Homework Set 3: ECE6550
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Due Oct. 26, 2005 (+ two weeks for DL students)

1
Show that the controllable canonical realization does in fact result in a completely controllable system.

2
Consider a particle whose acceleration is controlled directly through
\[ \ddot{y} = u. \]
Let \( x_1 = y \) and \( x_2 = \dot{y} \). Find the transition matrix \( \Phi(t, t_0) \) and reachability Gramian \( W(t_0, t) \) associated with this system.

3
Recall that the optimal \( u \), i.e. that minimizes
\[ \int_{t_0}^{t_1} u^T(t)u(t)dt \]
while driving the system from \( x(t_0) = x_0 \) to \( x(t_1) = x_1 \) is given by
\[ u(t) = B^T \Phi^T(t_1, t)W^{-1}(t_0, t_1)(x_1 - \Phi(t_1, t_0)x_0), \]
as long as the system is completely controllable.

Use the system you obtained in Question 2 and simulate your solution (e.g. using a variation of your previous code - don’t worry about exact discretizations. Just make sure that \( dt \) is small, e.g. \( dt=0.01 \)) when driving the system from \( x_1(0) = 0, \ x_2(0) = 0 \) to \( x_1(1) = 1, \ x_2(1) = 0 \) optimally.

4
This question concerns the duality between controllability and observability. Show that the system
\[ \dot{x} = Ax \\ y = Cx \]
is completely observable iff the system
\[ \dot{x} = A^T x + C^T u \]
is completely controllable.
Let

\[
\begin{align*}
\dot{x}_1 &= x_N - 2x_1 + x_2 \\
\dot{x}_2 &= x_1 - 2x_2 + x_3 \\
\dot{x}_3 &= x_2 - 2x_3 + x_4 \\
&\vdots \\
\dot{x}_{N-1} &= x_{N-2} - 2x_{N-1} + x_N \\
\dot{x}_N &= x_{N-1} - 2x_N + x_1.
\end{align*}
\]

This is (a version of) the so-called consensus algorithm for multi-agent control systems.

a

Let \( N = 10 \). Use the Matlab commands `null` and `eig` to find \( \mathcal{N}(A) \) as well as the stability properties of this system.

b

Simulate the behavior of this system from some different initial conditions and comment on how your results tie in with your answers to Question 5a.