linear dynamical systems.

In order to still provide some level of robustness in the design, a hybrid feedforward-feedback strategy may be used. The first step that must be taken towards finding the feedforward-feedback signal is to obtain the nominal trajectory pair ($x_n(t)$,$u_n(t)$). The aircraft model may be then linearized around this nominal trajectory, i.e. let

$$\delta x(t) = x(t) - x_n(t) \quad \text{and} \quad \delta u(t) = u(t) - u_n(t)$$

The derivative of these differential states can then be computed as

$$\begin{aligned}\delta\dot{x} &= \dot{x} - \dot{x}_n \\ &= f(x,u) - f(x_n,u_n) \\ &= f(\delta x + x_n, \delta u + u_n) - f(x_n,u_n).\end{aligned}$$

Taylor expansion may performed on the first term to find

$$\begin{aligned}&f(\delta x + x_n, \delta u + u_n) - f(x_n,u_n) \\ &= f(x_n,u_n) + \frac{\partial f}{\partial x}(x_n,u_n)\delta x + \frac{\partial f}{\partial u}(x_n,u_n)\delta u \\ &- f(x_n,u_n) + H.O.T(\delta x,\delta u),\end{aligned}$$

where *H.O.T.* denotes the Higher-Order Terms in the variations δx and δu. Keeping only the linear terms in the differential state and control results in

$$\begin{aligned}\delta\dot{x}(t) &= \frac{\partial f}{\partial x}(x_n(t),u_n(t))\delta x(t) + \frac{\partial f}{\partial u}(x_n(t),u_n(t))\delta u(t) \\ &= A(t)\delta x(t) + B(t)\delta u(t).\end{aligned}$$

It is of interest to find the control δu that minimizes the cost

$$\frac{1}{2}\int_0^T \left[\delta u^T \hat{R}\delta u + \delta x^T \hat{Q}\delta x\right]\mathrm{d}t + \frac{1}{2}\delta x(T)^T \hat{S}\delta x(T)$$

subject to

$$\begin{cases}\delta\dot{x}(t) = A(t)\delta x(t) + B(t)\delta u(t). \\ \delta x(0) = 0\end{cases}$$

where, as before, by selecting the positive definite matrix $\hat{R} = \hat{R}^T$ and positive semi-definite matrices $\hat{Q} = \hat{Q}^T$ and $\hat{S} = \hat{S}^T$ it is possible to tune the gain of the control signal, the states between the current waypoint and the next waypoint, and the state at the waypoint with an RTA, respectively. Fortunately this is now a finite horizon Linear Quadratic regulator which is known to have the solution

$$\delta u(t) = -\hat{R}^{-1}B(t)^T S(t)\delta x(t)$$

where $S(t)$=$S(t)^T$ is the solution to the Riccati differential equation

$$\dot{S}(t) = -A(t)^T S(t) - S(t)A(t) + S(t)B(t)\hat{R}^{-1}B(t)^T S(t) - \hat{Q},$$
$$S(T) = \hat{S}.$$

The control signal can be restated in terms of the actual state to have

$$u(t) = u_n(t) - \hat{R}^{-1}B(t)^T S(t)(x(t) - x_n(t))$$

where, as desired, the control signal possesses a feedforward term $u_n(t) + \hat{R}^{-1}B(t)^T S(t)x_n(t)$ and a feedback term $-\hat{R}^{-1}B(t)^T S(t)x(t)$. It is noteworthy that whenever the current state $x(t) = x_n(t)$ the feedback term cancels out and the control signal becomes the nominal control signal. However when the system deviates from its nominal trajectory, be it because of disturbances or other uncertainties, the feedback gain will change the control effort to force the aircraft back into its nominal trajectory.

Simulation Results

The approach described above was simulated to demonstrate its feasibility. In order to obtain results that are relevant to real-world scenarios, a descent scenario was chosen for Hartsfield-Jackson Atlanta International Airport. In particular, a descent trajectory was performed for a lateral path segment generated from the ERLIN NINE STAR chart at KATL airport. The route was constructed from the consecutive connection of waypoints DEVAC-CALCO-ROME-ERLIN.

For the purpose of the simulation, a simple model for the wind was assumed. The model chosen is a polynomial fit of the wind data samples generated from NOAA METAR data of KATL airport. METAR data was processed to obtain first and second order statistics for both direction and strength of the wind. These results were used to randomly generate data points, to which an n^{th} order polynomial representing East wind and North wind velocity were fitted.

To generate the nominal trajectory for the linearization, the use a descent algorithm with respect to the quadratic cost is proposed. Gradient descent with Armijo step size was chosen for its stability and speed [6]. Through the use of this algorithm, the nominal trajectory that complies with RNP and RTA was obtained using a wind model as previously discussed. Additionally, by a careful selection of weights all other aerodynamical constraints are also met. The nonlinear aircraft model was then linearized around the nominal trajectory as described above.

In order to demonstrate the robustness gained from feedback, an m^{th} order polynomial wind model that acted on the aircraft is used instead of the n^{th} order model used to generate the nominal trajectory. This new wind model was fitted to random data points of a chosen mean and covariance, as opposed to the wind model used in the generation of the nominal trajectory, which was obtained by processing the METAR data. This corresponds to the case where the wind forecast was wrong. Monte Carlo simulations of the flight trajectory with feedback were performed with the new wind model.

In the simulation, the origin was placed at DEVAC at sea level. x represents East, y represents North, and z represents upward direction. It was assumed that the aircraft started at DEVAC at 35,000 feet above Mean Sea Level and at a true airspeed of 450 knots, oriented 105◦ from the North and leveled to the Earth. The aircraft is to continuously descend to 20,000 feet above sea level at CALCO, which is located 72.1 nmi away from DEVAC at 105◦ from the North. In particular, the focus was on the first 180 second segment of this flight. In order to evaluate RTA compliance performance, a virtual RTA was set up at the 180s point along the continuous descent trajectory.

Monte Carlo simulations of the 180 second flight segment were run for 1,000 iterations for two cases – in case 1, the mean and covariance for the wind sample was set to be 5m/s and 15, respectively. In case 2, the mean is set to 10m/s while the covariance is set to 30. The RNP value is 2nmi for lateral deviation and 1,000ft for vertical deviation. RTA compliance was defined as whether the aircraft was within 1 nmi of the desired point at final time. Minimum descent angle is defined as −6◦. The nominal true airspeed is to decrease linearly as a function of altitude, 450 knots at 35,000 feet, and 300 knots at 10,000 feet. The allowable deviation from this nominal speed is defined to be ±30 knots. The following tables and plots show the nominal inputs and trajectories and results from the Monte Carlo simulations.

Figures 3 through 5 show the nominal input and state trajectories obtained using gradient descent with Armijo step size. With an initial wind forecast, nominal control input u_n is found such that the

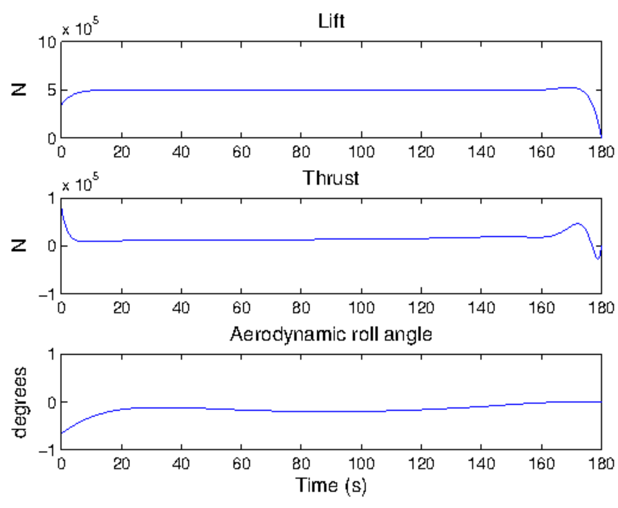

Figure 3. Nominal Optimal Inputs

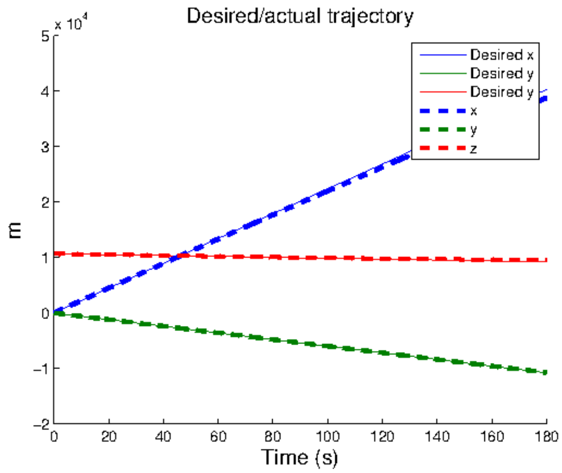

Figure 4. Nominal Path Tracking

nominal state trajectory x_n complies with RNP, RTA, and other performance requirements.

Figure 6 visually represents the effectiveness of our approach using feedback to robustify the control. It can be clearly seen that the trajectory without feedback starts to deviate from the nominal trajectory due to the mismatch of the forecast wind and actual wind disturbances. However, the trajectory with feedback is able to reject the disturbance and stay close to the nominal trajectory. Results from Monte Carlo simulations will be presented to quantitatively reinforce this point.

Monte Carlo simulations were run on a MSI FX620DX laptop computer with Intel i5-2410M

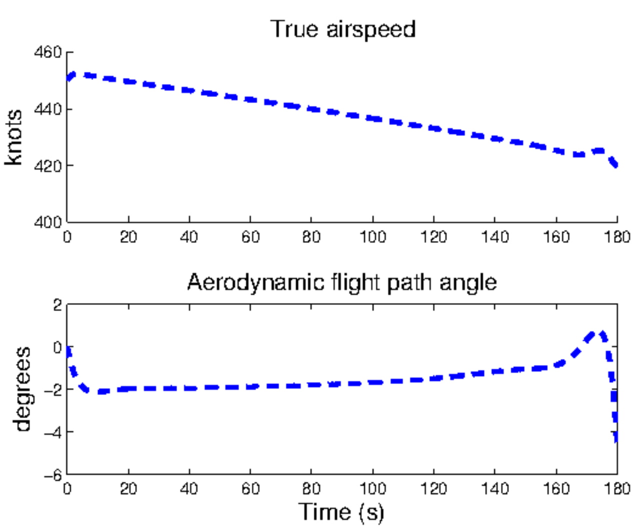

Figure 5. Other States of Interest

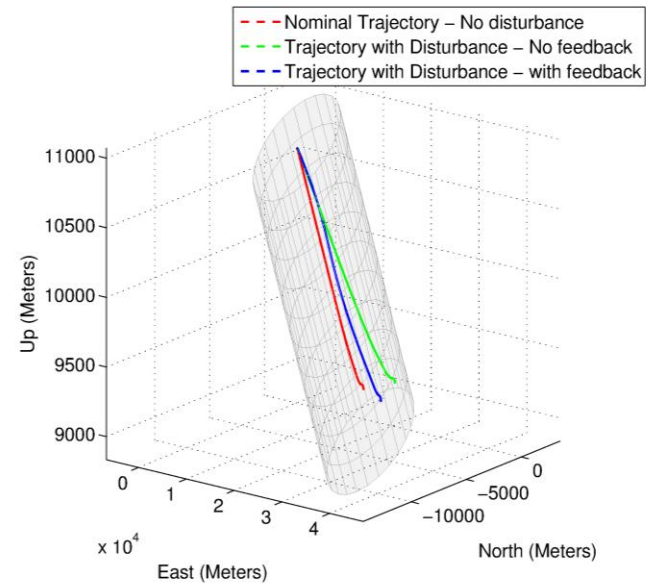

Figure 6. Trajectories in RNP Tube

processor and 4GB of RAM, running on Windows 7 operating system using MATLAB. The average time for each iteration was 1.41 seconds with 2001 sample points. The time for each iteration not only includes the time taken to compute the gain matrices at each time instance, but also the time required to linearize the nonlinear aircraft model around the nominal trajectory and simulate the trajectory. This shows that robustification using feedback in this way is computationally tractable in that it is possible to run the algorithm in real-time to compensate for various disturbances, which is one of the key features of this approach. The feedback phase of this approach – linearization, gain computation – is computationally efficient, especially the gain computation phase due to the linear nature of the computations required. The linearization phase needs to be done only once for any nominal trajectory segment. Thus recomputing for feedback gains becomes an entirely linear process once the linearization is done, enabling us to recompute the feedback gains quickly as necessary. It should be noted that the computer used to perform the Monte Carlo simulations was an "average" laptop computer.

Tables 1, 2 and figures 7 and 8 show that the inclusion of feedback clearly improves the RNP compliance of the aircraft, where RNP values are, as defined above, 2nmi for lateral deviation and 1000ft for vertical deviation. Very rarely the RNP is met if

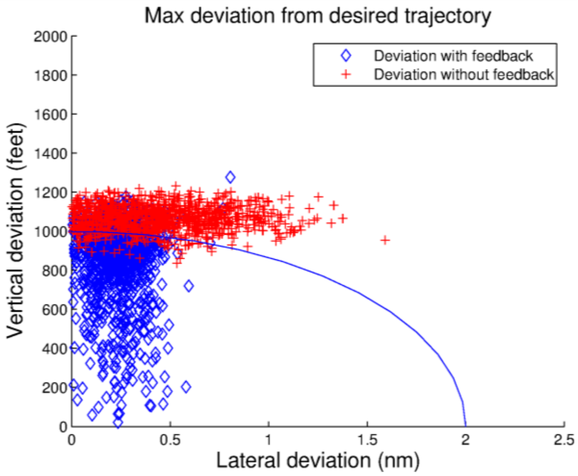

Figure 7. Max Deviation for Monte Carlo Run 1

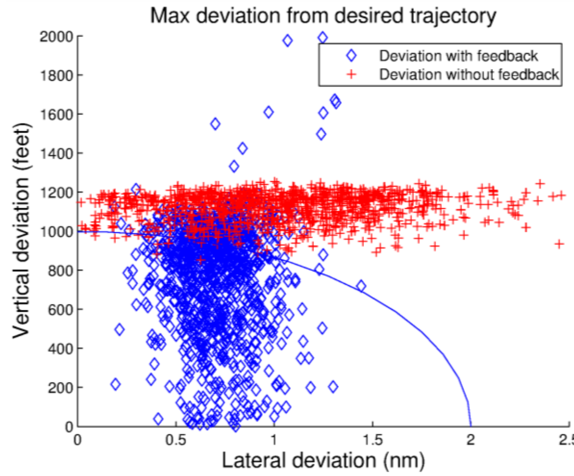

Figure 8. Max Deviation for Monte Carlo Run 2

the aircraft is controlled without feedback. Although the RNP compliance does not reach 95% even with feedback, in both cases, RNP compliance does not even reach 10% without feedback. Also, it should be noted that the performance with feedback is a function of how well the gains are tuned, and that it is potentially possible to tune the gains well enough such that necessary statistical confidence levels are achieved.

Table 1. Percentage of Achieved RNP & RTA, Run 1

	With Feedback	Open Loop
RNP compliance	74.9%	9.5%
RTA compliance	99.3%	98.8%

Table 2. Percentage of Achieved RNP & RTA, Run 2

	With Feedback	Open Loop
RNP compliance	68.8%	1.3%
RTA compliance	98.3%	99.8%

Table 3. Other Performance Criteria, Run 1

Minimum Descent Angle Compliance	100%
True Airspeed Compliance	97.7%

Table 4. Other Performance Criteria, Run 2

Minimum Descent Angle Compliance	98.5%
True Airspeed Compliance	77.6%

Tables 3 and 4 show minimum descent angle and true airspeed compliance rates for feedback case, out of many performance constraints of an aircraft. It can be seen that with weaker wind the compliance is achieved with very high confidence levels, although the confidence level deteriorates as the wind disturbance gets stronger.

Conclusions

This paper presents an optimal control approach to generating RNP compliant yet fuel effective flight trajectories within the 4D trajectory framework. In particular, necessary optimality conditions are derived for the full, nonlinear aircraft model as a generator of nominal trajectories. These trajectories are further robustified through a state feedback control scheme associated with the corresponding linearized problem.

The proposed approach is novel along three distinct dimensions, namely (1) it is computationally efficient in the sense that intense, off-line computations are avoided. Instead, the computations can be performed onboard the aircraft. (2) By introducing a reparameterization of time along the nominal path, and by allowing this parameterization be an explicit part of the decision variables, greater fuel efficiency is obtained. (3) Robustness to disturbances, and implicitly RNP compliance, is achieved through a state-feedback controller.

Simulation results highlight the efficacy and computational feasibility of the proposed approach.

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