2013 American Control Conference (ACC)
Washington, DC, USA, June 17-19, 2013

Leader Selection and Network Assembly for Controllability of Leader-Follower Networks

A. Yasin Yazıcıoğlu and Magnus Egerstedt
Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta, GA 30332, USA
yasin@gatech.edu, magnus@gatech.edu

Abstract— **In this paper, we study leader selection and network assembly for controllability of networked systems using tools from graph theory. Each agent in the network acts in a decentralized fashion by updating its state in accordance with a nearest-neighbor averaging rule. In order to control the system, external control inputs are injected into the dedicated leader nodes, and their influence is propagated throughout the network. In this paper, we devise a topological leader selection scheme and a network assembly scheme, both achieving complete controllability by utilizing the invariance of system controllability to the links between the leaders.**

I. Introduction

Decentralized control of networked, multi-agent systems has received considerable attention during the last decade. Numerous applications of decentralized control laws have been studied, including flocking (e.g., [2]), alignment and formation control (e.g., [1]-[4]), distributed estimation (e.g., [6]), sensor coverage (e.g., [5]), and distributed control of robotic networks (e.g., [7]), to name a few. In a distributed framework, a global task is achieved by local interactions of agents. In this setting, a fundamental question is whether such a decentralized system can be controlled by directly manipulating only some of the agents. Moreover, what is the smallest set of nodes that should be manipulated directly so that complete controllability of the overall system is achieved? Furthermore, how should distinct leader-follower networks be united to form a single completely controllable network? These questions motivate our study of the leader selection problem and the network assembly problem for networked control systems.

Controllability of networked systems was initially addressed in [8] via a spectral analysis. More topological analysis of the problem was later presented in [9], with an emphasis on how the symmetry with respect to the leader node affects the controllability of the system. More recently, equitable partitions [10], [11] and distances of followers to leaders [12], [13] were used to obtain tight upper and lower bounds on the dimension of the controllable subspace. Furthermore, structural controllability of networks was presented in [14].

Leader selection for leader-follower networks may aim to maximize different performance measures, such as convergence rate, robustness or controllability. For instance, in [15], the authors present a leader selection scheme that aims to maximize the convergence rate of the network. In [16], [17], the authors present how leaders may be selected to maximize the robustness of the consensus dynamics to additive noise by searching for a certain number of leaders resulting in the minimum H_2 norm for the system. Recently, in [18], the authors focus on structural controllability, and they present a topological leader selection scheme that provides a structurally controllable network that is robust to the partial failures in the communication links.

Distributed self-assembly has been extensively studied for multi-agent applications, and a number of assembly approaches have been proposed. For example, a programming language that specifies a robust shape formation in 2D is presented in [19]. Cellular automata are utilized in [20] to present generic self-assembly algorithms. In [21], authors introduce the use of graph grammars for distributed self-assembly. Note that self-assembly schemes may aim to achieve certain global properties as well as specific formations. For instance, in [22] authors study self assembly mechanisms that maintain connectivity of the system. Recently in [23], the authors present a self assembly scheme to build leader-asymmetric networks, which is a necessary condition for complete controllability of leader-follower networks.

In this paper, we present a topological leader selection scheme and a network assembly scheme for leader-follower networks in which the agents utilize a nearest-neighbor averaging rule. Some agents, called the *leaders,* support external control inputs that ultimately influence the dynamics of all other agents namely *followers* by spreading throughout the network. Under this setting, we tackle the problems of selecting leaders and assembling networks such that the resulting systems are completely controllable.

Our main results are a graph topological leader selection scheme and a network assembly scheme that can be realized in a decentralized fashion via methods such as graph grammars. We first show that the dimension of the controllable subspace is invariant under the addition/removal of edges between the leaders. Using this invariance, the network assembly is achieved by obtaining a connected structure via connecting the leaders of disjoint networks. For the leader selection problem, we present a divide and conquer method. In this method, we constrain the solution to be equivalent to an assembly of smaller substructures that are completely controllable under their respective leaders, only via adding some edges between their leaders. The resulting solution, in general, is suboptimal in terms of the number of leaders. However, complete controllability of the overall system is guaranteed, and the solution is obtained through analysis of rather simpler components.

The organization of this paper is as follows: Section II presents some preliminaries related to the system dynamics and algebraic graph theory. Section III provides the problem definitions for the network assembly, and the leader selection. In Section IV, we present our network assembly and leader selection schemes. In Section V, we present how the proposed leader selection scheme can be employed to select leaders based only on the controllability of paths. Finally, Section VI provides the concluding remarks.

II. PRELIMINARIES

Consider a networked system of n agents that utilize the same nearest neighbor averaging rule, known as the consensus equation, to govern their dynamics. For each particular agent i, the consensus equation is given as

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i), \tag{1}$$

where x_i is the state of agent i, and $\mathcal{N}_i$ is the set of agents neighboring agent i. Without loss of generality, let us assume that $x_i \in \mathbb{R}$, and the interactions among the agents are encoded via a static undirected graph $\mathcal{G} = (V, E)$. In this graph, each node in the node set, $V = \{1, 2, \dots, n\}$, corresponds to a particular agent, and the edge set, $E \subseteq V \times V$, is a set of unordered pairs (i, j), denoting that nodes i and j are neighbors. In this context, neighbor nodes are the ones that have the measurements of each other's relative states.

The consensus equation provides a simple, yet powerful foundation for decentralized control strategies that can be utilized in various tasks, including coverage control, containment control, distributed filtering, flocking and formation control. With all agents utilizing the consensus equation, their states asymptotically converge to the stationary mean,if and only if the underlying graph is connected [3].

Assume that we would like to control this network by applying external control signals to some of the nodes. Without loss of generality, let the first m nodes be the leaders taking the external control inputs, and let the remaining $(n - m)$ nodes be the followers, whose dynamics are governed by (1). Let the m dimensional control input be represented by the vector u. Then, the dynamics of the leader nodes satisfy

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i) + [u]_i, \text{ for } i = 1, 2 \dots, m. \tag{2}$$

where, $[u]_i$ denotes the i^{th} entry of the control vector u. When the external control signals are applied to the leader nodes, their effect on the dynamics propagates to the rest of the nodes through the underlying network.

In order to relate the system dynamics to the topology of the underlying network, we use some basic tools from algebraic graph theory, in particular the degree matrix, the adjacency matrix, and the graph Laplacian.

Let Δ be the $n \times n$ degree matrix associated with the graph. The entries of Δ are given as

$$[\Delta]_{ij} = \begin{cases} |\mathcal{N}_i| & \text{if } i = j \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $|\mathcal{N}_i|$ denotes the cardinality of $\mathcal{N}_i$, and it is equal to the number of neighbors of node i.

The adjacency matrix, $\mathcal{A}$, is an $n \times n$ symmetric matrix with its entries given as

$$[\mathcal{A}]_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The graph Laplacian, L, is simply given as the difference of the degree and the adjacency matrices,

$$L = \Delta - \mathcal{A}. \tag{5}$$

In light of (1) and (2), the dynamics of the leader-follower network with m leaders can be given as

$$\dot{x} = -Lx + Bu, \tag{6}$$

where $x = [x_1, x_2, \dots, x_n]^T$ is the state vector obtained by stacking the states of each individual node, and B is an $n \times m$ matrix with the following entries

$$[B]_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Note that (6) represents a standard, linear time-invariant system and it relates the system dynamics to the graph topology through the graph Laplacian.

III. PROBLEM DEFINITION

In this section, we present the definitions for the leader selection and the network assembly problems tackled in this paper. For both problems, we consider networked systems with the dynamics in (6) and aim to achieve complete controllability.

Definition (Network Assembly Problem): Let $\mathcal{G}_p = (V_p, E_p)$, $p = 1, 2, \dots, k$, represent disjoint completely controllable leader-follower networks. The network assembly problem is forming a connected completely controllable network, $\bar{\mathcal{G}}$, by adding some edges only between the nodes belonging to disjoint networks.

Note that, in general, combining two disjoint systems can reduce, increase, or preserve controllability. A simple example is shown in Fig. 1, where different assemblies result in different controllability properties. For cases where the smaller components being assembled are completely controllable, the network assembly problem boils down to finding an assembly that preserves the controllability.

Definition (Leader Selection Problem): For a given networked system, let $\mathcal{G} = (V, E)$ be the interaction graph. Leader selection problem is the problem of finding a set of nodes, $V_l \subseteq V$, with minimal cardinality such that if the nodes in V_l are chosen as leaders, then the system is completely controllable.

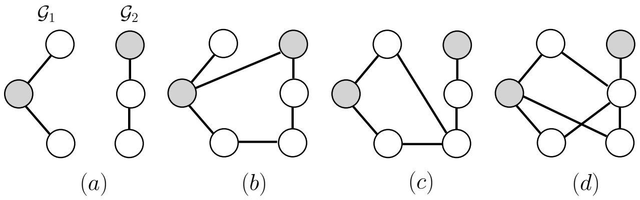

Fig. 1. Two disjoint leader-follower networks $\mathcal{G}_1$ and $\mathcal{G}_2$, where their leaders are represented as gray nodes. Their controllability matrices have $rank(\Gamma_1) = 2$, and $rank(\Gamma_2) = 3$ respectively. The assembly shown in (b) has $rank(\bar{\Gamma}) = 6$. On the other hand, the assembly in (c) has $rank(\bar{\Gamma}) = 5$. Finally, the assembly in (d) depicts a case where $rank(\bar{\Gamma}) = 4$.

Clearly, both problems can be solved via tools from linear systems theory, such as rank tests, without exploiting the relationships between system controllability and graph structure. However, such approaches come with various drawbacks. For instance, they may rely on a brute force search among a high number of possibilities. Furthermore, such methods are mostly centralized, and they require some information about the whole graph. Moreover, they are generally not efficient in dealing with changes in the network topology, such as addition or removal of nodes and edges, and in such cases the whole computation needs to be repeated. On the other hand, these issues can be tackled by utilizing some relationships between the graph topology and the system controllability.

IV. Network Assembly and Leader Selection

We start our derivations by proving that the system controllability is invariant to the addition or removal of edges between the leaders. This invariance property is utilized in our leader selection and network assembly schemes.

Proposition 4.1 *Let $\mathcal{G} = (V, E)$ be a leader-follower network with the dynamics in (6), and let Γ be the corresponding controllability matrix. Then, $rank(\Gamma)$ is invariant to the addition or removal of edges between the leaders in $\mathcal{G}$.*

Proof:
Here, we only prove that the addition of edges between leaders does not change the rank of the controllability matrix, as the proof for removal immediately follows. Let the dynamics for agents in the graph $\mathcal{G} = (V, E)$ be given as (6). Once the new edges are added, resulting dynamics can be represented as

$$\dot{x} = -(L + \tilde{L})x + Bu. \tag{8}$$

If the new edges are only formed between the leaders, $\tilde{L}$ has nonzero entries only in its rows corresponding to the leaders. Without loss of generality let the first m nodes be leaders. Then,

$$\tilde{L} = B\tilde{L}_m, \tag{9}$$

where $\tilde{L}_m$ is an $m \times n$ matrix consisting of only the first m rows of $\tilde{L}$. Let the control signal be in the form of

$$u = \tilde{u} + \tilde{L}_m x, \tag{10}$$

for some $\tilde{u}$. Then the dynamics in (8) can equivalently be represented as

$$\begin{aligned} \dot{x} &= -(L + \tilde{L})x + B(\tilde{u} + \tilde{L}_m x), \\ &= -(L + \tilde{L})x + B\tilde{L}_m x + B\tilde{u}, \\ &= -Lx + B\tilde{u}. \end{aligned} \tag{11}$$

Since the dynamics in (6) and (11) have the same controllability matrix, Γ, $rank(\Gamma)$ is preserved. ■

A. Network Assembly

The proposed network assembly scheme is based on the following corollary to Proposition 4.1.

Corollary 4.2 *Let $\mathcal{G}_p = (V_p, E_p)$, $p = 1, 2, \ldots, k$, represent disjoint leader-follower networks. Let Γ_p denote the controllability matrix for each such network $\mathcal{G}_p$. If these networks are assembled to obtain a connected graph, $\bar{\mathcal{G}}$, by adding some edges only between their leaders, then the rank of the controllability matrix of the resulting network satisfies*

$$rank(\bar{\Gamma}) = \sum_{p=1}^{k} rank(\Gamma_p). \tag{12}$$

Proof: The initial system can be considered as a single networked system consisting of k disjoint components. Stacking the states of nodes into a single vector x, we can write the dynamics before assembly as

$$\dot{x} = -\mathbf{L} + \mathbf{B}u, \tag{13}$$

where $\mathbf{L}$ is block diagonal matrix with the Laplacians of the disjoint graphs, $L_1, \ldots, L_k$, on the main diagonal, and similarly $\mathbf{B}$ is block diagonal matrix with the input matrices of disjoint networks, $B_1, \ldots, B_k$, on the main diagonal. Let $\mathbf{\Gamma}$ be the controllability matrix for the dynamics in (13). Since, initially the networks, $\mathcal{G}_1, \ldots, \mathcal{G}_k$, are disconnected we have

$$rank(\mathbf{\Gamma}) = \sum_{p=1}^{k} rank(\Gamma_p). \tag{14}$$

From Proposition 4.1, we know that addition of edges between the leaders does not affect the controllability of the system. Hence, we conclude that if the disconnected graphs are connected only through edges between leaders, then the resulting system has the controllability matrix, $\bar{\Gamma}$, satisfying

$$rank(\bar{\Gamma}) = rank(\mathbf{\Gamma}) = \sum_{p=1}^{k} rank(\Gamma_p). \tag{15}$$

■

Note that, with the initial disjoint components being completely controllable under their corresponding leaders, Corollary 4.2 implies that any connected graph that is obtained by adding some edges only between their leaders is completely controllable. Fig. 2 shows an example of completely controllable assembly for 3 disjoint networks.

This assembly scheme can be realized in a decentralized fashion via methods such as graph grammars [21]. Different

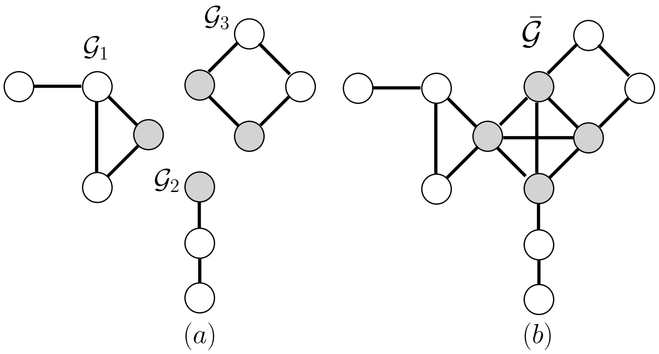

Fig. 2. Disjoint, completely controllable graphs (a), and a completely controllable assembly (b).

assemblies can be achieved by designing the rule set accordingly. As an example, we present a simple grammar that realizes the assembly shown in Fig. 2. For a given k disjoint networks, let us have $k+1$ labels, where all the followers are labelled with 0, and leaders of each disjoint component are given a unique label identifying the disjoint component they are contained in. Then the following rule achieves the assembly shown in Fig. 2.

$$\Phi_A = \quad (r_0): \quad p \quad q \quad \rightarrow \quad p-q, \; p>q>0, \qquad (16)$$

where p and q denote the node labels. According to this rule, whenever two unconnected nodes with distinct positive labels, $p>q>0$, meet, they link to each other. Note that such nodes are leaders that were not initially members of the same connected component. Hence, this rule achieves the edge formation only between the leaders of disjoint (initially) components.

B. Leader Selection

In this paper we present a suboptimal (in terms of the number of leaders) solution to the leader selection problem by utilizing the interaction between the topology of the interaction graph and the system controllability. Note that, in general, a large networked system may have a very complex topology, and obtaining the optimal solution for the leader selection problem can be a very challenging task to achieve. However, such a graph can be considered as a sum of rather smaller and simpler graphs with some edges added between certain nodes. Using this perspective in the leader selection problem, one can achieve the controllability of the overall system through the controllability of its smaller components. Here we follow this approach and present a divide and conquer scheme, which in general produces a suboptimal solution, for the leader selection problem. This scheme is based on Proposition 4.1 and the fact that the addition of more leaders to a controllable system can not reduce the dimension of the controllable subspace.

Proposition 4.3 *Let $\mathcal{G}=(V,E)$ be a leader-follower network with the dynamics in (6), and Γ be the corresponding controllability matrix. Then, assigning additional leaders can not reduce $rank(\Gamma)$.*

Proof:
Proof is trivial since the new leaders can always be controlled to act as followers. Using the leader dynamics in this paper given in (2), this can be done by injecting a constant control signal of zero to the new added leaders. ■

Our leader selection scheme constrains the solution to the leader selection problem to be equivalent to an addition of edges between the leaders of possibly disjoint smaller components that are controllable under their respective leader assignments. This way, we achieve the leader selection for large systems by using some available controllability and optimal leader selection results for smaller structures. We refer to these available graph structures (along with their optimal leader selections) as building blocks.

The proposed scheme introduces a trade-off between the optimality and the complexity of the solution. As the number of the nodes having an edge involved in the 'divide' step increases, more leaders are added to the leader set. The amount of deviation from the optimal solution depends on the topology of the given graph, and the available set of building blocks. For instance, if the given graph is already an available building block, then we already have an optimal leader selection. Otherwise, the proposed approach provides a leader selection just by using the information available through the building blocks. An example is depicted in Fig. 3, where this approach is applied to a graph by using a single building block and two building blocks.

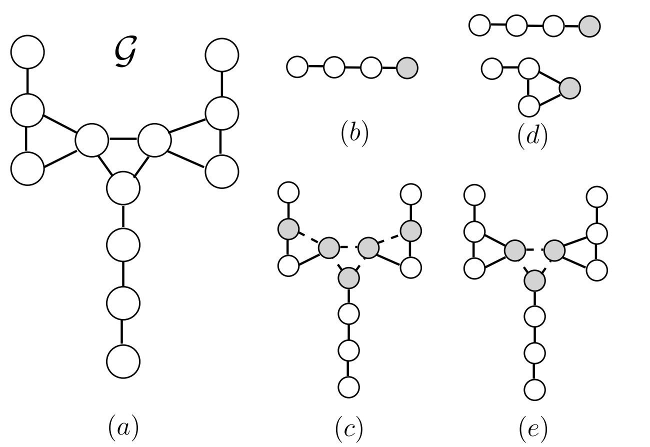

Fig. 3. For the complete controllability of a sample graph $\mathcal{G}$ (a), two sets of leaders (shown in gray), as shown in (c) and (e), are derived using the proposed scheme. The leader selection in (c) is obtained using Proposition 4.3 along with the information that the structure in (b) is completely controllable. If it is further given that the additional structure in (d) is also completely controllable, then one can obtain a smaller set of leaders as given in (e) using the same scheme. The dashed edges in (c) and (e) show how the corresponding building blocks in (b) and (d) are extracted from $\mathcal{G}$.

V. Leader Selection Using Paths

In this section, we elaborate further on our leader selection scheme by presenting an example that only uses paths as building blocks. Path graphs are simple structures for controllability analysis, and it is known that a path graph is controllable if one of its terminal nodes is assigned as a leader [8].

Proposition 5.1 *[8] A path graph with the dynamics in (6) is completely controllable if one of its terminal nodes is a leader.*

We present how a completely controllable system can be achieved with the proposed scheme by only utilizing the controllability of paths as given in Proposition 5.1.

Definition (Path Inducing Edge Removal) (PIER): Let $\mathcal{G} = (V, E)$ be connected graph. Removal of a set of edges, $E_r \subseteq E$, from $\mathcal{G}$ is called the PIER defined by E_r, if the resulting graph, $\mathcal{G}_r = (V, E \backslash E_r)$, consists only of disjoint path graphs.

Note that here we consider disconnected nodes as paths containing a single node. Examples of PIER are depicted in Fig. 4.

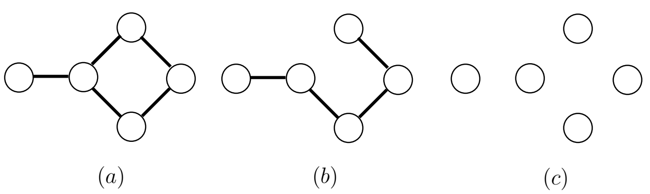

Fig. 4. A connected graph (a), and PIER examples are shown. In (b) one edge is removed and the resulting graph comprises a single path of 5 nodes. In (c), all the edges are removed, and the resulting graph consists of five disjoint single-node paths.

Theorem 5.2 *Let $\mathcal{G} = (V, E)$ be connected graph, let $E_r \subseteq E$ define a PIER, and let $P_1, P_2, \cdots, P_M$ be the disjoint path graphs in $\mathcal{G}_r = (V, E \setminus E_r)$. $\mathcal{G} = (V, E)$ is completely controllable if the set of leaders, V_l satisfy:*

$$(i, j) \in E_r \Rightarrow i, j \in V_l, \tag{17}$$

$$\forall P_k, \exists v_k \in V_l, \tag{18}$$

where v_k is a terminal node of the path P_k.

Proof:
Let us start the proof by considering the controllability of network $\mathcal{G}_r = (V, E \setminus E_r)$. Let a set of nodes, V_l, satisfying (17) and (18) be assigned as leaders of this network. Using (18) along with Proposition 5.1, we know $\mathcal{G}_r$ is completely controllable under this leader selection since each disjoint path is completely controllable. Note that $\mathcal{G}$ is obtained from $\mathcal{G}_r$ by adding the edges in E_r. In light of (17), addition of edges E_r to G_r is only adding edges between the leaders of $\mathcal{G}_r$. From Corollary 4.2, we know that controllability is preserved under this operation, hence G is also completely controllable through the leader set V_l. ■

In order to use this approach to assign as few as possible leaders, one needs to find a PIER defining edge set resulting in minimum number of nodes in the leader set to satisfy Theorem 5.2.

Definition (Minimal Leader Selection): Let $\mathcal{G} = (V, E)$, be a connected graph, let E_r define a PIER, and let $\mathcal{V}(E_r)$ be the set of all possible leader sets that satisfy the conditions in Theorem 5.2. A leader set, $V_l^* \in \mathcal{V}(E_r)$, is called a minimal leader selection (MLS) defined by E_r, if

$$|V_l^*| = \min_{V_l \in \mathcal{V}(E_r)} |V_l|. \tag{19}$$

For a given E_r, we denote this minimum number of leaders as $\mathcal{L}(E_r) = |V_l^*|$.

Definition (Minimal PIER): Let $\mathcal{G} = (V, E)$, be a connected graph, and let $\mathcal{P}$ denote the set of all PIER defining edge sets. A PIER defined by an edge set E_r^* is called a minimal PIER, if E_r^* satisfies

$$\mathcal{L}(E_r^*) = \min_{E_r \in \mathcal{P}} \mathcal{L}(E_r). \tag{20}$$

Fig. 5 depicts some examples for which the leader selection is done using a minimal PIER. As it is depicted in this figure, the PIER based implementation of the proposed scheme provides results that are optimal or very close to the optimal for some graphs, whereas for some other topologies its results can deviate significantly from the optimal. The following proposition helps in characterizing the type of graphs for which the PIER based implementation deviates significantly from the optimal.

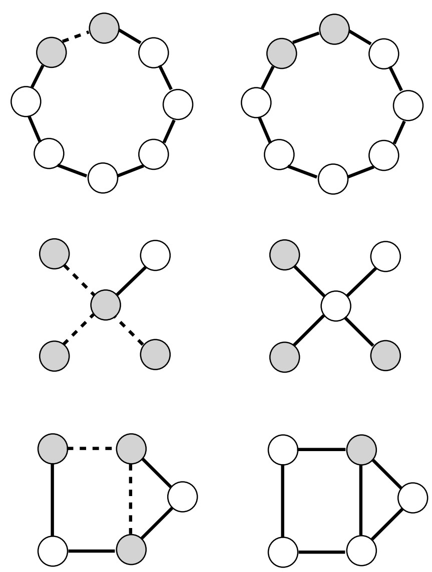

Fig. 5. Leader selection examples for different networks. First column of figures shows some leader selections obtained using the PIER based implementation of the proposed scheme, whereas the second column depicts some optimal (minimum number of leaders) leader selections for these graphs.

Proposition 5.3 *Let $\mathcal{G} = (V, E)$, be a connected graph, and let E_r define a PIER. If $i \in V$ is a node with a degree $d_i > 2$, then there exists at least $d_i - 2$ edges in E_r that are incident to i.*

Proof: Proof follows immediately from the definition of PIER. As the resulting graph, $\mathcal{G}_r = (V, E \setminus E_r)$, consists only of paths, node degrees in $\mathcal{G}_r$ are upper-bounded by 2. Hence, any node $i \in V$ having a degree $d_i > 2$ must have at least $(d_i - 2)$ of edges incident to it disconnected to obtain a PIER. ■

Corollary 5.4 *For a given network, $\mathcal{G} = (V, E)$, let $i \in V$ be a node with a degree $d_i > 2$. Then, the PIER based implementation of the proposed leader selection scheme assigns i, and at least $d_i - 2$ of its neighbors as leaders.*

In light of Corollary 5.4, one can say that the efficiency of the PIER based implementation mostly depends on the number of nodes having a degree greater than 2, and as the number of more connected nodes increases the efficiency is reduced.

Note that the PIER based implementation is only presented as a simple example for the proposed scheme, and the results can be significantly improved by incorporating additional local structures. The main advantage of this method is that it utilizes some available controllability and leader selection results in small structures to achieve controllability in more complex systems. Moreover, while in this paper we do not present a decentralized implementation, selecting leaders by searching for small local structures has a significant potential to be achieved in a decentralized fashion.

VI. CONCLUSION

In this paper, we presented a self assembly scheme and a leader selection scheme, both achieving complete controllability for leader-follower networks. These schemes are based on the invariance of the rank of the controllability matrix to the addition or removal of edges between leaders.

For the self assembly problem, we showed that for a given set of distinct, completely controllable networks, any assembly obtained by forming edges between their leaders is completely controllable. Furthermore, such an assembly can be achieved via local rules defined as graph grammars.

For the leader selection problem, we presented a suboptimal divide and conquer method guaranteeing the complete controllability. The solution (network with assigned leaders) is constrained to be equivalent to an addition of edges between the leaders of possibly disjoint smaller components that are completely controllable under their respective leaders. As such, it achieves the controllability of the overall system through the controllability of its smaller components, and it introduces a trade-off between the optimality and the simplicity of the solution. We also presented how leader selection can be achieved via this approach by only using the information that a path graph is completely controllable if one of its terminal nodes is a leader.

REFERENCES

[1] A. Fax and R. M. Murray, "Graph Laplacian and Stabilization of Vehicle Formations", IFAC World Congress, 2002.

[2] H. Tanner, A. Jadbabaie, and G. Pappas, "Flocking in Fixed and Switching Networks", *IEEE Trans. Autom. Control*, 52(5): 863–868, 2007.

[3] A. Jadbabaie, J. Lin and A. S. Morse, "Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules", *IEEE Trans. Autom. Control*, 48(6): 988–1001, 2003.

[4] Z. Lin, M. Broucke, and B. Francis, "Local Control Strategies for Groups of Mobile Autonomous Agents", *IEEE Trans. Autom. Control*, 49(4): 622–629, 2004.

[5] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage Control for Mobile Sensing Networks", *IEEE Trans. on Robot. and Automat.*, 20(2): 243–255, 2004.

[6] A. Speranzon, C. Fischione, and K. H. Johansson, "Distributed and Collaborative Estimation over Wireless Sensor Networks", *IEEE Conf. Decision and Control*, pp. 1025–1030, 2006.

[7] F. Bullo, J. Cortes, and S. Martinez, *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*, Princeton University Press, 2009.

[8] H. G. Tanner, "On the Controllability of Nearest Neighbor Interconnections", *IEEE Conf. Decision and Control*, pp. 2467–2472, 2004.

[9] A. Rahmani and M. Mesbahi, "On the Controlled Agreement Problem", *American Control Conf.*, pp. 1376–1381, 2006.

[10] S. Martini, M. Egerstedt, and A. Bicchi, "Controllability Decompositions of Networked Systems Through Quotient Graphs", *IEEE Conf. Decision and Control*, pp. 5244–5249, 2008.

[11] A. Rahmani, M. Ji and M. Egerstedt, "Controllability of Multi-Agent Systems from a Graph Theoretic Perspective", *SIAM J. Control Optim.*, 48(1): 162–186, 2009.

[12] S. Zhang, M. K. Camlibel, and M. Cao, "Controllability of Diffusively-Coupled Multi-Agent Systems with General and Distance Regular Coupling Topologies", *IEEE Conf. on Decision and Control*, pp. 759–764, 2011.

[13] A. Y. Yazıcıoğlu, W. Abbas, and M. Egerstedt, "A Tight Lower Bound on the Controllability of Networks with Multiple Leaders", *IEEE Conf. on Decision and Control*, 2012.

[14] Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabási, "Controllability of Complex Networks", *Nature*, 473: 167-173, 2011.

[15] A. Franchi, H. H. Bulthoff, and P. R. Giordano, "Distributed Online Leader Selection in the Bilateral Teleoperation of Multiple UAVs", *IEEE Conf. on Decision and Control*, pp. 3559–3565, 2011.

[16] S. Patterson and B. Bamieh, "Leader Selection for Optimal Network Coherence, *IEEE Conf. on Decision and Control*, pp. 2692–2697, 2010.

[17] F. Lin, M. Fardad, and M. R. Jovanovic, "Algorithms for Leader Selection in Large Dynamical Networks: Noise-corrupted Leaders, *IEEE Conf. on Decision and Control*, pp. 2932–2937, 2011.

[18] S. Jafari, A. Ajorlou, and A. G. Aghdam, "Leader Selection in Multi-Agent Systems Subject to Partial Failure", *American Control Conf.*, pp. 5330–5335, 2011.

[19] R. Nagpal, "Programmable Self-Assembly Using Biologically Inspired Multiagent Control", *Intl. Conf. on Autonomous Agents and Multi-Agent Systems*, pp. 418-425, 2002.

[20] K. Kotay and D. Rus, "Generic Distributed Assembly and Repair Algorithms for Self-Reconfiguring Robots", *IEEE/RJS Intl. Conf. on Intelligent Robots and Systems*, pp. 2362-2369, 2004.

[21] E. Klavins, R. Ghrist, and David Lipsky, "Graph Grammars for Self-Assembling Robotic Systems", *IEEE Intl. Conf. on Robotics and Automation*, pp. 5293-5300, 2004.

[22] K. Sohrabi, W. Merrill, J. Elson , L. Girod, F. Newberg, and W. Kaiser, "Scalable Self-Assembly for Ad Hoc Wireless Sensor Networks", *IEEE Trans. on Mobile Computing*, 3(4): 317-331, 2004.

[23] W. Abbas and M. Egerstedt, "Hierarchical Assembly of Leader-Asymmetric, Single-Leader Networks", *American Control Conf.*, pp. 1082-1087, 2011.