Homework Set 2: ECE6550
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Due Sept. 21, 2005 (+ two weeks for DL students)

1

Given the linear system
\[ \dot{x}(t) = A(t)x(t). \]
Show that
\[ \Phi(t, t_0) = e^{\int_{t_0}^{t} A(s)ds} \]
if \( A(t) \) is a diagonal matrix.

*Hint: You only need to show that certain matrices commute.*

2
Let
\[ A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ -1 & -1 & 1 \end{bmatrix}. \]
Use the Cayley-Hamilton theorem to express \( A^p \) as a linear function of \( I, A, A^2 \).

3
Let
\[ A = \begin{bmatrix} 3.25 & -1.15 & -1.45 & 0.8 & 0 \\ 0.75 & -0.45 & -2.35 & 0.4 & 0 \\ -1 & 1.2 & 2.6 & -0.4 & 0 \\ -4 & 2.2 & 1.6 & 0.6 & 0 \\ -3.25 & -0.05 & -2.15 & -0.4 & -1 \end{bmatrix}. \]
Use the Matlab command `eig(A)` to compute
\[ e^{At}. \]

*Hint: The command “help eig” will tell you how to use this command.*

4
Let
\[ \dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u. \]
We want to implement this as a discrete time, sampled-data system. In other words, find the matrices \( \hat{A} \) and \( \hat{B} \) such that the discrete-time system is given *exactly* by
\[ x(k+1) = \hat{A}x(k) + \hat{B}u(k). \]
You may assume that the sample time is \( \Delta \).
5

Use the $A, B, \hat{A}, \hat{B}$ matrices from question 4. Let $y(k) = (1,0)x(k)$, $\Delta = 0.4$, and let $u(k) = e^{-\Delta}$ and $x(0) = (0.1, 0.1)^T$. Use your matlab program from HW1 to compare the exact discretization with the Euler method, i.e. first run it with the previous code (where you have to change $dt$ as well as $x0$), where

\[
\text{while (t<=tf);} \\
\text{u=exp(-t); % Control signal} \\
x=x+dt.*(A*x+B*u); % Euler approximation \\
y=C*x; \\
t=t+dt; \\
X=[X,x]; Y=[Y,y]; T=[T;t]; \\
\text{end;}
\]

Then use the exact discretization

\[
\text{while (t<=tf);} \\
\text{u=exp(-t); % Control signal} \\
x=hatA*x+hatB*u; % Exact discretization \\
y=C*x; \\
t=t+dt; \\
X=[X,x]; Y=[Y,y]; T=[T;t]; \\
\text{end;}
\]

Comment on the difference between your results.