A Control Theoretic Model of the Muscular Actions in Human Head-Eye-Coordination *

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Abstract

In this article we investigate the problem of how to model and control the combined motion of the human head and eye. We develop a model of the muscles, based on a simplified physical model and an assumption that the muscles can be modeled as damped springs with a second order linear dynamics. We then find control laws that both make the combined pupil-movement follow a given trajectory, and make the separate head and eye trajectories three times continuously derivable. Our controls also make the energy produced in the movement small, since we believe that to be a reasonable, physical control-criterion.

1 HEAD AND EYE ROTATION

1.1 DYNAMICS OF HEAD ROTATION

Human head movements are controlled by more than 20 pairs of muscles that link the skull, spinal column and shoulder girdle in a complex variety of configurations.

What we want to do in this paper is to model the complex behavior of the muscles which control the horizontal rotation of the head in such a way that the rotation of the head is given account for in a simple way. Therefore we chose to model the muscles as just one pair of muscles, conducting the same actions as all

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of the muscles involved in the action. This is because we are more interested in the principles of the controls behind the muscular contractions, than in finding an exact muscular model at the price of clarity.

We chose to model these muscles as damped springs with a second order linear dynamics of the form

$$\ddot{x} = -k(x - L) - g\dot{x} + v(t), \tag{1}$$

where L and x are the lengths of the unstretched and the stretched spring respectively, and k and g are frequency and damping parameters of the spring. A controller, $v(t)$, is added to the spring, and the control term is only added to one of the two muscles at a time, since only one muscle is active when the head is rotating.

If the angle θ is chosen to be the system state variable, we then, after some calculations, end up with

$$\ddot{\theta} = \mathrm{sign}(\theta) f(|\theta|, \mathrm{sign}(\theta)\dot{\theta}) + u(\theta)v(t), \tag{2}$$

where

$$u(\theta) = \begin{cases} \mathrm{sign}(\theta) g(|\theta|) & \text{if } \mathrm{sign}(\theta) = \mathrm{sign}(\dot{\theta}) \\ \mathrm{sign}(\theta)\frac{1}{R} & \text{if } \mathrm{sign}(\theta) \neq \mathrm{sign}(\dot{\theta}) \end{cases} \tag{3}$$

and

$$v(t) = \begin{cases} v_1(t) & \text{if } \dot{\theta} > 0 \\ v_2(t) & \text{if } \dot{\theta} < 0. \end{cases} \tag{4}$$

1.2 DYNAMICS OF OCULAR MOTION

The *external* and *internal recti*, the muscles behind the rotation, both attach on the so called Annulus of Zinn, behind the eye, and they also attach rather high up on the eye itself, which makes the modeling a bit easier than in the head case, since the geometry is simplified by the fact that the forces, produced by the two muscles, can be assumed to always be tangential to the eye itself.

In almost the same way as in the neck case, we get the system describing the eye rotation to be

$$\ddot{\phi} = -2(g\dot{\phi} + k\phi) - \mathrm{sign}(\dot{\phi})\frac{1}{r}v(t), \tag{5}$$

with

$$v(t) = \begin{cases} v_1(t) & \text{if } \dot{\phi} > 0 \\ v_2(t) & \text{if } \dot{\phi} < 0. \end{cases} \tag{6}$$

1.3 THE COMBINED DYNAMICS

So if we return to our initial problem; How do we combine the movements of the head and the eye in order to follow an object with a given trajectory, $\psi(t)$, at a constant distance from the head?

Geometrical considerations directly gives us

$$\ddot{\phi} = -\text{sign}(\gamma)\frac{1}{\sqrt{1-\eta(\gamma)^2}}\left(\frac{d^2\eta(\gamma)}{d\gamma^2}\dot{\gamma}^2 + \frac{d\eta(\gamma)}{d\gamma}\ddot{\gamma} + (\frac{d\eta(\gamma)}{d\gamma}\gamma)^2\frac{\eta(\gamma)}{1-\eta(\gamma)^2}\right), \quad (7)$$

with

$$\gamma(t) = \psi(t) - \theta(t) \quad (8)$$

and

$$\eta(\gamma) = \frac{d\cos\gamma - h}{\sqrt{d^2 + h^2 - 2hd\cos\gamma}}. \quad (9)$$

This can be stated in a more compact form as

$$\ddot{\phi} = F(\theta, \psi, \dot{\theta}, \dot{\psi}, \ddot{\theta}, \ddot{\psi}), \quad (10)$$

but we still have equation 5, which gives us

$$v_{eye}(t) = -\text{sign}(\dot{\phi})r[F(\theta, \psi, \dot{\theta}, \dot{\psi}, \ddot{\theta}, \ddot{\psi}) + 2(g\dot{\phi} + k\phi)]. \quad (11)$$

This way of letting the main tracking be done by the eye is a product of the so called *occulocentric view*. This means that the main tracking is performed by the eye, while the head is just moving in a general way.

2 CONTROL LAWS

Now that we have a model for the combined process of activating both the muscles of the neck and of the eye, the next task is to find the control laws. A reasonable approach is to try to minimize the energy produced in the movement, and since the mass of the head, M, is so much larger than the mass of the eye, m, one criterion for finding our control could be that it should make the angular acceleration of the head as small as possible.

In order to accomplish this, we divide the trajectory of the head into subparts, where in some parts the head accelerates, and in others the angular acceleration is zero. This is because one obvious control that makes $|\ddot{\theta}|$ small is the one that makes $\ddot{\theta} = 0$. Therefore we want the major part of the trajectory to be of this type.

If we recall equation 2 - 4, we directly see that to achieve this, we simply let

$$v_{lin}(t) = -\frac{\text{sign}(\theta)f(|\theta|, \text{sign}(\theta)\dot{\theta})}{u(\theta)}. \quad (12)$$

This linear approach is unfortunately not enough. First of all, we assume that we start following the object when the head and the eye both are at rest at some fixed angle, and therefore we need to find controls that can accelerate the systems up to some suitable velocity when the tracking is initiated. Secondly, when the followed trajectories are not well behaved, we have to take into account that that the eye may rotate out of bound if no modification of the head's zero acceleration trajectory is being made. These two cases show that we need to be able to accelerate the head in a controlled way in some situations.

The general idea behind the control laws we chose to use, can be illustrated by the block chart in Figure 1. If we want to find the control producing the

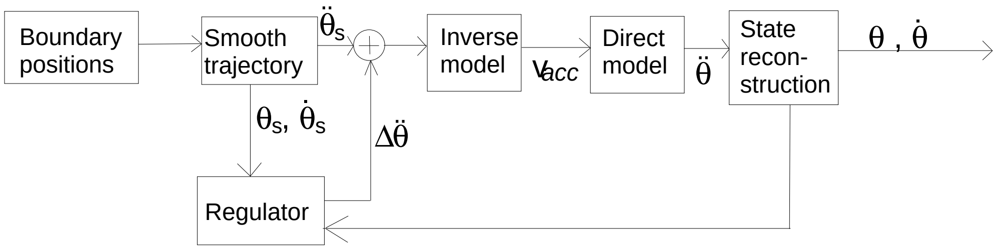

Figure 1: Block chart for the feed forward acceleration case.

desired trajectory, we simply use the inverse for $\ddot{\theta}$. We chose to model the feedback on the form

$$\Delta\ddot{\theta}(t) = C_1(\theta_{calc}(t) - \theta_{actual}(t)) + C_2(\dot{\theta}_{calc}(t) - \dot{\theta}_{actual}(t)), \tag{13}$$

where θ_{calc} is the desired, calculated trajectory.

3 CONCLUSIONS

When it comes to the developed model, the weakest part is probably that of trying to model muscles as second order springs, since an actual muscle has a dynamics that is much more complicated than that. However, this approach has the major advantage that it makes the mathematics reasonably simple. It is also sufficiently complete when it comes to actually start thinking about how to control the head and the eye muscles simultaneously.

The control strategy we chose to use was based on a desire to keep the energy produced in the movement small, since we believed this to be a physically reasonable approach. We therefore let the angular acceleration of the head be zero most of the time, since this would make the energy small.

When it comes to physical adequacy, it can be worth comparing our results to the trajectories found in Guitton's *Eye-Head Coordination in Gaze Control*. It turns out that our piecewise linear approach is not so bad after all, since an actual combined movements seem to have somewhat of the same piecewise linear characteristics as our trajectories, even though they, of course, are more complex. This should not, however, disqualify our model as not being an interesting step towards an understanding of the complex behavior of human head-eye-coordination.