Towards Power-Aware Rendezvous

Hassan Jaleel, Shaunak Bopardikar and Magnus Egerstedt

***Abstract*— In this paper we connect the power consumed by agents in a network to how much they move. In other words, by moving around, the agents' power levels decrease, which in turn impacts the range at which they can sense/communicate with neighboring agents. This work constitutes the first, explicit connection between mobility and power consumption from the point of view of being able to carry out the coordinated controls task and we, in particular, investigate the effect of power consumption on the rendezvous problem, i.e., the problem of having the agents meet at a common location. Conditions are given for when the rendezvous problem can be achieved for a two-agent problem as well as for a network of agents organized in a directed cycle graph.**

I. Introduction

The problem of controlling distributed, mobile networks has received considerable attention during the last few years. A broad class of problems has been successfully addressed relating to the definition of decentralized mobility laws for achieving certain global, geometric shapes, such as formation control ([12], [7], [8], [17], [11]), coverage control ([4], [3], [15]), rendezvous ([10], [1]), and flocking and swarming ([20], [21], [16], [26]). Fundamental to all of these results is the idea that the interaction laws must be scalable so that the developed algorithms can be deployed over a large collection of mobile nodes, e.g., sensor and communication devices.

One key aspect of such large-scale systems is that they are inherently power sensitive. In order to deploy a large collection of nodes, payload issues (e.g., battery sizes) become a significant problem and in the sensor networks community, this has been addressed through the development of strategies for maximizing the life-time of the network by using various power aware resource algorithms, e.g.,([2], [6], [9], [25], [27]).

From a mobility vantage-point, power-awareness is arguably even more important than from a sensing vantage-point. It is estimated that "communications" are orders of magnitude more expensive than "computations and sensing" when it comes to power consumption, while mobility is orders of magnitude more expensive than all of them. In other words, it is cheaper to "think" than to "talk", and it is cheaper to "talk" than to "walk". Despite this rather informal and crude power consumption taxonomy, it is clear that mobile nodes use power as they move. However, very little has been said about how to design power-aware mobility strategies.

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In this paper, we take a first stab at remedying this omission. In particular, we study the by-now classic rendezvous problem [1], i.e., the problem of having all the nodes meet in a common, a priori unspecified location using only relative position information. Our take on this problem is to use a sensor footprint model that depends on the current power levels and that shrinks as the power level decreases. Moreover, the rate at which the power level decreases is proportional to the input to the system. As a result, the more the agents move, the less power they have, and, subsequently, the smaller their sensor footprints become. The reason why a shrinking sensor footprint is important is that the agents can only sense the relative positions of other agents within their sensory range. This formulation of the power aware mobility problem thus affords a natural formulation of the trade-offs between mobility and power consumption.

The particulars of the problem under consideration in this paper might be considered somewhat limited. We will first study a simple two-agent rendezvous problem followed by an N-agent problem with the interaction network being given by a directed cycle topology. It should be noted though that this paper constitutes a first step towards understanding the connections between mobility, power levels, and sensory footprints. As such, its novelty and merit should be mainly understood as providing this link in a formally precise manner rather than an effective solution to particular, practical problem.

The outline of this paper is as follows: In Section II, we describe the interaction, power consumption, and sensing models and discuss what these models imply for the construction of general, power-aware mobility strategies. In Section III, we study the simplified problem of having two planar mobile robots meet at a common location subject to connectivity, and hence also power consumption constraints. This discussion is generalized, in Section IV, to the case where the interaction topology is given by a directed cycle graph. Finally, in Section V, the implications for general, power aware mobility problems are discussed and promising future research directions are identified.

II. Power, Sensing, and Mobility Models

Consider a network of N planar, mobile agents, with positions $x_1, \ldots, x_N \in \mathbb{R}^2$. We assume that each of these agents' dynamics are given by single integrators, i.e,

$$\dot{x}_i = u_i,\ i = 1, \ldots, N. \tag{1}$$

Now, each of the agents has a corresponding non-negative power level $p_i \in \mathbb{R}_+,\ i = 1, \ldots, N$, and as the agents move around, this power level is depleted. In this paper, we simply assume a direct proportional decay rate

$$\dot{p}_i = -c\|u_i\|, \tag{2}$$

where $c > 0$ is the power loss coefficient [23]. It should be noted that much more elaborate power loss models can be constructed (see for example [5], [18], [19]) but for the purpose of the initial developments in this paper, we stick with this first order model.

The way we tie the effect of the power levels to what the agents can do is by relating the power levels to the sensor footprints. In fact, the way the agents interact with each other is by measuring their displacements relative to neighboring agents, i.e., agents that are in their sensor ranges. In other words, if we let Δ_i be the sensor range associated with agent i and if $\|x_i - x_j\| \leq \Delta_i$, i.e., agent j is within agent i's sensory range, u_i is allowed to depend on the relative displacement $x_i - x_j$, which we assume is what the agents can in fact measure. Tallying up the contribution from all agents within sensory range of agent i, the control law in Equation (1) is assumed to be of the form

$$u_i = \sum_{j \mid \|x_i - x_j\| \leq \Delta_i} f(x_i - x_j)\sigma_{ij}, \tag{3}$$

for some interaction law $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. Here $\sigma_{ij} \in \{0, 1\}$ is an indicator function that dictates whether or not agent j should be effecting the movement of agent i. (To paraphrase [15], "just because you're neighbors doesn't mean you're friends.") Note, by letting $\sigma_{ij} = 1$ and $f(x_i - x_j) = x_j - x_i$, we recover the standard consensus equation ([4], [17], [20]).

The final part of the construction relates the power levels to the effective sensor footprint. This connection depends on what type of sensor is used and, for the purpose of the discussion in this paper, we follow the model developed in [13]. Here, the sensor range model is based on the RF power density function for an isotropic antenna, with the sensor footprint being proportional to the available power of the sensor node. But, as the footprint (disk) is quadratic in the sensor range (radius), we get that

$$\Delta_i^2 = \gamma_i p_i, \tag{4}$$

where $\gamma_i > 0,\ i = 1, \ldots, N$, is a constant that depends on various factors such as the transmission medium and source. Putting all of these individual components together, we obtain the main object of study in this paper, namely an agent model that we chose to call a **MoPS** (**Mo**bility, **P**ower, and **S**ensing) agent.

Definition 2.1 (MoPS Agent): A *MoPS* agent is a first-order **Mo**bility, **P**ower, and **S**ensing agent, whose dynamics are given in Equation (1), whose power decay is given in Equation (2), whose sensory footprint is given by Equation (4), and whose control law satisfies the restrictions given in Equation (3).

The key question under consideration in this paper is what effect the shrinking footprints have on the performance of the agent team. For instance, if two agents are to meet at a common location, they need to be "visible" to each other (or at least one of them to the other). But, even though they may both be within each other's sensory ranges initially, by moving around, the power consumption may indeed cause the agents to lose track of each other since the sensor ranges may become too small. This is an issue, of course, also in more elaborate cooperative control scenarios and what is needed is a systematic approach to designing coordinated controllers that take power consumption into account already at the design stage.

III. Power Aware Rendezvous: Two Agent Case

As already stated, the rendezvous problem involves moving a collection of mobile agents to the same spatial location. This, moreover, should be accomplished with only local information given in terms of the relative inter-agent displacements.

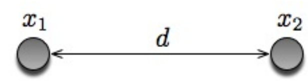


Fig. 1. Rendezvous between two MoPS.

To start the discussion, we will first consider the rendezvous problem for a pair of MoPS agents and, in particular, investigate what the implications are in terms of power consumption. We will make two additional, simplifying assumptions about the two agents, depicted in Figure (1), namely (1) that they do not act stupidly, in the sense that they do indeed move towards each other, and (2) that they act symmetrically and have the same initial power levels and power decay rates. A consequence of the first simplifying assumption is that we can restrict the problem to a 1-dimensional problem in which the agents are moving on the line between them. As such, without loss of generality, we restrict the formulation to the case where $x_i \in \mathbb{R},\ i = 1, 2$.

The second assumption implies that the two agents are executing the same anti-symmetric control strategy in that

$$\dot{x}_1 = f(x_1 - x_2) = -f(x_2 - x_1) = \dot{x}_2,$$

where f is the particular control strategy used.

If we assume, again, without loss of generality, that $x_1, x_2 \in \mathbb{R}$ and that $x_1 \leq x_2$, we can let u constitute the control action applied, in the sense that

$$\dot{x}_1 = -u,\ \dot{x}_2 = u.$$

Under this formulation, with the assumption that the agents do not act stupidly, we immediately see that $u \leq 0$. As a consequence, we get that the distance between the agents, $d = x_2 - x_1$, has the dynamics

$$\dot{d} = 2u, \tag{5}$$

with solution

$$d(t) = d_0 + 2\int_0^t u(s)ds = d_0 - 2\int_0^t |u(s)|ds = d_0 - 2\mathcal{U}_t, \tag{6}$$

where d_0 is the initial distance between the two agents, and where $\mathcal{U}_t = \int_0^t |u(s)|ds$ is the total control energy used by an agent over the interval $[0,t]$.

As we assume the agents to act symmetrically and have the same initial power levels, we can use $p(t)$ to denote this level, which thus satisfies

$$\dot{p} = cu. \tag{7}$$

As such, we directly see, in light of Definition 2.1, that in order for rendezvous to be successfully executed by these two agents, they need to be able to sense each other, i.e., we need to ensure that

$$d^2(t) \leq \gamma p(t) \tag{8}$$

throughout the duration of the movement. We let $e(t)$ denote the power gap

$$e(t) = \gamma p(t) - d^2(t), \tag{9}$$

with the interpretation that $e(t) \geq 0$ means that the agents can sense each other while $e(t) < 0$ means that they are not within range of each other. One natural question to ask now is how much control energy can be injected into the system without rendering e negative, i.e., without causing the underlying interaction network to become disconnected.

Lemma 3.1: *The maximum energy that can be injected into a two MoPS system, with initial separation and power conditions satisfying $\gamma p_0 - d_0^2 > 0$, over a given time interval $[0,t]$, without rendering the underlying network disconnected is*

$$\mathcal{U}_t^\star = \frac{d_0 - c\gamma/4}{2} + \sqrt{\left(\frac{d_0 - c\gamma/4}{2}\right)^2 + e_0}, \tag{10}$$

where $e_0 = \gamma p_0 - d_0^2$.

Proof: The solution to Equation (7) is given by

$$p(t) = p_0 - c\mathcal{U}_t. \tag{11}$$

Replacing the expressions for $d(t)$ and $p(t)$ in Equation (9) with the explicit solutions for p and d yields,

$$e(t) = \gamma(p_0 - c\mathcal{U}_t) - (d_0 - 2\mathcal{U})^2.$$

To find the maximum energy that can be injected while maintaining connectivity, we need to put $e(t) = 0$ and solve the resulting quadratic equation to find

$$\mathcal{U}_t^\star = \frac{d_0 - c\gamma/4}{2} + \sqrt{\left(\frac{d_0 - c\gamma/4}{2}\right)^2 + e_0},$$

and the proof follows. ■

Now, we need to relate this maximal energy injection to the achievement of rendezvous. In particular, we need to ensure that if the agents move as much as they possible can without causing the network to get disconnected, they do in fact end up at the same location. The subsequent theorem establishes conditions on the initial power level that ensures that this is in fact doable.

Theorem 3.2: *For a two MoPS system, rendezvous can be achieved if*

$$p_0 \geq \frac{6d_0^2 + \gamma c d_0}{8\gamma}, \tag{12}$$

Proof: At any time, the distance between the two agents is given by Equation (6). And, if rendezvous is achieved at time t, we thus have $d(t) = 0$, which, in light of Equation (6) implies that

$$d_0 = 2\mathcal{U}_t. \tag{13}$$

Replacing $\mathcal{U}_t$ by $\mathcal{U}_t^\star$ in the above expression gives the most restrictive conditions when this can in fact be achieved. And, straightforward algebraic manipulation gives the condition on p_0 that

$$p_0 = \frac{6d_0^2 + \gamma c d_0}{8\gamma}.$$

As $\mathcal{U}_t^\star$ is the maximum energy that can be injected without loosing connectivity, this means for any $U_t \leq \mathcal{U}_t^\star$ the inequality in (12) is satisfied. Moreover, from Equation (13), we have that rendezvous is achieved, and the theorem follows. ■

One consequence of Lemma 3.1 and Theorem 3.2 is that for a system of two MoPS agents, where the rendezvous problem is reduced to a 1-dimensional problem, the type of controller or the time t needed to solve the problem, does not matter. The only thing that matters is the total energy, $\mathcal{U}_t$, supplied to the system, which depends completely on the initial conditions. For example, if we want to achieve rendezvous in T time units and the condition in Equation (12) is satisfied, then a constant u given by

$$u(t) = -\frac{1}{T}\mathcal{U}_T^\star$$

will solve the problem.

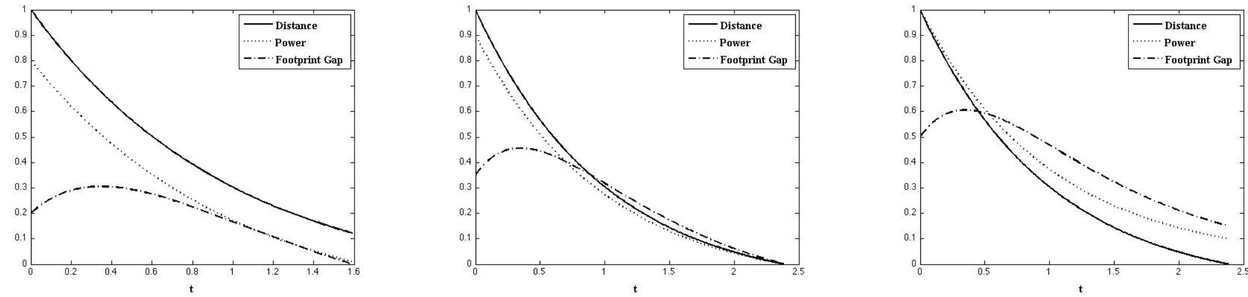
This controller is used in Figure (2), where three different initial power levels are used. In face, we let

$$p_0 = \frac{6d_0^2 + \gamma c d_0}{8\gamma} + \epsilon,$$

and in the left figure, $\epsilon < 0$ resulting in rendezvous not to be achieved before connectivity is lost. In the middle figure $\epsilon = 0$ and rendezvous is achieved exactly at the time when footprint becomes zero, while the right figure shows the case when $\epsilon > 0$, with the result that rendezvous is achieved with power left over.

From these observations, one would be tempted to draw the conclusion that the condition in Equation (12) is not only sufficient but also necessary. It is in fact also necessary under the assumption that connectivity is maintained. But, the agents may, by pure luck or in some other open-loop fashion, still be able to achieve rendezvous despite not being able to "see" each other, which is why we formulate this condition as a sufficient but not necessary condition.

The rather surprising fact that the actual control law does not matter in this case is of course not true in general. If there are more than two agents (as we will see in the next



(a) Rendezvous fails (b) Rendezvous barely succeeds (c) Rendezvous succeeds

Fig. 2. Depicted are the distance between agents (solid), the available power (dotted), and $e(t)$, i.e., the power gap (dashed-dotted) which corresponds to how close agents are to becoming disconnected. In the left figure, $e = 0$ before rendezvous is achieved. In the middle figure, rendezvous is achieved at the very moment when e becomes zero. The right figure shows a case when rendezvous is achieved with $e > 0$.

section) or if the dynamics is double integrator rather than single integrator, we are no longer this fortunate.

IV. Rendezvous Under Directed Cycle Topologies

In this section we consider the more involved situation where we have a network of N MoPS agents. We assume that the interaction topology, i.e., the underlying graph that dictates the information flow, is given by a directed cycle which remains static throughout the motion [14]. The number of agents, N, is greater than 2 and every agent $i \in \{1, \dots, N\}$, with position x_i is thus connected to $(i+1)$-th agent at position x_{i+1} (modulo N), as shown in Figure (3). We moreover assume that all the agents have the same initial power levels.

Since the graph representing the system is balanced and has a rooted out branching, all the agents will meet at $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i(0)$, i.e., centroid of their initial positions they were to execute the standard consensus algorithm (e.g., [20], [22]). However for this to work in the presence of decaying power levels, the graph must remain connected, which, from Equation (8), implies that,

$$\|x_i(t) - x_{i+1}(t)\|^2 = d_i^2(t) \leq \gamma_i p_i(t), \tag{14}$$

for all time t. Here $p_i(t)$ is the power level of agent i at time t and $d_i(t)$ is the distance between agents i and $i+1$.

As we can no longer hope for a situation where the results do not depend on what particular control laws we use, we chose to work with the consensus equation over a directed cycle, i.e., the interaction law executed by the MoPS agents in this system is given by

$$u_i = k(x_{i+1} - x_i), \tag{15}$$

where $k > 0$ is a constant. Using the notation from Section II, $f(x_i - x_j) = k(x_j - x_i)$ and $\sigma_{ij} = 1 \;\Leftrightarrow\; j = i+1$ (modulo N) in Equation (3) .

The overall system can be written as

$$\dot{x} = Ax,$$

where A is an $N \times N$ circulant matrix,

$$A = \begin{pmatrix} -k & k & 0 & \cdots & \cdots & 0 \\ 0 & -k & k & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & -k & k \\ k & 0 & \cdots & \cdots & 0 & -k \end{pmatrix}$$

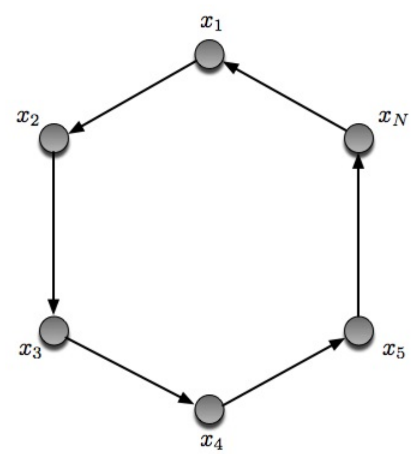


Fig. 3. System Model for Directed Cycle Graph

The following result characterizes a sufficient condition that ensures that rendezvous is achieved in the sense of all agents being within an ϵ distance of the initial centroid without rendering the network disconnected. This result is slightly more involved than the one in the previous section due to the fact that the network is more complex.

Theorem 4.1: *For a system consisting of N MoPS agents arranged in a directed cycle topology and executing the control law (15), suppose that:*

1) *the power loss coefficient c_i satisfies*

$$c_i \leq \frac{2\sqrt{2}\lambda e^{-\lambda T(\epsilon)}}{k\gamma_i} \max_{i\in\{1,\dots,n\}} \|x_i(0) - \bar{x}\|, \tag{16}$$

where

$$T(\epsilon) := \frac{1}{\lambda} \max_{i \in \{1,\dots,n\}} \ln \left(\frac{\|x_i(0) - \bar{x}\|}{\epsilon} \right), \qquad (17)$$

and λ is the real component of the second largest eigenvalue of A, which is a function of k, and

2) *the initial power levels satisfy*

$$p_i(0) \geq \frac{2}{\gamma_i} \max_{i \in \{1,\dots,n\}} \|x_i(0) - \bar{x}\|^2. \qquad (18)$$

Then, the connectivity constraint (14) *holds at all times $t \in [0, T(\epsilon)]$, and all agents are within a distance ϵ of $\bar{x}$ at time $t = T(\epsilon)$.*

Proof: Recall that without the connectivity constraint, the control law (15) ensures exponential convergence of all the agents at the centroid ([20], [22]). Thus, there exists a constant $\lambda > 0$ (the real component of the second largest eigenvalue of A), so that for every $i \in \{1, \dots, N\}$,

$$\|x_i(t) - \bar{x}\| \leq \|x_i(0) - \bar{x}\| e^{-\lambda t} \qquad (19)$$

Therefore, given an $\epsilon > 0$, after the time $T(\epsilon)$ satisfying Equation (17), all the agents are within a distance ϵ of the centroid of the initial locations.

It now remains to show that under the conditions of this proposition, the connectivity constraint (14) holds at all times $t \in [0, T(\epsilon)]$. Now,

$$\begin{aligned} d_i^2 &= \|x_i - x_{i+1}\|^2 = \|x_i - \bar{x} - (x_{i+1} - \bar{x})\|^2, \\ &\leq \|x_i - \bar{x}\|^2 + \|x_{i+1} - \bar{x}\|^2, \end{aligned}$$

Using (19)

$$d_i^2 \leq 2 \max_i \|x_i(0) - \bar{x}\|^2 e^{-\lambda t}. \qquad (20)$$

Assuming Equation (16) holds then from Equation (20),

$$d_i^2 \leq \frac{16 \lambda^2 e^{-4\lambda t}}{\gamma_i^2 c_i^2 k^2} \max_i \|x_i(0) - \bar{x}\|^4.$$

From the power decay model (2), the control law (15) and the above inequality we get

$$\dot{p}_i \geq -\frac{4\lambda e^{-2\lambda t}}{\gamma_i} \max_i \|x_i(0) - \bar{x}\|^2,$$

After integrating the above inequality and using the condition (18)

$$p_i(t) \geq \frac{2 e^{-2\lambda t}}{\gamma_i} \max_i \|x_i(0) - \bar{x}\|^2,$$

Comparing this with Equation (20) results in

$$p_i(t) \geq \frac{d_i^2}{\gamma_i}.$$

which proves that the connectivity constraint holds at all times $t \in [0, T(\epsilon)]$, thus proving the theorem.

■

A couple of observations can be made from Theorem 4.1. Firstly, to satisfy the condition on power loss coefficient (16), each agent needs to estimate the maximum initial distance of the centroid $\bar{x}$ from an agent. Secondly, keeping all the other parameters fixed, as ϵ becomes smaller, $T(\epsilon)$ becomes larger, and therefore the condition (16) implies that c_i needs to be smaller. This is intuitive because with a longer time to rendezvous, each agent is expected to spend more power, and therefore, the power-loss coefficient must be smaller.

V. Conclusions

In this paper, we connect the power consumption to mobility algorithms in an explicit way by establishing conditions under which rendezvous is achievable in the face of shrinking sensor/communications footprints. We study the particular cases of two-agent rendezvous and rendezvous over a directed cycle topology. It should be noted that although these two situations might be considered somewhat limited, they do in fact constitute a first attempt at achieving this tradeoff between mobility and power consumption. As such, the results in this paper should be thought of as providing a first installation in this general area of investigation rather than the final solution to the general problem.

In fact, a number of questions immediately follow from the results in this paper. For example, how can one achieve rendezvous for arbitrary and possible time-varying topologies? What happens when more elaborate models are used for connecting the sensor footprints to the power levels or for establishing power decays as a function of the control inputs? How should one handle agent dynamics that are not simply single integrators? All of these questions (and more) are potentially fruitful avenues for investigation and the real contribution in this paper should thus be understood in terms of allowing these questions to be asked rather than providing definitive answers.

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