From Algorithms to Architectures in Cyber-Physical Networks

Magnus Egerstedt

Cyber-physical systems (CPS) are, at their core, characterized by fundamentally different models of computation. On the physical side, the Laws of Physics apply, i.e., differential equations describe the dynamics of the systems. On the cyber side, discrete models dictate the evolution of the computations. The result is a hybrid dynamic system, and, by now, a rich body of work exists for characterizing, modeling, designing, and analyzing such systems, thus providing a general model for CPS. (For a representative sample, see [5, 10] and references therein.)

However, one aspect of CPS that has not yet received the same *systematic* treatment is the fact that such systems are oftentimes interconnected, e.g., as is the case in power grids, precision agriculture infrastructure, smart building controls, and mobile sensor and communication networks, just to name a few, [1, 4, 8]. There certainly is a vast literature on networked systems in terms of coordinated controls, e.g., [3, 7, 9], but an explicit focus on what the *cyber* and what the *physical* aspects of such networks entail has been somewhat absent. The purpose of this paper is to highlight one key feature of such networks, where physical interconnections between physical nodes have to co-exist with an overlaid computational, information-exchange network, thus creating a network (or really a network of networks) that also must be characterized by different computational models. We call such networks *Cyber-Physical Networks*, or CPN, and this short paper is to be understood as a small step towards a general theory of CPN, as opposed to a complete treatment of the subject; such a treatment does not yet exist.

A CPN is comprised of (at least) two interacting networks, G_P and G_C, where $G_P = V_P \times E_P$, with V_P being the set of physical nodes, and $E_P \subseteq V_P \times V_P$ encodes the existence of physical couplings between the nodes. The cyber-part of the network, $G_C = V_C \times E_C$, encodes the information flow among computational nodes, i.e., the edges in this graph denotes communication channels between cyber agents – as opposed to dynamical coupling terms. The way these two networks come together to form a CPN, G_{CP}, is through the coupling between cyber-nodes and physical nodes. And, there are two distinctly different ways in which these two types of nodes can interact, namely through sensing and actuation. As such, we define two more edge sets, $\tilde{E}_a \subseteq V_C \times V_P$ and $\tilde{E}_s \subseteq V_P \times V_C$, where the subscripts denote *sensing* and *actuation*, respectively. The interpretation is that cyber-node i can influence (directly) physical node j if and only if $(i, j) \in \tilde{E}_a$, while it can sense physical node j if and only if $(j, i) \in \tilde{E}_s$. The resulting CPN is obtained through the union of these constituent components, i.e.,

$$G_{CP} = (V_P \cup V_C, E_P \cup E_C \cup \tilde{E}_a \cup \tilde{E}_s).$$

Now, associate a state $x_i,\ i = 1, \ldots, N_P,\ (|V_P| = N_P)$, with each physical node and use $x_P = [x_1, \ldots, x_{N_P}]^T$ to denote the aggregate. Moreover, let $u_j,\ j = 1, \ldots, N_C,\ (|V_C| = N_C)$, be a decision variable/control signal associated with the cyber nodes, the physical constraints can be written on the form

$$\dot{x} = F(x, u),\ G(x, u) = 0.$$

But, the differential coupling constraints must respect the sparsity pattern of the underlying network, since they encode pairwise dynamic couplings, and we denote this physical sparsity pattern by

$$F \in \text{sparse}_P(G_{CP}),$$

which means that the phsyical nodes can only "affect" each other directly if they form an edge in E_P, while the decision variables can only "affect" the physical node states if they form an edge in $\tilde{E}_a$. Examples of such couplings are the Kuramoto coupled oscillator models [6] or the Bergen-Hill power exchange model [2], just to name a few.

This CPN model captures the ways in which a physical network interacts with a cyber-network through actuators and sensors. The dynamic coupling constraints as well as the physical interaction network are typically given *a priori* since the Laws of Physics are what they are, and the design task is to construct effective ways of controlling and coordinating such networks, i.e., design the cyber part. But, one can easily include other, more architectural questions, and what this means is really that we have only begun to scratch the surface of CPN, and significant work remains to be done in order to fully harness their expected utilities.

References

[1] I. F. Akyildiz, E. Cayirci, W. Su and Y. Sankarasubramaniam. A Survey on Sensor Networks, *IEEE Communications Magazine*, Vol. 40, No. 8, pp. 102-114, Aug. 2002.

[2] A. R. Bergen and D. J. Hill, A Structure Preserving Model for Power System Stability Analysis, *IEEE Transactions on Power Apparatus and Systems*, Vol. 100, No. 1, pp. 25-35, 1981.

[3] F. Bullo, J. Cortés, and S. Martínez. *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*, Princeton University Press, Princeton, New Jersey, U.S.A., 2009.

[4] G.N. Ericsson. Cyber Security and Power System Communication-Essential Parts of a Smart Grid Infrastructure, *IEEE Transactions on Power Delivery*, Vol. 25, No. 3, pp. 1501–1507, 2010.

[5] T. A. Henzinger. The Theory of Hybrid Automata, *Proceedings of the 11th Annual Symposium on Logic in Computer Science (LICS)*, IEEE Computer Society Press, pp. 278-292, 1996.

[6] Y. Kuramoto. *Chemical Oscillations, Waves, and Turbulence*. Springer- Verlag, Berlin and New York, 1984.

[7] M. Mesbahi and M. Egerstedt. *Graph Theoretic Methods in Multiagent Networks*, Princeton University Press, Princeton, New Jersey, U.S.A., 2010.

[8] M. Nazari, Z. Costello, J. Feizollahi, S. Grijalva, and M. Egerstedt. Distributed Frequency Control of Prosumer-Based Electric Energy Systems. *IEEE Transactions on Power Systems*, No. 99, pp. 1-9, March 2014.

[9] W. Ren and R. Beard. *Distributed Consensus in Multi-vehicle Cooperative Control*, Springer, Berlin, 2008.

[10] P. Tabuada. *Verification and Control of Hybrid Systems: A Symbolic Approach*, Springer, 2009.